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Research Article

EFFECT OF INITIAL STRESS AND GRAVITY ON RAYLEIGH WAVES PROPAGATING IN NON-HOMOGENEOUS ORTHOTROPIC ELASTIC MEDIA

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ABSTRACT
The influence of the gravity on the propagation of Rayleigh waves in a prestressed inhomogeneous, orthotropic elastic solid medium has been discussed. The method of variable of separation is used to find the frequency equations of the surface waves. The obtained dispersion equations are in agreement with the classical results when gravity, non-homogeneity and initial stress are neglected.

Key Words: *Inhomogeneity, Orthotropic Elastic Solid, Gravity Field and Initial Stress*

INTRODUCTION
The theory of elasticity is an approximation to the stress-strain behavior of real materials. An ideal elastic material regains its original configuration on the removal of deforming force. Therefore an ideal ‘‘elastic wave’’ is that wave which propagates through a material in such a way that the particles oscillates about their mean positions without causing any change. The earth has a layered structure, and this exerts a significant influence on the propagation of elastic waves. The simplest cases of influence exerted on the propagation of seismic waves by a single plane boundary which separates two half-spaces with different properties, and by two parallel plane boundaries forming a layer. Earth is being treated as an elastic body in which three types of waves can occur.

1. Dilatational and equivoluminal waves in the interior of the earth.
2. In the neighborhood of its surface known as Rayleigh waves (1885).
3. Third type of waves occurs near the surface of contact of two layers of the earth known as love waves (1944). The Rayleigh waves are observed far from the disturbance source near the surface. Since the energy carried by these waves is concentrated over the surface, its dissipation is slower than the dilatational and equivoluminal waves where the energy is dissipated over the volume of the disturbed region. Therefore, during earth quakes for an observer remote from the source of disturbance, the Rayleigh waves represent the greatest danger. In the case of Love waves, the energy is concentrated near the interface; hence they are dissipated more slowly. Rayleigh waves have been well recognized in the study of earthquake waves, seismology, geophysics and geodynamics. A large amount of literature is to be found in the standard books of (Bullen, 1965; Ewing *et al.*, 1957; Stoneley, 1924 and Jeffreys, 1959).

Haskell (1953) studied the dispersion of surface waves in multilayered media. Goda (1992) discussed the effect of inhomogeneity and anisotropy on Stoneley waves. Biot (1964) studied the influence of gravity on Rayleigh waves, assuming the force of gravity to create a type of initial stress of hydrostatic nature and the medium to be incompressible. Taking into account, the effect of initial stresses and using Biot’s theory of incremental deformations, Jones (1965) discussed many problems of elastic waves and vibrations under the influence of gravity field. Sengupta and Acharya (1979) also studied the influence of gravity on the propagation of waves in a thermoelastic layer. Brunelle (1973) considered the surface wave propagation under initial tension of compression. Abd-Alla *et al.*, (1996) analyzed the Rayleigh waves in an orthotropic thermoelastic medium under gravity field and initial stress. Recently, various studies on propagation of surface waves such as Love waves in a non-homogeneous elastic media, Rayleigh waves in a non-homogeneous granular media, Stoneley, Rayleigh and Love waves in

viscoelastic media, Love Waves in a non-homogeneous orthotropic layer under compression 'P' overlying semi-infinite non-homogeneous medium were studied by Kakar *et al.*, (2012-2013).
In the present study, the influence of gravity and initial stress on the propagation of Rayleigh type waves in a non-homogeneous, orthotropic elastic solid medium has been discussed. The Dispersion equation so obtained is in well agreement with the corresponding classical results.

FORMULATION OF THE PROBLEM

Let us consider an orthotropic, non-homogeneous elastic solid under an initial compression P along x-direction further it is also under the influence of gravity. Here we consider Oxyz Cartesian coordinates system where O be any point on the plane boundary and Oz be normal to the medium and Rayleigh wave propagation is taken in the +ve direction of x-axis. It is also assumed that at a great distance from center of disturbance, the wave propagation is two dimensional and is polarized in (x, z) plane. So displacement components along x and z directions. i.e. u and w are non-zero while v = 0.
Also it is assumed that wave is surface wave as the disturbance is extensively confined to the boundary. Let g be the acceleration due to gravity and ρ be the density of the material medium.
Here states of initial stresses are given by

$$\left.\begin{aligned}\sigma_{ij} &= \sigma \; ; \; i = j \\ &= 0 \; ; \; i \neq j\end{aligned}\right\} \text{, where i, j = 1, 2, 3} \tag{1}$$

