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Research Article

TWO-DIMENSIONAL SOLUTE TRANSPORT IN FINITE HOMOGENEOUS POROUS FORMATIONS

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ABSTRACT
An analytical approach to two-dimensional non-reactive solute transport in finite homogeneous porous formations is compared with the numerical result obtained from two-level explicit finite difference method. The analytical solution is derived with time-dependent point-source contamination expressed as logistic sigmoid functions for three different types of velocity expressions. First, the flow velocity in the aquifer is asymptotic in nature, second, the flow velocity is an exponentially decreasing function and third the flow velocity is sigmoid expression. These velocity expressions represent the groundwater flow in complex nature of geological formation. The geological formation is initially polluted. This may be represented by a mathematical expression i.e., exponentially decreasing function of space. A particular case is derived which validates the solution, and the analytical solution is illustrated using suitable input parameters.

Key Words: *Solute Transport; Aquifer; Analytical Solution*

NOTATIONS

c_i	Initial background solute concentration $\left[ML^{-3}\right]$
γ	Decay parameter $\left[L^{-1}\right]$
x	Space variable along x-axis $\left[L\right]$
y	Space variable along y-axis $\left[L\right]$
c	Contaminant concentration in the aquifer at any time $\left[ML^{-3}\right]$
t	Time variable $\left[T\right]$
u	Groundwater velocity component along x-axis $\left[LT^{-1}\right]$
v	Groundwater velocity component along y-axis $\left[LT^{-1}\right]$
D_x	Dispersion coefficient along x-axis $\left[L^2T^{-1}\right]$
D_y	Dispersion coefficient along y-axis
D_{xy}, D_{yx}	Off diagonal dispersion components $\left[L^2T^{-1}\right]$
u_0	Initial seepage velocity along x-axis $\left[LT^{-1}\right]$
v_0	Initial seepage velocity along y-axis $\left[L^2T^{-1}\right]$

f	Temporally dependent function
a	Dispersivity $\left[L\right]$
D_{x0}	Initial dispersion coefficient along x-axis $\left[L^2T^{-1}\right]$
D_{y0}	Initial dispersion coefficient along y-axis $\left[L^2T^{-1}\right]$
c_0	Solute concentration at the source $\left[ML^{-3}\right]$
q	Contaminant decay rate coefficient $\left[T^{-1}\right]$
T^*	New time variable $\left[T\right]$
m	Flow resistance coefficient $\left[T^{-1}\right]$
C	Non-dimensional solute concentration in the aquifer
X	Non-dimensional space variable along x-axis
Y	Non-dimensional space variable along y-axis
T	Non-dimensional time variable
D_1	Non-dimensional dispersion coefficient along x-axis
D_2	Non-dimensional dispersion coefficient along y-axis
U	Non-dimensional groundwater velocity along x-axis
V	Non-dimensional groundwater velocity along y-axis
Q	Non-dimensional contaminant decay rate coefficient
h_1, h_2, α	Constant parameters
Z	Non-dimensional space variable
η	Non-dimensional parameter
β	Outer boundary of the axis symmetry in cylindrical region
K^*	Non-dimensional parameter
J_0	Bessel function of first kind of zeroth order
p	Root of Bessel's function of zeroth order
c_1, K	Arbitrary constant
J_1	Bessel function of first kind of first order
$\overline{K}(p,T)$	Hankel transform of $K(\eta,T)$
X_i	Value of X at i^{th} interval
Y_j	Value of Y at j^{th} interval
T_k	Value of T at k^{th} interval
ΔX	Length of sub-interval in X domain
ΔY	Length of sub-interval in Y domain
ΔT	Length of sub-interval in T domain
$C_{i,j,k}$	The contaminant concentration at a point (x_i, y_j) at k^{th} sub-interval of time T

INTRODUCTION

In natural resources management, solute transport modeling is helpful to predict the solute concentration in aquifers, rivers, lakes and streams. The solute transport modeling is very much relevant describing advection dispersion equation. This advection-dispersion equation is solved by analytical approach and or numerical approach. Analytical approach provides physical insights into the solute transport phenomena and present benchmark solutions against which numerical approach can be tested. Analytical solutions are usually derived from the basic physical principles and are free from numerical and other truncation errors that often occur in numerical simulations (Zheng and Benett, 1995). In solute transport modeling, the advection-dispersion equation is widely used to describe the non-reactive solute dispersion in geological formations such as aquifer. The flow against dispersion in non-adsorbing porous formation has been investigated by Marino (1978), Aral and Tang (1992), Kumar and Kumar (2002), Chen *et al.,* (1996), Singh *et al.,* (2010) among others.

There have been a multitude of investigations on one-, two- and three- dimensional solute transport modeling. Yeh (1981) presented an analytical solution for various types of sources release and aquifer configuration with the Green function approach on one-, two-, and three- dimensional solute transport.

