International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2014 Vol. 4 (1) January-March, pp. 202-208/Ghate et al.
Research Article

(n + 2) DIMENSIONAL KALUZA-KLEIN STRING COSMOLOGICAL MODEL IN BRANS-DICKE THEORY OF GRAVITATION

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ABSTRACT
We have studied Kaluza-Klein universe in the presence of cosmic strings as a source in a scalar tensor theory of gravitation proposed by Brans and Dicke (1961) which is the extension of five dimensional Kaluza-Klein string cosmological models investigated by Reddy *et al.*, (2007). Some physical properties of the model are also discussed.

Keywords: *Brans-Dicke Theory, Cosmic Strings, (n+2) Dimensional Kaluza-Klein Metric*

INTRODUCTION

Einstein's general theory of relativity is successful in describing gravitational phenomena. It has also served as a basis for model of the universe. However, since Einstein first published his theory of gravitation, there have been many criticisms on general relativity because of the lack of certain 'desirable' features in the theory. For example Einstein himself pointed out that general relativity does not account satisfactorily for inertial properties of matter, i.e. Mach's principle is not substantiated by general relativity. So in recent years, there has been lot of interest in several alternative theories of gravitation. The most important among them are scalar–tensor theories of gravitation formulated by Brans and Dicke (1961), Nordvedt (1970) and Saez and Ballester (1985). All versions of the scalar–tensor theories are based on the introduction of a scalar field φ into the formulation of general relativity, this scalar field together with the metric tensor field then form a scalar–tensor field representing the gravitational field.

Brans–Dickeformulated a scalar-tensor theory of gravitation in which gravity is mediated by a scalar field φ in addition to the usual metric tensor field g_{ij} present in Einstein theory. In this theory, the long range scalar field φ is generated by the whole of matter in the universe according to Mach's Principle and has the dimension of the universe of the gravitational constant G. Number of relativists (Singh *et al.*, 1983; Singh and Rai, 1983; Banerjee and Santosh, 1982; Banerjee and Banerjee, 1983; Ram, 1983; Ram and Singh, 1997; Petzold, 1984; Reddy *et al.*, 2007; Adhav *et al.*, 2009; Rao *et al.*, 2008; Singh and Sharma, 2010; Singh and Kale, 2011; Sharif and Waheed, 2012; Ghate and Sontakke, 2013) had made detailed discussion on several aspects of Brans Dicke theory of gravitation.

The concept of string theory was developed to describe events at the early stages of the evolution of the universe. Kibble (1976) and Vilenkin (1985) believed that strings may be one of the sources of density perturbations that are required for the formation of large scale structures in the universe. The study of string cosmological models was initiated by Vilenkin (1981), Letelier (1983), Krori *et al.*, (1990). Relativistic string models in the context of Bianchi space times have been obtained by Krori *et al.*, (1994), Banerjee *et al.*, (1990), Tikekar *et al.*, (1994), Bhattacherjee & Baruah (2001), Mahanta & Mukherjee (2001). Numbers of researchers *viz*. Gundlach & Ortiz (1990), Barros & Romero (1995), Sen & Banerjee (1997), Barros *et al.*, (2001), Reddy *et al.*, (2003a,b), Adhav *et al.*, (2008), Rao & Neelima (2013), Rathore *et al.*, (2009), Katore *et al.*, (2011), Pradhan *et al.*, (2012), Reddy *et al.*, (2013) have studied several aspects of cosmic strings in Brans–Dickescalar-tensor theories of gravitation.

The possibility that space time has more than four dimensions has attracted many researchers to the field of higher dimensions (Witten, 1984). Study of higher dimensional space-times is also important because of the underlying idea that the cosmos at its early stage of evolution might have had a higher dimensional era. It is believed that a consistent unification of all fundamental forces in nature would be possible within the space-time with an extra dimension beyond those four observed so far. Higher dimensional theories of Kaluza-Klein type have been considered to study some aspects of early universe (Chodos and Detweiler,