Further σ is a function of z
$\therefore$ equation of equilibrium of initial compression are

$$\frac{\partial \sigma}{\partial x} = 0 = \frac{\partial \sigma}{\partial y}, \tag{2}$$

$$\frac{\partial \sigma}{\partial z} - \rho g = 0.$$

SOLUTION OF PROBLEM

Considering eq (1) and eq (2) and conditions for compressibility, the dynamical equations in three dimensions of an elastic medium under initial compression and gravity are given by

$$\sigma_{11,x} + \sigma_{12,y} + \sigma_{13,z} + P(w_{z,y} - w_{y,z}) - \rho g\, u_{3,x} = \rho u_{,tt}, \tag{3}$$

$$\sigma_{12,x} + \sigma_{22,y} + \sigma_{23,z} - P w_{z,x} = \rho v_{,tt}, \tag{4}$$

$$\sigma_{13,x} + \sigma_{23,y} + \sigma_{33,z} - P w_{y,x} + \rho g\, u_{1,x} = \rho w_{,tt}, \tag{5}$$

Where u, v, w are displacement components in x, y and z direction and w_x, w_y, w_z are rotational components and are given by

$$w_x = \frac{1}{2}(w_{,y} - v_{,z}) \; ; \; w_y = \frac{1}{2}(u_{,z} - w_{,x})$$

$$w_z = \frac{1}{2}(v_{,x} - u_{,y}). \tag{6}$$

Further dynamical eqs in (x, z) directions are given by

$$\sigma_{11,x} + \sigma_{13,z} - P w_{y,z} - \rho g\, u_{3,x} = \rho u_{,tt},$$

$$\sigma_{13,x} + \sigma_{33,z} - P w_{y,x} + \rho g\, u_{1,x} = \rho w_{,tt}, \tag{7}$$

Where stress components are given by

$$\sigma_{11} = (C_{11} + P)\, u_{1,x} + (C_{13} + P)\, u_{3,z} ,$$
$$\sigma_{33} = C_{31}\, u_{1,x} + C_{33}\, u_{3,z} ,$$
$$\sigma_{13} = C_{44}\, (u_{1,z} + u_{3,x}), \tag{8}$$

Where C_{ij} are elastic constants.

Let us take the assumption that $C_{44} = \frac{1}{2}(C_{11} - C_{13})$.

Substituting equation (6) and equation (8) in equation (7); we have

$$(C_{11} + P)(2u_{1,xx} + u_{1,zz} + u_{3,xz}) + C_{13}(u_{3,xz} - u_{1,zz}) + (u_{1,z} + u_{3,x})$$
$$(C_{11} - C_{13})_{,z} + 2u_{1,x}(C_{11} + P)_{,x} + 2u_{3,z}(C_{13} + P)_{,x} - 2\rho g\, u_{3,x} = 2\rho u_{1,tt}, \tag{9}$$
$$C_{11}(u_{1,xz} + u_{3,xx}) + (C_{13} + P)(u_{1,xz} - u_{3,xx}) + 2C_{33}\, u_{3,zz}$$
$$+ 2\rho g\, u_{1,x} + (u_{1,z} + u_{3,x})(C_{11} - C_{13})_{,x} + u_{1,x}\, C_{13,z} + u_{3,z}\, C_{33,z} = 2\rho u_{3,tt} . \tag{10}$$

Now we assume the non-homogeneity for the elastic half space, density and compression are given by

$$C_{ij} = \alpha_{ij}\, e^{mz},\ \rho = \rho_0\, e^{mz},\ P = P_0\, e^{mz}, \tag{11}$$

Where $\lambda_{i,j}$, ρ_0, P_0 and m are constants.

Substituting eq (11) in eqs (9) and (10) we get

$$e^{mz}(\alpha_{11} + P_0)(2u_{1,xx} + u_{1,zz} + u_{3,xz}) + \alpha_{13}(u_{3,xz} - u_{1,zz})\, e^{mz} \tag{12}$$
$$+ (u_{1,z} + u_{3,x})(\alpha_{11} - \alpha_{13})\, m\, e^{mz} - 2\rho_0\, g\, u_{3,x}\, e^{mz} = 2\rho_0 u_{1,tt} ,$$
$$\alpha_{11}(u_{1,xz} + u_{3,xx}) + (\alpha_{13} + 2P_0)(u_{1,xz}) - (\alpha_{13} + 2P_0)\, u_{3,xx} + 2\alpha_{33}\, u_{3,zz} + 2\rho_0 g\, u_{1,x}$$
$$+ 2\alpha_{13}\, m\, u_{1,x} + 2\alpha_{33}\, m u_{3,z} = 2\rho u_{3,tt} . \tag{13}$$