Latinopoulos *et al.,* (1988) presented a method for obtaining analytical solutions for chemical transport in two-dimensional aquifers assuming a constant velocity field. They obtained the solution by integrating the solution of a modified dimensional differential equation considering the source as continuous and instantaneous injection. The analytical solution of a two-dimensional solute transport equation was studied by Aral and Liao (1996). The solution was obtained using superposition principle for uniform, linear, asymptotic and exponential varying dispersion coefficient. Shan and Javandel (1997) obtained solutions of a two-dimensional solute transport equation in a vertical section of a homogeneous aquifer with steady uniform groundwater flow. The solutions were derived for both constant-flux and constant-concentration sources in a finite as well as semi-infinite domain. Park and Zhan (2001) provided analytical solutions of contaminant transport from one-, two-, and three dimensional finite sources in a finite-thickness aquifer using the Green function method. Using the Hankel transform technique (HTT), Kumar and Kumar (2002) obtained an analytical solution for pollutant transport in groundwater in a homogeneous finite aquifer. Here, the uniform input source concentration was taken at the far end from the origin, i.e. against the flow.

Chen (2007) derived an analytical solution of two-dimensional advection-dispersion equation in cylindrical co-ordinates for non-axisymmetrical solute transport in a tracer test system using a power series technique coupled with the Laplace and finite Fourier cosine transform techniques. Here, the longitudinal and transverse dispersivities were assumed to be a linear function of solute distance. Chen *et al.,* (2008) also presented an analytical approach to the two-dimensional advection-dispersion equation for describing solute transport in a uniform flow field with linear distance-dependent longitudinal and transverse dispersivities. Here, the extended power series method coupled with the Laplace and finite Fourier cosine transforms was used. Considering an aquifer-aquitard system, Zhan *et al.,* (2009) presented an analytical solution of two-dimensional solute transport using the Laplace transform technique in which the first and third type boundary conditions were considered. Singh *et al.,* (2010a) explored an analytical solution of two-dimensional solute transport in a homogeneous finite aquifer using the Hankel Transform Technique in which the input source concentration was taken at the far end away from the origin. Chen *et al.,* (2011) developed an analytical solution of two-dimensional solute transport in porous media in the form of cylindrical co-ordinate system in finite domain. The first type and third type inlet boundary conditions were assumed to define the problem. The analytical solution was obtained by using the Finite Hankel Transform of second kind and general integral transform technique successively for different ranges of Peclet number. In most of the cases, the aquifer was initially assumed to be solute free which means no initial background concentration existed in the aquifer. This does not always happen in the aquifer in nature and, therefore, some initial concentration may exist in the aquifer.

In the present study, a two-dimensional solute transport equation for a homogeneous finite aquifer was considered. Initially, the aquifer was clean, meaning that some initial background concentration exist in the aquifer and it was represented by an exponentially decreasing function of space instead of uniform or zero concentration, and, zero initial background concentration in the groundwater system was assumed by Singh *et al.*, (2010). The uniform initial concentration is not always the case in the complex nature of geological formations and hence this work is extended with variable initial concentration may be with respect to time or space. The reason for considering an initial condition of this type was explored in the case of transient model (Franke and Reilly, 1987). At the other end of the aquifer, the time dependent input source concentrations is assumed as logistic sigmoid function which is different from time-dependent boundary condition taken by Singh *et al.*, (2010). In most of the investigations, groundwater velocity has been considered steady. However, when the groundwater table rises and falls, the velocity of flow in the aquifer may be transient or unsteady. In the present problem three different types of velocity expression have been taken as of Aral and Liao (1996). The dispersion coefficient as directly proportional to the seepage velocity is used. This dispersion theory was established by Ebach and White (1958), Scheidegger (1961), Rumer (1962), Bruce (1970), and explored by Kumar (1983). The Hankel Transform Technique (HTT) of first kind of zeroth order is used to find an analytical solution for solute transport and it is compared with the numerical solution obtained from two-level explicit finite difference method to validate the solution.

Mathematical Formulation

Consider a homogeneous porous formation for example, aquifer that has an initial background concentration, which is assumed as a function of space, say, $c_i \exp\left(-\gamma x\right)$, where γ $[L^{-1}]$ is the decay parameter and $c_i \left[ML^{-3}\right]$ is the solute concentration. The uniform initial concentration or zero concentration which is not remains same throughout the geological formations taken by many researchers. But the initial concentration may vary with time or space. Hence, the exponential decreasing function of space dependent term is taken into consideration in the present problem. The longitudinal and lateral directions at the origin are taken as x and y axes, respectively. Let $c\left[ML^{-3}\right]$ denote the contaminant concentration in the aquifer at any time $t\,[T]$; $u\,[LT^{-1}]$ and $v\,[LT^{-1}]$ denote the x and y groundwater velocity components, respectively; and D_x $[L^2T^{-1}]$ and D_y $[L^2T^{-1}]$ denote the dispersion coefficients along the x and y axes, respectively. The source of contamination is introduced as the point source and it reaches the water table at the point $x = L,\ y = H$ which is diametrically opposite from the origin, whereas the direction of flow is from origin toward the other end of the aquifer. The problem is one of determining c as a function of space (x, y) and time t . The physical system is graphically shown in Figure 1.

