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Research Article

BOUNDARY LAYER FLOW OVER A STRETCHING VERTICAL SHEET WITH SURFACE HEAT FLUX

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ABSTRACT
This paper considers the problem of two-dimensional boundary layer flow adjacent to a vertical, continuously stretching sheet in a viscous incompressible fluid. It is assumed that the sheet is stretched with a power-law velocity and is subjected to a variable surface heat flux. The governing boundary layer equations are transformed into ordinary differential equations using similarity transformation, which are than solved using maple software. The influence of velocity exponent parameter m (or temperature exponent parameter n) is presented and discussed.

Key Words: *Boundary Layer Flow, Stretching Sheet, Surface Heat Flux, Similarity Transformation*

Nomenclature
f dimensionless stream function
g acceleration due to gravity
k thermal conductivity
Pr Prandtl number
Re_x local Reynolds number
Gr_x Grashof number
q_w surface heat flux
$u_w(x)$ velocity of the stretching surface
m velocity exponent parameter
n temperature exponent parameter
T fluid temperature
T_∞ ambient temperature
u, v velocity component of the fluid along the x and y directions, respectively
x, y Cartesian coordinates along the surface and normal to it, respectively

Greek symbols
α thermal diffusivity
β thermal expansion coefficient
η dimensionless similarity variable
υ kinematic viscosity
Ψ stream function
λ buoyancy parameter
θ dimensionless temperature

Superscript
$'$ derivative with respect to η

INTRODUCTION
The fluid dynamics over a stretching surface is important in extrusion process. The production of sheeting material arises in a number of industrial manufacturing process and includes both metal and polymer sheets. Examples are numerous and they include the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets, the

boundary layer along a liquid film in condensation process, paper production, glass blowing, metal spinning, and drawing plastic films, to name just a few. The quality of the final product depends on the rate of heat transfer at the stretching surface. The pioneering study by Crane (1970) who presented an analytical solution for the steady two-dimensional stretching of a surface in a quiescent fluid, many authors have considered various aspects of this problem and obtained similar solutions. Since then, many authors have studied various aspects of this problem. For instance, Magyari and Keller (1999; 2000). Sriramulu *et al.* (2001) studied steady flow and heat transfer of a viscous incompressible fluid through porous medium over a stretching sheet. Partha *et al.* (2005) studied the similar problem, by considering exponentially stretching surface. The temperature field in the flow over a linearly stretching surface subject to a variable surface temperature was studied by Grubka and Bobba (1985), while Dutta *et al.* (1985) reported the temperature distribution for the uniform surface heat flux condition. Elbashbeshy (1998) and Lin and Chen (1998) considered the heat transfer characteristics on a stretching horizontal surface subject to a power-law velocity and variable surface heat flux.
Motivated by works mentioned above and practical applications, the main concern of the present paper is to study the problem of heat transfer characteristics adjacent to a stretching vertical sheet with a power-law velocity subjected to a variable surface heat flux.

MATHEMATICAL FORMULATION

We consider a steady, two-dimensional flow of a viscous and incompressible fluid adjacent to a vertical, continuously stretching sheet placed in the plane $y = 0$ of a Cartesian system of coordinates xy with the x-axis along the sheet, while the y-axis is measured normal to the surface of the sheet. It is assumed that the surface heat flux and the stretching velocity vary in a power-law with the distance from the leading edge, i.e. $q_w(x) = ax^n$ and $u_w(x) = bx^m$ respectively, where a and b are constants and m and n are the exponents. Under these assumptions along with the Boussinesq and boundary-layer approximations, the equations which model the problem under consideration are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots(1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \qquad \ldots(2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad \ldots(3)$$

Along with the boundary conditions for the problem are given by:

$$y = 0 : u = u_w(x), v = 0, \frac{\partial T}{\partial y} = -\frac{q_w}{k}$$

$$y \to \infty : u = 0, T = T_\infty \qquad \ldots(4)$$

The continuity equation can be satisfied by introducing a stream function Ψ such that $u = \frac{\partial \Psi}{\partial y}$ and $v = -\frac{\partial \Psi}{\partial x}$, then the momentum and energy equations can be transformed into the corresponding nonlinear ordinary differential equations by the following transformation [10, 11]:

$$\eta = \left(\frac{u_w}{\upsilon x}\right)^{1/2} y,\ f(\eta) = \frac{\Psi}{(\upsilon x u_w)^{1/2}},\ \theta(\eta) = \frac{k(T - T_\infty)}{q_w}\left(\frac{u_w}{\upsilon x}\right)^{1/2} \qquad \ldots(5)$$

Where η is the independent similarity variable. The transformed nonlinear ordinary differential equations are:

$$f''' + \frac{m+1}{2} f f'' - m f'^2 + \lambda\theta = 0 \qquad \ldots(6)$$

$$\frac{1}{Pr}\theta'' + \frac{m+1}{2} f\theta' - n f'\theta = 0 \qquad \ldots(7)$$

where primes denote differentiation with respect to η, m is the velocity exponent parameter, n is the temperature exponent parameter, Pr = $\frac{\upsilon}{\alpha}$ is the Prandtl number and $\lambda = \frac{Gr_x}{Re_x^{5/2}}$ is the buoyancy or mixed convection parameter with $Gr_x = \frac{g\beta\, q_w x^4}{k\upsilon^2}$ and $Re_x = \frac{u_w x}{\upsilon}$ are the local Grashof number and local

Reynolds number, respectively. It can be shown that λ is independent of x if $n = (5m-3)=2$. Thus, in the presence of buoyancy force, similarity is achieved under this limitation.
For $n = (5m - 3)=2$, equation (7) becomes:

$$\frac{1}{Pr}\theta'' + \frac{m+1}{2} f\theta' - \frac{5m-3}{2} f'\theta = 0 \qquad \ldots(8)$$

Then the boundary conditions becomes

$$f(0) = 0, f'(0) = 1, \theta'(0) = -1$$
$$f'(\infty) \to 0, \theta(\infty) \to 0. \qquad \ldots(9)$$

NUMERICAL SOLUTION

The non- liner ordinary differential equation (6) and (8) subject to boundary condition (9) are solve for different values of velocity exponent parameter m, with fixed values of buoyancy parameter λ and Prandtl number Pr, numerically using Runge-Kutta-Fehlberg forth-fifth order method. To solve these equations we adopted symbolic algebra software Maple.