To investigate the surface wave propagation along Ox, we introduce displacement potentials in terms of displacements components are given by

$$u = \phi_{,x} - \psi_{,z};\ w = \phi_{,z} + \psi_{,x} \tag{14}$$

Introducing eq (14) in eqs (12) and (13) we get

$$2(\alpha_{11} + P_0)\nabla^2\phi - 2\rho_0\, g\, \psi_{,x} + m(\alpha_{11} - \alpha_{13})(2\phi_{,z} + \psi_{,x}) = 2\rho_0\, \phi_{,tt} , \tag{15}$$
$$(\alpha_{11} + P_0 - \alpha_{13})\nabla^2\psi + 2\rho_0\, g\phi_{,x} - m(\alpha_{11} - \alpha_{13})\, \psi_{,z} = 2\rho_0\, \psi_{,tt} , \tag{16}$$

and

$$\alpha_{11}\, \phi_{,xx} + \alpha_{33}\, \phi_{,zz} - \rho_0 g\, \psi_{,x} - 2\alpha_{13}\, m\, \psi_{,x} + 2\alpha_{33}\, m\phi_{,z} = 2\rho_0\phi_{,tt} , \tag{17}$$
$$(\alpha_{11} - \alpha_{13} - 2P_0)\, \psi_{,xx} + (2\alpha_{33} - \alpha_{13} - \alpha_{11} - 2P_0)\, \psi_{,zz} + (2\rho_0 g + 2\alpha_{13}\, m)\, \phi_{,x}$$
$$+ 2\alpha_{33}\, m\, \psi_{,z} = 2\rho_0\, \psi_{,tt} , \tag{18}$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$.

Since the velocity of waves are different in x and z direction. Now eq (15) and (16) represent the compressive wave along x and z-direction while eq (17) and (18) represents the shear waves along these directions. Since we consider the propagation of Rayleigh waves in x-direction

$\therefore$ we consider only equation (15) and equation (18).

To solve equation (15) and equation (18) we introduce

$$\phi(x, y, z) = f(z)\, e^{i\alpha(x - ct)} ,$$

and ψ (x, y, z) = h(z) $e^{i\alpha(x-ct)}$. (19)

putting eq (19) in eq (15) and eq (18) we get

$f_{,zz} + Af_{,z} + Bf + Ch = 0,$ (20)

$h_{,zz} + A'h_{,z} + B'h + C'f = 0,$ (21)

Where

$$A = \frac{m\left(\alpha_{11}-\alpha_{13}\right)}{\alpha_{11}+P_0},\ B = \frac{\alpha^2\left(\rho_0 c^2-\alpha_{11}-P_0\right)}{\alpha_{11}+P_0},\ C = \frac{\left[-2\rho_0 g+m\left(\alpha_{11}-\alpha_{13}\right)\right] i\alpha}{2\left(\alpha_{11}+P_0\right)},$$

$$A' = \frac{2m\alpha_{33}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0},\ B' = \frac{\alpha^2\left(2c^2\rho_0-\alpha_{11}+\alpha_{13}+2P_0\right)}{2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0},$$

$$C' = \frac{\left(2\rho_0 g+2m\alpha_{13}\right) i\alpha}{2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0}. \quad (22)$$

Now eq (20) and eq (21) have exponential solution in order that f (z) and h (z) describe surface waves and also they varnish as z $\rightarrow \infty$ hence eq (15) takes the form,

$$\phi\,(x, z, t) = \left[C_1 e^{-\lambda_1 z} + C_2 e^{-\lambda_2 z}\right] e^{i\alpha(x-ct)},$$

$$\text{and } \psi\,(x, z, t) = \left[C_3 e^{-\lambda_1 z} + C_4 e^{-\lambda_2 z}\right] e^{i\alpha(x-ct)}, \quad (23)$$