The partial differential equation (PDE) describing the two-dimensional advection and dispersion in a homogeneous, isotropic aquifer can be written as

$$\frac{\partial c}{\partial t} = D_x(t)\frac{\partial^2 c}{\partial x^2} + D_y(t)\frac{\partial^2 c}{\partial y^2} - u(t)\frac{\partial c}{\partial x} - v(t)\frac{\partial c}{\partial y} \tag{1}$$

Here, off diagonal dispersion components D_{xy} and D_{yx} are not taken into account as one of the axes coincides with the direction of average uniform velocity and accordingly D_{xy} and D_{yx} both are supposed to be zero (Bear and Verruijt, 1998).

Let u and v be expressed as

$$u = u_0 f(t) \quad and \quad v = v_0 f(t) \tag{2}$$

where u_0 $[LT^{-1}]$ *and* v_0 $[LT^{-1}]$ = initial values of u *and* v, respectively, and $f(t)$ is assumed to be different type of algebraic function.

For many types of porous media, the dispersion coefficient often varies proportionally to the seepage velocity. Therefore, let

$$D_x = au_0 f(t) \quad and \quad D_y = av_0 f(t) \tag{3}$$

where a =dispersivity$[L]$ that depends on the distribution of aquifer heterogeneities and scale of the field problem (Bedient *et al.*,1999). Using Eq. (2), Eq. (3) can be written as

$$D_x = D_{x0} f(t) \quad and \quad D_y = D_{y0} f(t) \tag{4}$$

where $D_{x0} = au_0$ *and* $D_{y0} = av_0$ are the initial values of D_x *and* D_y, respectively.

As stated earlier, the initial contaminant concentration is a function of space say $c_i \exp(-\gamma x)$ at $t = 0$ and at $x = L,\ y = H$. The input point source concentration has been taken as time-dependent in the form of logistic sigmoid function different from time dependent boundary condition taken by Singh *et al.*, (2010). The logistic sigmoid function is horizontally asymptotic in nature, i.e., it increases continuously for $t > 0$ and tends to 1 as $t \to \infty$. In the solute transport modeling context, the input point source concentration can be taken as of this form assuming that input concentration would initially increase with time and after a certain time period it would stabilize at an asymptotic value. Hence, the initial and boundary conditions are expressed as

$$c(x,y,t) = c_i \exp(-\gamma x), \quad x \ge 0, y \ge 0, t = 0 \tag{5}$$

$$c(x,y,t) = \frac{c_0}{[1+\exp(-qt)]} \quad x = L, y = H, t > 0 \tag{6}$$

Here, $q\left[T^{-1}\right]$ is the contaminant decay rate coefficient and $c_0\left[ML^{-3}\right]$ is solute concentration.

Using Eqs. (2) and (4), Eq. (1) can be written as follows:

$$\frac{1}{f(t)}\frac{\partial c}{\partial t} = D_{x0}\frac{\partial^2 c}{\partial x^2} + D_{y0}\frac{\partial^2 c}{\partial y^2} - u_0\frac{\partial c}{\partial x} - v_0\frac{\partial c}{\partial y} \tag{7}$$

Using the following transformation (Crank 1975), a new time variable T^* is introduced as:

$$T^* = \int_0^t f(t)\,dt$$

(8)

Now with the use of Eq. (8), Eq. (7) can be written as follows:

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$$\frac{\partial c}{\partial T^*} = D_{x0}\frac{\partial^2 c}{\partial x^2} + D_{y0}\frac{\partial^2 c}{\partial y^2} - u_0\frac{\partial c}{\partial x} - v_0\frac{\partial c}{\partial y} \tag{9}$$

Now a set of non-dimensional variable is defined as follows:

$$C = \frac{c}{c_0},\ X = \frac{x}{L},\ Y = \frac{y}{H},\ T = mT^*$$

$$D_1 = \frac{D_{x0}}{mL^2};\quad D_2 = \frac{D_{y0}}{mH^2};\ U = \frac{u_0}{mL};\quad V = \frac{v_0}{mH};\ Q = \frac{q}{m} \tag{10}$$

In terms of non-dimensional variables, the governing Eq. (9) can be written as:

$$\frac{\partial c}{\partial T} = D_1\frac{\partial^2 C}{\partial X^2} + D_2\frac{\partial^2 C}{\partial Y^2} - U\frac{\partial C}{\partial X} - V\frac{\partial C}{\partial Y} \tag{11}$$

The initial and boundary conditions can be written as

$$C(X,Y,T) = \frac{c_i}{c_0}\exp(-\gamma XL), \qquad X \ge 0,\quad Y \ge 0,\quad T = 0 \tag{12}$$

$$C(X,Y,T) = \frac{1}{2}\left[1 + \frac{QT}{2}\right], \qquad X = 1,\quad Y = 1,\quad T > 0 \tag{13}$$

The source of contamination in the horizontal plane is the time dependent input source at the far end from the origin, i.e., at $x = L,\ y = H$, i.e., against flow.