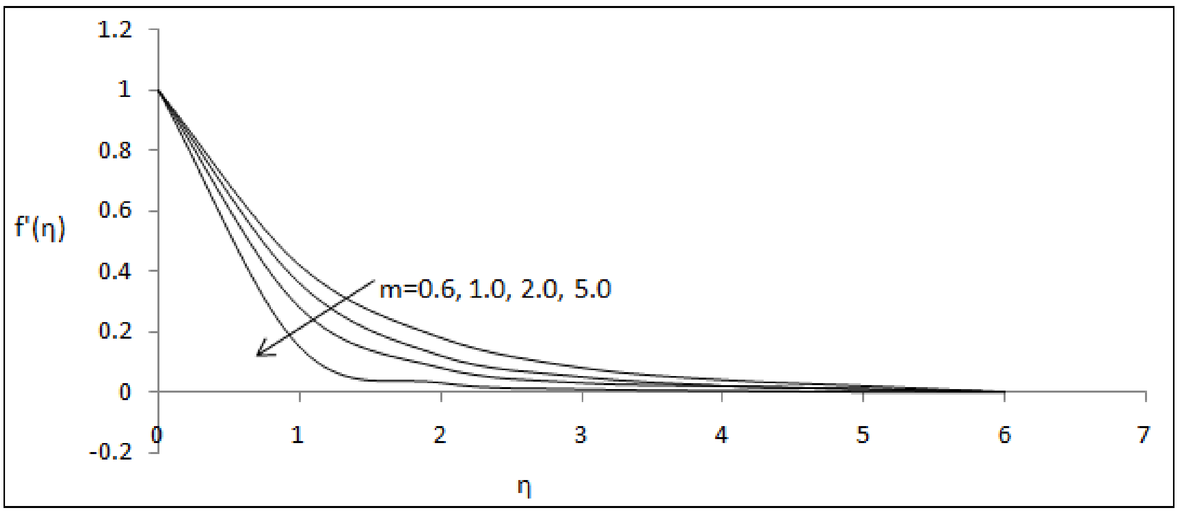

Figure 1: Velocity distribution f'(η) for the different values of m (for Pr=1.0 and λ=0.0).

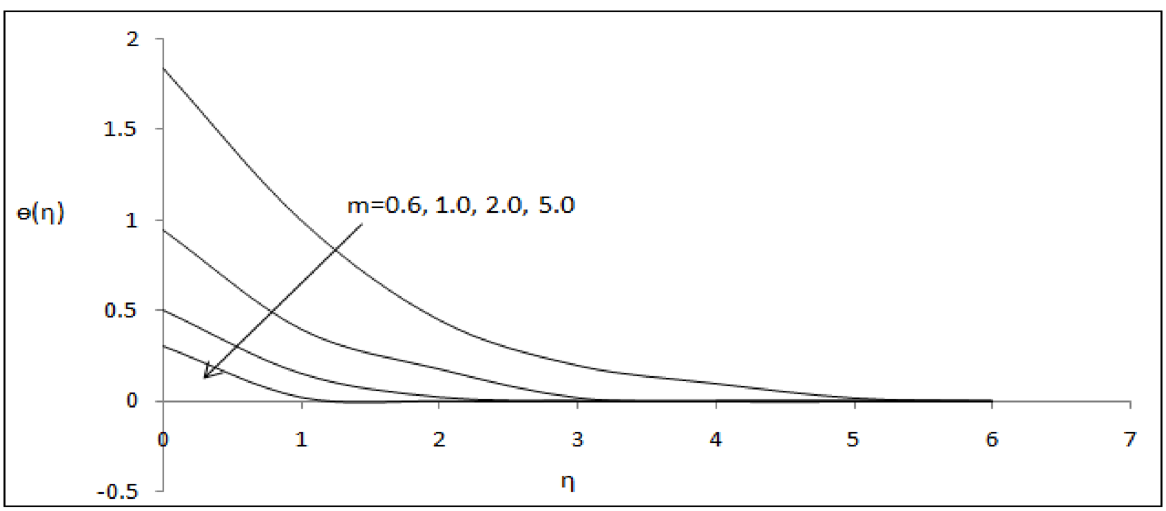

Figure 2: Temperature distribution θ(η) for different values of m (for Pr=1.0 and λ=0.0).

Maple uses the well known Runge-Kutta-Fehlberg Fourth-fifth order (RFK45) method to generate the numerical solution of a boundary value problem. The boundary condition $\eta = \infty$ was replaced by those at $\eta = 6$ in accordance with standard practice in the boundary layer analysis. The effects of the m on the velocity distribution and temperature distribution are shown in figures 1 and 2 respectively.

RESULTS AND DISCUSSION

Figures 1 and 2 shows the velocity and temperature distribution for different values of velocity exponent parameter m (or temperature exponent parameter n) with fixed values of Pr=1.0 and λ=0.0, respectively. It is seen that the boundary layer thickness of both velocity and temperature profiles decrease as the values of m increases.

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