Where C_1, C_2, C_3, C_4 are arbitrary constants and λ_1, λ_2 are the roots of the equation

$$\lambda^4 + \left[\frac{m\left(\alpha_{11}-\alpha_{13}\right)}{\alpha_{11}+P_0} + \frac{2m\alpha_{33}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0}\right]\lambda^3$$
$$+\ \alpha^2\left[\frac{2\rho_0c^2-\alpha_{11}+\alpha_{13}+2P_0}{2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0} + \frac{\rho_0c^2-\alpha_{11}-P_0}{\alpha_{11}+P_0}\right]\lambda^2$$
$$+\ m\alpha^2\left[\frac{\left(\alpha_{11}-\alpha_{33}\right)\left(2\rho_0c^2-\alpha_{11}+\alpha_{13}+2P_0\right)+\left(\rho_0c^2-\alpha_{11}-P_0\right)2\alpha_{33}}{\left(\alpha_{11}+P_0\right)\left(2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0\right)}\right]\lambda$$
$$+\left[\frac{\alpha^4\left(\rho_0c^2-\alpha_{11}-P_0\right)\left(2\rho_0c^2-\alpha_{11}+\alpha_{13}+2P_0\right)}{\left(\alpha_{11}+P_0\right)\left(2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0\right)}\right.$$
$$\left.+\alpha^2\,\frac{\left\{m\left(\alpha_{11}-\alpha_{13}\right)-2\rho_0 g\right\}\left(2\rho_0 g+2m\alpha_{13}\right)}{2\left(\alpha_{11}+P_0\right)\left(2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0\right)}\right] = 0. \quad (24)$$

Here we consider only real roots of eq (24). Now the constants C_1, C_2 and C_3, C_4 are related by the eqs (20) and eq (21).

By equating the co-efficients of $e^{-\lambda_1 z}$ and $e^{-\lambda_2 z}$ to zero, eq (20) gives,

$C_3 = \gamma_1 C_1$, $C_4 = \gamma_2 C_2$, (25)

$$\text{Where } \gamma_j = \frac{2i[(\alpha_{11}+P_0)\lambda_j^{\,2} - m\,(\alpha_{11}-\alpha_{13})\lambda_j + (\rho_0 c^2 - \alpha_{11} - P_0)]}{\alpha\left[m\left(\alpha_{11}-\alpha_{13}\right)-2\rho_0 g\right]}\ ;\ j = 1, 2. \quad (26)$$

BOUNDARY CONDITIONS

The plane z = 0 is free from stresses i.e. $\sigma_{13} = \sigma_{33} = 0$ at z = 0, (27)

Where $$\sigma_{13} = \frac{1}{2}(\alpha_{11} - \alpha_{13})\,[2\,\phi_{,xz} - \psi_{,zz} + \psi_{,xx}]\,e^{mz}, \qquad (28)$$

$$\sigma_{33} = \alpha_{31}\,[\phi_{,xx} - \psi_{,xz}]\,e^{mz} + \alpha_{33}[\phi_{,zz} + \psi_{,zx}]\,e^{mz}. \qquad (29)$$

Introducing eq (28) and (29) in eq (27) we have

$$C_1\,(2\lambda_1\,i\alpha + \gamma_1\lambda_1^2 + \alpha^2\gamma_1) + C_2\,[2\,\lambda_2\,i\alpha + \gamma_2\lambda_2^2 + \alpha^2\,\gamma_2] = 0, \qquad (30)$$

$$C_1\,[-\alpha^2\,\lambda_{13} + \lambda_1^2\,\alpha_{33} - \alpha_1\gamma_1 i\alpha\,\,(\alpha_{33} - \alpha_{13})] + C_2\,[-\alpha^2\lambda_{13} + \lambda_2^2\,\alpha_{33} - \lambda_1\,\gamma_1 i\alpha\,(\alpha_{33} - \alpha_{13})] = 0. \qquad (31)$$

Eliminating C_1 and C_2 from eq (30) and eq (31); we have

$$[2\lambda_1\,i\alpha + \gamma_1\lambda_1^2 + \alpha^2\gamma_1]\,[-\alpha^2\lambda_{13} + \lambda_2^2\,\alpha_{33} - \lambda_1\,\gamma_1 i\alpha\,(\alpha_{33} - \alpha_{13})]$$
$$-\,[2\,\lambda_2\,i\alpha + \gamma_2\lambda_2^2 + \alpha^2\,\gamma_2]\,[-\alpha^2\,\lambda_{13} + \lambda_1^2\,\alpha_{33} - \alpha_1\gamma_1 i\alpha\,\,(\alpha_{33} - \alpha_{13})] = 0, \qquad (32)$$

Where γ_j (j = 1, 2) are given by equation (26) and λ_j (j = 1, 2) are roots of eq. (24).