Analytical Solution

To obtained the analytical solution of Eq. (11) subject to initial and boundary conditions given in Eqs. (12) and (13) applying the Hankel transform technique of first kind of zeroth order, the solution in non-dimensional form can be written as

$$C(X,Y,T) = \exp\left[h_1(X-1) + h_2(Y-1)\right]$$
$$\times\left[\frac{1}{2}\left(1+\frac{QT}{2}\right) - \frac{2}{\beta}\sum_p \frac{J_0(p\eta)}{pJ_1(\beta p)}\left\{\begin{array}{l}\left[\left(\frac{\alpha}{2}+\frac{Q}{4}\right)\frac{1}{\left(\alpha+p^2D_2\right)} + \frac{QT\alpha}{4\left(\alpha+p^2D_2\right)} - \frac{\alpha Q}{4\left(\alpha+p^2D_2\right)^2}\right] \\ +\left\{\frac{1}{2} - \left(\frac{\alpha}{2}+\frac{Q}{4}\right)\frac{1}{\left(\alpha+p^2D_2\right)} + \frac{\alpha Q}{4\left(\alpha+p^2D_2\right)^2}\right\}\times\exp\left[-\left(\alpha+p^2D_2\right)T\right] \\ -\frac{c_i}{c_0}\left\{1-\left(\gamma L+h_1\right)\sqrt{\frac{D_1}{D_2}} - h_2\frac{\beta}{3}\right\}\times\exp\left[h_1+h_2-\left(\alpha+p^2D_2\right)T\right]\end{array}\right\}\right] \tag{14}$$

where, $\alpha = D_1h_1^2 + D_2h_2^2,\quad h_1 = \dfrac{U}{2D_1}$, $h_2 = \dfrac{V}{2D_2}$ and $\beta = \sqrt{1+\dfrac{D_2}{D_1}}$.

Numerical Solution

The Numerical solution of Eq. (11) with Eqs. (12) and (13) is obtained with the help of two-level explicit finite difference method. The $X, Y\ and\ T$ domains are divided into equal number of subinterval and represented as

$$X_i = X_{i-1} + \Delta X,\ i = 1, 2, ..., M,\ X_0 = 0,\ \Delta X = 0.1$$
$$Y_j = Y_{j-1} + \Delta Y,\ j = 1, 2, ..., N,\ Y_0 = 0,\ \Delta Y = 0.1 \quad (15)$$
$$T_k = T_{k-1} + \Delta T,\ k = 1, 2, ..., I,\ T_0 = 0, \Delta T = 0.001$$

The contaminant concentration at a point (x_i, y_j) at k^{th} sub-interval of time T is denoted as $C_{i,j,k}$.
The first and second order derivative in Eq. (11) is approximated as forward difference approximation and central difference approximation respectively. Using two-level explicit finite difference method, Eq. (11) with Eqs. (12) and (13) become

$$\begin{aligned} C_{i,j,k+1} = C_{i,j,k} &+ \frac{D_1 \Delta T}{\Delta x^2}\left(C_{i+1,j,k} - 2C_{i,j,k} + C_{i-1,j,k}\right) + \frac{D_2 \Delta T}{\Delta y^2}\left(C_{i,j+1,k} - 2C_{i,j,k} + C_{i,j-1,k}\right) \\ &- \frac{U \Delta T}{\Delta x}\left(C_{i+1,j,k} - C_{i,j,k}\right) - \frac{V \Delta T}{\Delta y}\left(C_{i,j+1,k} - C_{i,j,k}\right) \end{aligned} \quad (16)$$

$$C_{i,j,0} = \frac{c_i}{c_0}\exp\left(-\gamma X_i L\right), i \ge 0,\ j \ge 0 \quad (17)$$

$$C_{M,N,k} = \frac{1}{2}\left[1 + \frac{QT_k}{2}\right] \quad i = M,\ j = N, k > 0 \quad (18)$$

The limitation of an explicit scheme is that there is a certain stability criterion associated with it, so that the size of time step cannot exceed a certain value. For the present problem the stability analysis has been done to improve the accuracy of the numerical solution (Bear and Verrujit 1998) and the stability condition for the size of time step is obtained as

$$0 < \ \Delta T \le \frac{1}{2\left(\frac{D_1}{(\Delta X)^2} + \frac{D_2}{(\Delta Y)^2} + \frac{U}{2\Delta X} + \frac{V}{2\Delta Y}\right)} \quad (19)$$

which satisfy the results and conditions obtained by Ashtiani and Hosseini (2005).

RESULTS AND DISCUSSION

We consider three different time-dependent forms of velocity expression followed by Aral and Liao (1996) and expressed as follows:

1. Exponentially decreasing form of velocity

$$u = u_0 f(t),\ f(t) = 1 - \exp\left(\frac{-mt}{K}\right) \Rightarrow T = \left(mt + K\left(\exp\left(-\frac{mt}{K}\right) - 1\right)\right) \quad (20)$$

2. Asymptotic form of velocity

$$u = u_0 f(t),\ f(t) = \frac{mt}{(mt + K)} \Rightarrow T = \left(mt - K\frac{mt}{(mt + K)}\right) \quad (21)$$

3. Algebraic Sigmoid form of velocity

$$u = u_0 f(t),\ f(t) = \frac{mt}{\sqrt{(mt)^2 + K^2}} \Rightarrow T = \left(\sqrt{(mt)^2 + K^2} - K\right) \quad (22)$$

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where K is the arbitrary constant. Considering $K=0$ in Eq. (20) to (22) results as $f(t)=1$and it represent the problem with uniform velocity and dispersion coefficient.
The values of mt has been considered as 29, 32 and these values of mt yield $t=1758$, 1940 and 2122 days at a regular interval of approximately 182days. If the value $t=1758$ days represents some date in the month of December when groundwater velocity is maximum and then $t=1940$ days represents some date in June when groundwater velocity is minimum. Again, the next value of t, represents the month of December in the next year and so on.

The two-dimensional solution for the different form of velocity expressions are computed for the values u_0=0.01km/year, v_0=0.001km/year, D_{x0}=6.0km^2/year, D_{y0}=0.6km^2/year, m =0.0165(/year), q=0.0001(/year), $\gamma=0.001(/km)$, c_i=0.01, c_0=1, x =100 km, y=50 km, L =100 km and H=50 km. Figure2 represents the analytical and numerical solution of the problem for the above set of input values with asymptotic type of velocity expression at mt=26 and K=0 i.e. for uniform seepage velocity and dispersion coefficient. It is observed that, the contaminant concentration decreases with distance. It is also observed that the numerical solution shows the same pattern as observed in the analytical solution which validates the analytical method used to solve the problem. Due to presence of some numerical error approximation, the numerical solution slightly deviates from analytical solution. Figure 3 represents the contaminant concentration profile for different values of mt with asymptotic type velocity expression with K =5. It shows that the contaminant concentration decreases with distance and increases with time throughout the aquifer. Figure 4 shows the contaminant concentration pattern for asymptotic type velocity expression with mt=26, $D_{x0}=7.0\ km^2/year$ and $D_{y0}=0.7\ km^2/year$ for different values of K. Here, it is observed that the contaminant concentration decreases on increasing the value of K.

Figure 5 represents the contaminant concentration pattern for different values of longitudinal and transverse dispersion coefficient with K=5, mt=26. It shows that for asymptotic type of velocity expression the contaminant concentration decreases on increasing the dispersion co-efficient. Figure 6 shows the concentration pattern for varying initial seepage velocity along longitudinal and lateral direction for K=5, mt=26, D_{x0}=6.0km^2/year and D_{y0}=0.6km^2/year.

Here, it is observed that the contaminant concentration decreases on increasing the initial seepage velocity components. Figure 7 represents the concentration pattern for different types of velocity expression for the values K=5, mt=26, u_0=0.01km/year, v_0=0.001km/year D_{x0} =6.0km^2/year and D_{y0}=0.6km^2/year. It is observed that the contaminant concentration decreases with distance and the rate of fall of contaminant concentration is faster in case of exponential type of velocity expression as compared to sigmoid and asymptotic type velocity expression. From the entire figure, it is also observed that the contaminant concentration decreases with distance in both longitudinal and lateral direction.