Now eq (32) gives the wave velocity equation for Rayleigh waves in a non-homogeneous elastic half space of orthotropic material under the initial compression and influence of gravity.

From eq (32), it follows that Rayleigh waves depends on gravity, initial compression, non-homogeneous character of the medium and nature of the material.

From equation (32),we conclude that if α is large i.e. length of wave i.e. $\frac{2\pi}{\alpha}$ is small then gravity and compression have small effects on Rayleigh waves in non-homogeneous orthotropic half space and if α is small i.e. $\frac{2\pi}{\alpha}$ is large then gravity and compression plays a vital role for finding out the wave velocity c.

When the medium is isotropic, eq (32) becomes

$$[2\lambda_1\,i\alpha + \gamma_1\lambda_1^2 + \alpha^2\gamma_1]\,[K_1^2\,(\lambda_2^2 - \alpha^2) + 2\,K_2^2\,(1 - i\alpha\,\gamma_2\lambda_2)]$$
$$-\,[2\,\lambda_2\,i\alpha + \gamma_2\lambda_2^2 + \alpha^2\,\gamma_2]\,[K_1^2\,(\lambda_1^2 - \alpha^2) + 2\,K_2^2\,(1 - i\alpha\,\gamma_1\lambda_1)] = 0, \qquad (33)$$

Where $K_1^2 = \dfrac{\lambda + 2\mu + P}{\rho}$, $K_2^2 = \dfrac{\mu - P/2}{\rho}$, ($\lambda$, μ are Lame's constants). (34)

Eq (34) determines the Rayleigh waves in a non-homogeneous isotropic elastic solid under the influence of gravity and compression.

When initial compression is absent i.e. $P_0 = 0$, then equation (33) reduces to,

$$[2\lambda_1\,i\alpha + \gamma_1\lambda_1^2 + \alpha^2\gamma_1]\,[K_1^2\,(\lambda_2^2 - \alpha^2) + 2\,K_2^2\,(1 - i\alpha\,\gamma_2\lambda_2)]$$
$$-\,[2\,\lambda_2\,i\alpha + \gamma_2\lambda_2^2 + \alpha^2\,\gamma_2]\,[K_1^2\,(\lambda_1^2 - \alpha^2) + 2\,K_2^2\,(1 - i\alpha\,\gamma_1\lambda_1)] = 0, \qquad (35)$$

Where $K_1^2 = \dfrac{\lambda + 2\mu}{\rho}$, $K_2^2 = \dfrac{\mu}{\rho}$.

Eq (35) determines the Rayleigh surface waves in a non-homogeneous isotropic elastic solid under the influence of gravity which is similar to corresponding classical result given by Das *et al.*,

When non-homogeneity of the material is absent, we get same dispersion eq. as (32) with

$$\gamma_j = \frac{-i[(\alpha_{11}+P_0)\lambda_j^2+(\rho_0 c^2-\alpha_{11}-P_0)]}{\alpha\rho_0 g} \quad ; j = 1, 2,$$

Where λ_1, λ_2 are the roots of the equation

$$\lambda^4+\alpha^2\left[\frac{2\rho_0c^2-\alpha_{11}+\alpha_{13}+2P_0}{2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0}+\frac{\rho_0c^2-\alpha_{11}-P_0}{\alpha_{11}+P_0}\right]\lambda^2$$
$$+\left[\frac{\alpha^4\left(\rho_0c^2-\alpha_{11}-P_0\right)\left(2\rho_0c^2-\alpha_{11}+\alpha_{13}+2P_0\right)-2\alpha^2\rho_0^2g^2}{\left(P_0+\alpha_{11}\right)\left(2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0\right)}\right]=0. \qquad (36)$$

When gravity field is absent, we get same velocity eq. for Rayleigh waves in non-homogeneous elastic solid under initial compression as eq (32) with

$$\gamma_j = \frac{2i[(\alpha_{11}+P_0)\lambda_j^2-m(\alpha_{11}-\alpha_{13})\lambda_j+(\rho_0 c^2-\alpha_{11}-P_0)]}{\left[\alpha m\left(\alpha_{11}-\alpha_{13}\right)\right]} \quad ; j = 1, 2,$$