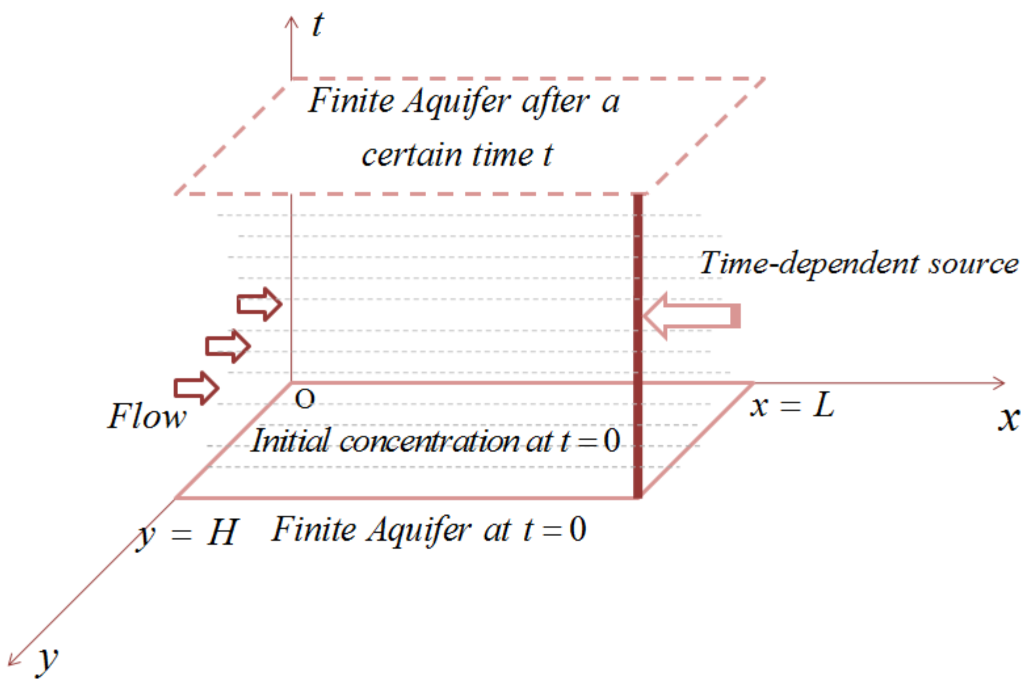

Figure 1: Physical model of the problem

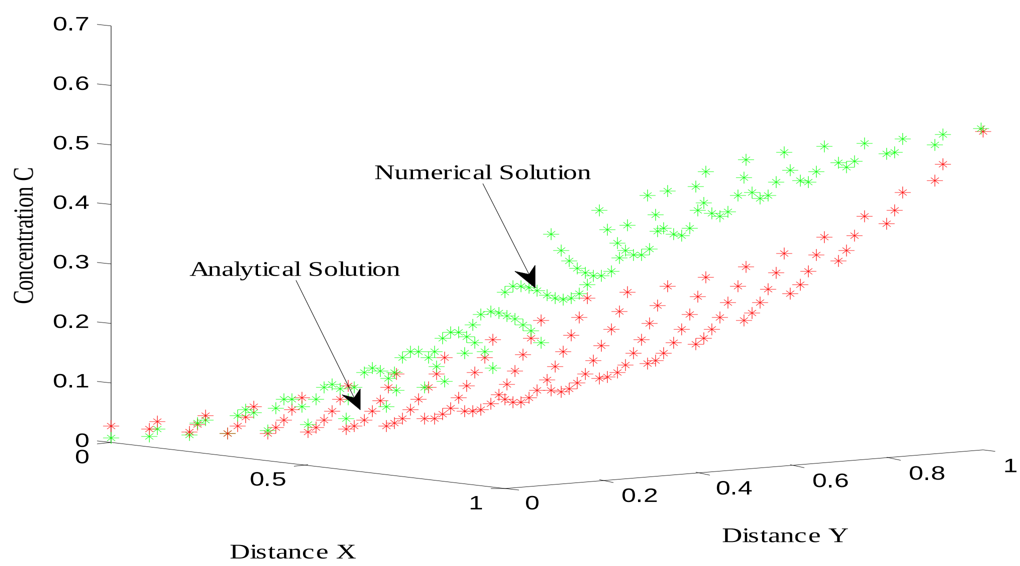

Figure 2: Concentration profile for asymptotic velocity expression for $mt=26, K=0, D_{x0}=6\ D_{y0}=0.6, u_0=0.01\ and\ v_0=0.001$

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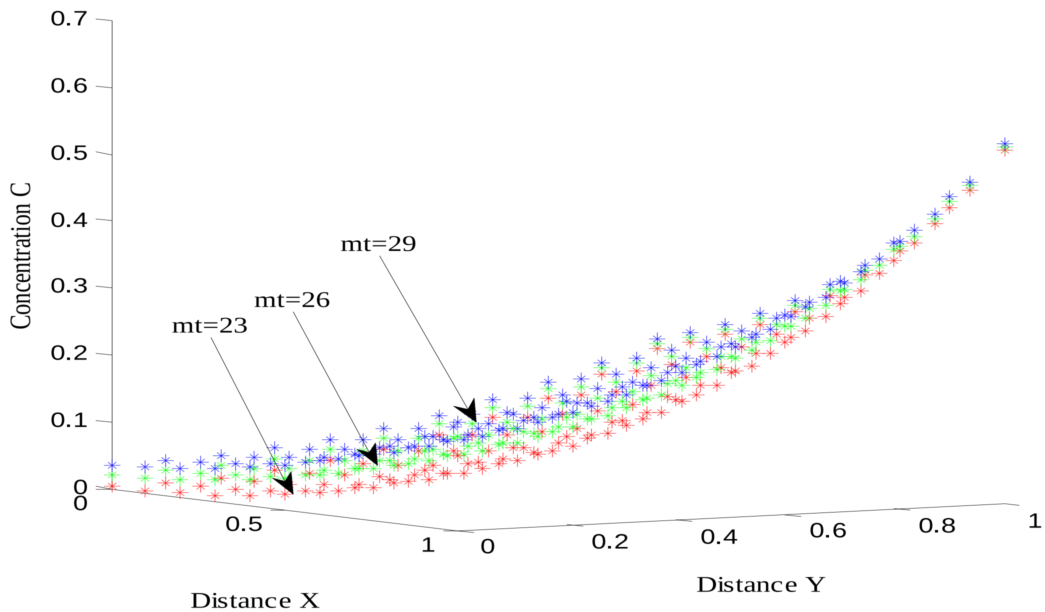

Figure 3: Concentration profile for asymptotic velocity expression for $K=5, D_{x0}=6, D_{y0}=0.6,\ u_0=0.01\, and\ v_0=0.001$

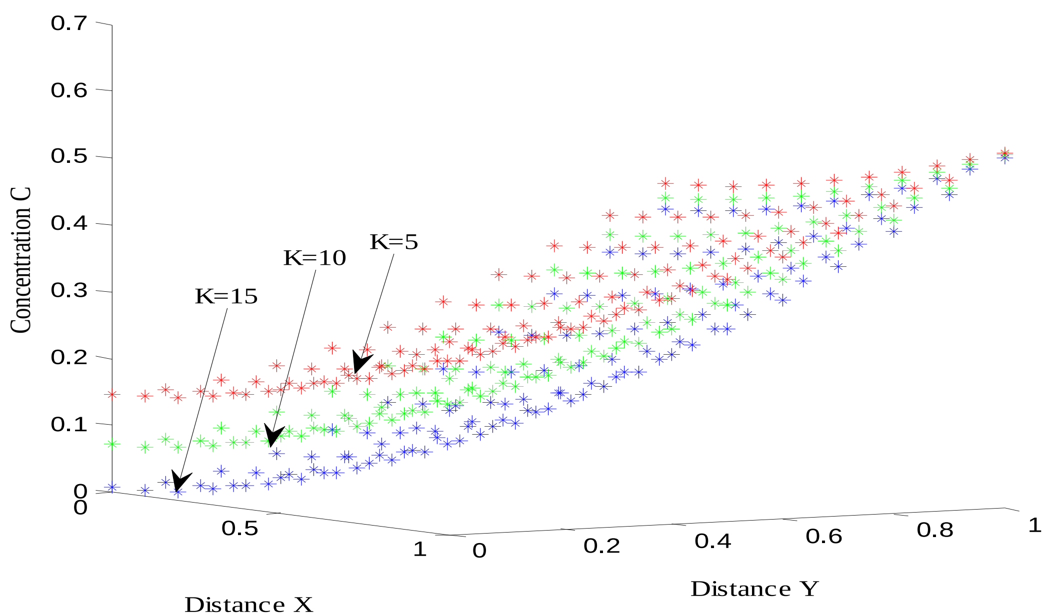

Figure 4: Concentration profile for asymptotic velocity expression for $mt=26,\ D_{x0}=7, D_{y0}=0.7,\ u_0=0.01\, and\ v_0=0.001$

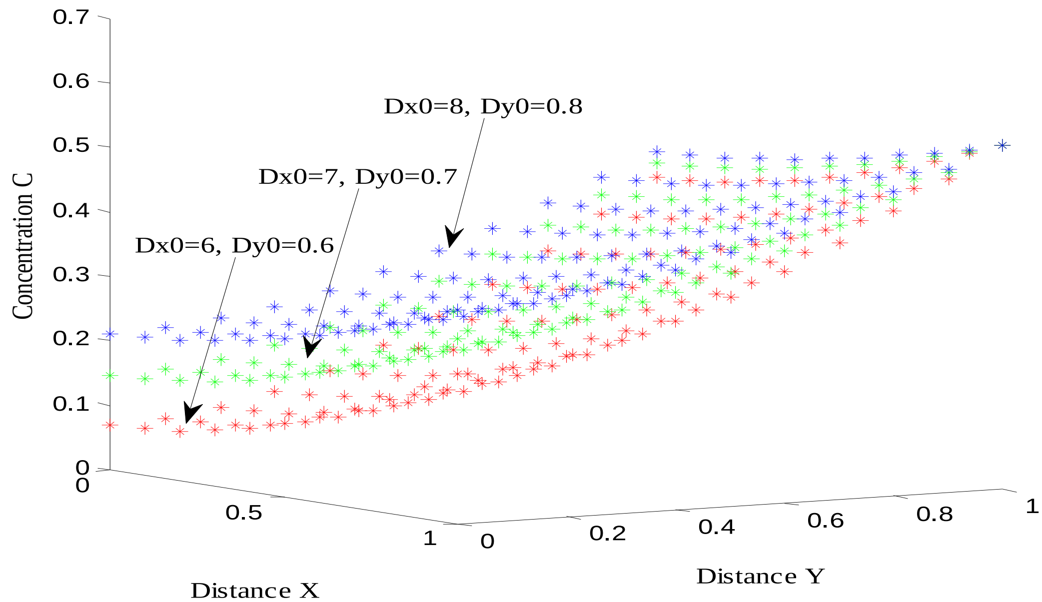

Figure 5: Concentration profile for asymptotic velocity expression for $mt = 26,\ K = 5$**,** $u_0 = 0.01$ *and* $v_0 = 0.001$