Where λ_1, λ_2 are roots of the equation

$$\lambda^4+\left[\frac{m\left(\alpha_{11}-\alpha_{13}\right)}{\alpha_{11}+P_0}+\frac{2m\alpha_{33}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0}\right]\lambda^3$$
$$+\alpha^2\left[\frac{2\rho_0c^2-\alpha_{11}+\alpha_{13}+2P_0}{2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0}+\frac{\rho_0c^2-\alpha_{11}-P_0}{\alpha_{11}+P_0}\right]\lambda^2$$
$$+m\alpha^2\left[\frac{\left(\alpha_{11}-\alpha_{33}\right)\left(2\rho_0c^2-\alpha_{11}+\alpha_{13}+2P_0\right)+\left(\rho_0c^2-\alpha_{11}-P_0\right)2\alpha_{33}}{\left(\alpha_{11}+P_0\right)\left(2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0\right)}\right]\lambda$$
$$+\left[\frac{\alpha^4\left(\rho_0c^2-\alpha_{11}-P_0\right)\left(2\rho_0c^2-\alpha_{11}+\alpha_{13}+2P_0\right)}{\left(\alpha_{11}+P_0\right)\left(2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0\right)}\right.$$
$$\left.+\alpha^2\frac{\left\{m\left(\alpha_{11}-\alpha_{13}\right)\right\}\left(m\alpha_{13}\right)}{\left(\alpha_{11}+P_0\right)\left(2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0\right)}\right]=0. \qquad (37)$$

When medium is initially unstressed i.e. $P_0 = 0$,

We get, velocity equation for Rayleigh waves is similar to equation (32) with

$$\gamma_j = \frac{2i[(\alpha_{11}\lambda_j^2-m(\alpha_{11}-\alpha_{13})\lambda_j+(\rho_0 c^2-\alpha_{11})]}{\alpha\left[m\left(\alpha_{11}-\alpha_{13}\right)-2\rho_0 g\right]} \quad ; j = 1, 2,$$

Where λ_1, λ_2 are roots of the equation

$$\lambda^4+\left[\frac{m\left(\alpha_{11}-\alpha_{13}\right)}{\alpha_{11}}+\frac{2m\alpha_{33}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}}\right]\lambda^3$$
$$+\alpha^2\left[\frac{2\rho_0c^2-\alpha_{11}+\alpha_{13}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}}+\frac{\rho_0c^2-\alpha_{11}}{\alpha_{11}}\right]\lambda^2$$

$$+\ m\alpha^2\left[\frac{(\alpha_{11}-\alpha_{33})\left(2\rho_0c^2-\alpha_{11}+\alpha_{13}\right)+\left(\rho_0c^2-\alpha_{11}\right)2\alpha_{33}}{(\alpha_{11})(2\alpha_{33}-\alpha_{11}-\alpha_{13})}\right]\lambda$$

$$+\left[\frac{\alpha^4\left(\rho_0c^2-\alpha_{11}\right)\left(2\rho_0c^2-\alpha_{11}+\alpha_{13}\right)}{(\alpha_{11})(2\alpha_{33}-\alpha_{11}-\alpha_{13})}+\alpha^2\ \frac{\left\{m(\alpha_{11}-\alpha_{13})-2\rho_0 g\right\}\ (2\rho_0 g+2m\alpha_{13})}{2(\alpha_{11})\,(2\alpha_{33}-\alpha_{11}-\alpha_{13})}\right]=0. \quad (38)$$

When the non-homogeneity of the material and gravity field are absent further medium is initially unstressed and isotropic, eq (32) reduces to,

$$4\sqrt{\left(1-\frac{c^2}{K_1{}^2}\right)\left(1-\frac{c^2}{K_2{}^2}\right)}=\left(2-\frac{c^2}{K_2{}^2}\right)^2, \qquad (39)$$

Where $K_1{}^2=\dfrac{\lambda+2\mu}{\rho}$, $K_2{}^2=\dfrac{\mu}{\rho}$.

Equation (39) is similar to the equation given by Rayleigh.

DISCUSSION AND CONCLUSION

Equation (32) represents the wave velocity equation for the Rayleigh waves in a non-homogeneous, orthotropic elastic solid medium under the influence of gravity and initial compression. It depends upon the wave number and confirming that waves are dispersive. Moreover, the dispersion equation contains terms involving gravity, initial compression and non-homogeneity, so the phase velocity ‘c’ not only depends upon the gravity field and initial compression but also on the non-homogeneity of the material medium.

The explicit solutions of this wave velocity equation cannot be determined by analytical methods. However, these equations can be solved with the help of numerical method, by a suitable choice of physical parameters involved in medium.

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