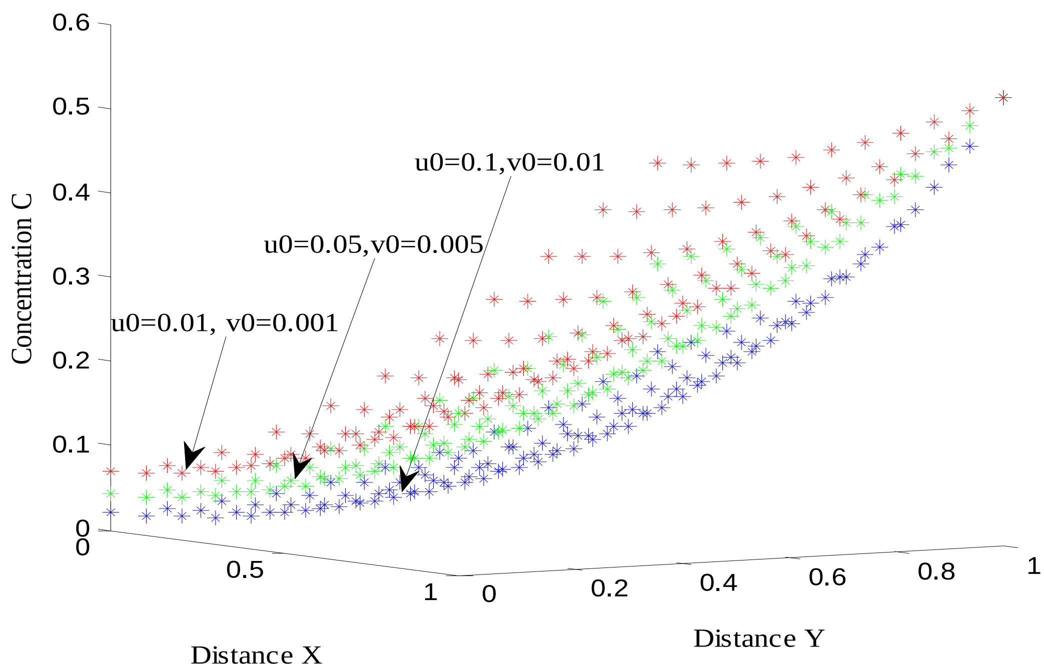

Figure 6: Concentration profile for asymptotic velocity expression for $mt = 26,\ K = 5,$ $D_{x0} = 6$ *and* $D_{y0} = 0.6$

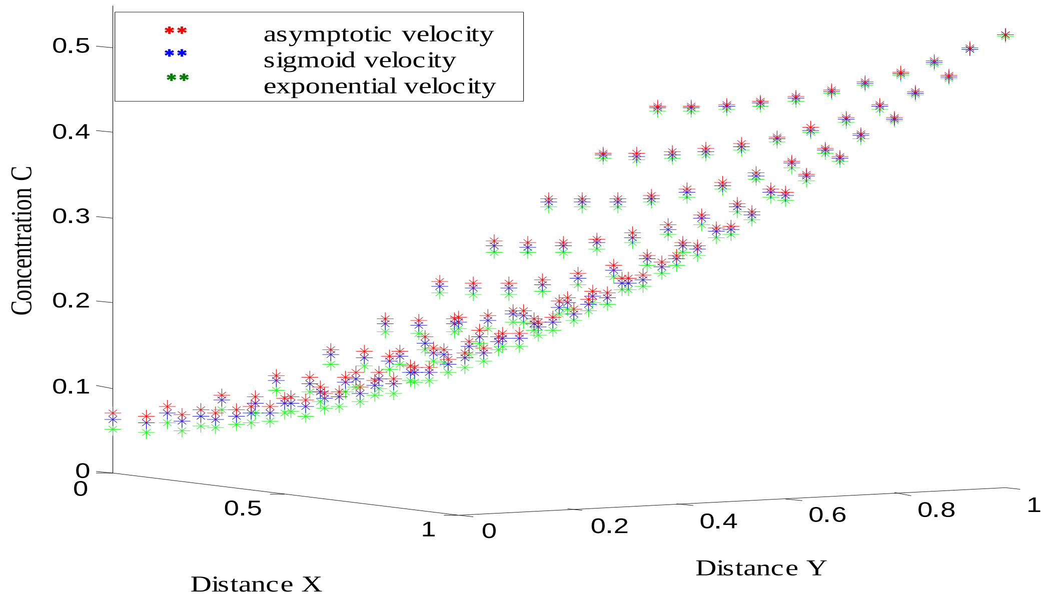

Figure 7: Concentration profile for different velocity expression for $mt = 26,\ K = 5,\ D_{x0} = 6,\ D_{y0} = 0.6,\ u_0 = 0.01$ *and* $v_0 = 0.001$**.**

Conclusion

An analytical solution to solute transport in a two-dimensional homogeneous finite porous formation is obtained. The Hankel Transform Technique (HTT) of first kind of zeroth order is employed. The obtained analytical result is compared with that of the numerical one obtained with the help of two-level explicit finite difference method. The initial concentration is taken as space dependent exponentially decreasing function. The time-dependent input boundary condition is considered at the other end as logistic sigmoid function to address the advective-dispersive equation. These solutions may help to determine the contaminants concentration distribution pattern in homogeneous porous formations due to the release of a time-dependent source. It may also be applicable to benchmark the numerical codes and solution to the problem. The following conclusions can be drawn from this study:

1. The numerical solution obtained for uniform seepage velocity and dispersion coefficient follows almost the same pattern as that of the pattern observed for analytical solution which validates the analytical method used in the problem up to 90% in agreement.
2. The contaminant concentration decreases with distance but increases with time for asymptotic type time-dependent velocity expression.
3. The solute concentration values decreases on increasing the arbitrary constant K used in different types of velocity expression.
4. On increasing the longitudinal and lateral dispersion coefficient the contaminant concentrations increases.
5. The contaminant concentration decreases rapidly on increasing the initial seepage velocity along longitudinal and lateral direction i.e. on increasing the initial seepage velocity the contaminant concentration reduces.

6. The contaminant concentration values are compared for three different forms of seepage velocity expression and it has been observed that the exponential form of velocity expression decreases more gradually as compared to asymptotic and sigmoid type of velocity expression.
7. The solute concentration decreases rapidly for the exponentially decreasing form than for the asymptotic and sigmoid form of velocity.

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