1980; Freund, 1982; Sahdev, 1984). The unification of gravity with other fundamental forces in nature is still a challenging problem today. Nordtstorm (1987) and Kaluza-Klein (1921; 1926) suggested the idea of one extra space dimension in order to explain the unification of gravity with other fundamental forces. Gross and Perry (1983) have shown that the five dimensional Kaluza-Klein theory of unified gravity and electromagnetism admits soliton solutions. Further they explained the inequality of the gravitational and inertial masses due to violation of Birkoff's theorem in Kaluza-Klein theory, which is considered with the principle of equivalence. Numbers of authors such as Ponce (1988), Chi (1990), Fukui (1993), Liu and Wesson (1994), Coley (1994) have studied higher dimensional cosmological models in different context. Adhav *et al.*, (2008) studied string cloud and domain walls with quark matter in N dimensional Kaluza-Klein cosmological model. Khadekar *et al.*, (2010) have obtained higher dimensional cosmological model with variable equation of state in presence of G and Λ. Katore (2011) have studied extra dimensional FRW cosmology with variable G and Λ. Recently, Ram & Priyanka (2013) have investigated some Kaluza-Klein cosmological models in *f(R,T)* gravity.
In this paper (n+2) dimensional Kaluza-Klein string cosmological model has been investigated in Brans Dicke theory (1961), which is the extension of five dimensional Kaluza-Klein string cosmological model Reddy *et al.*, (2007). To get the deterministic solution, the equation of state for Reddy string given by $\rho + \lambda = 0$ has been considered.

Metric and Field Equations

We consider (n+2) dimensional Kaluza-Klein metric of the form

$$ds^2 = -dt^2 + R^2(t) \sum_{i=1}^{n} dx_i^2 + A^2(t)\, du^2 , \tag{1}$$

where (n+2)th co-ordinate is taken to be space-like.
Brans Dicke (1961) field equations are

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}(\phi_{i;j} - g_{ij}\Box\varphi \ ) \tag{2}$$

and

$$\Box\phi \equiv \phi^{k}_{;k} = 8\pi\phi^{-1}T(3+2\omega)^{-1}, \tag{3}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, ω is the dimensionless coupling constant, T_{ij} is the stress energy of the matter. Comma and semicolon denote partial and covariant differentiation respectively.
The energy momentum tensor for cosmic strings is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j ,$$

(4)

where $\rho = \rho_p + \lambda$ is the rest energy density of cloud of strings with particles attached to them, λ is the tension density of the string and ρ_p is the rest energy density of the particles, u^i the cloud velocity and

$$x^i = \left(0, 0, ..., 0, A^{-1}\right). \tag{5}$$

The direction of string will satisfy

$$u_i u^i = -1 = -x_i x^i \qquad \text{and} \qquad u_i x^j = 0 \tag{6}$$

in the co-moving co-ordinate system.
We have from (5)

$$T_0^0 = -\rho, \quad T_1^1 = T_2^2 = ... = T_n^n = 0, \quad T_{n+1}^{n+1} = -\lambda. \tag{7}$$

Here the quantities ρ, λ and ϕ depend on t only.

With the help of (4-7), the field equations (2) and (3) for the metric (1) reduce to

$$\frac{n(n-1)}{2}\frac{\dot{R}^2}{R^2}+n\frac{\dot{R}\dot{A}}{RA}=\frac{8\pi\rho}{\phi}+\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}+\frac{\ddot{\phi}}{\phi}+\square\frac{\phi}{\phi} \tag{8}$$

$$(n-1)\frac{\ddot{R}}{R}-\frac{(n-1)}{2}\frac{\dot{R}^2}{R^2}+(n-1)\frac{\dot{R}\dot{A}}{RA}+\frac{\ddot{A}}{A}=-\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}-\frac{\dot{\phi}\dot{R}}{\phi R}+\square\frac{\phi}{\phi} \tag{9}$$

$$n\frac{\ddot{R}}{R}-\frac{n(n-1)}{2}\frac{\dot{R}^2}{R^2}=8\pi\lambda-\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}-\frac{\dot{\phi}\dot{A}}{\phi A}+\square\frac{\phi}{\phi} \tag{10}$$

$$\square\phi=\ddot{\phi}+\dot{\phi}\left(n\frac{\dot{R}}{R}+\frac{\dot{A}}{A}\right)=\frac{8\pi(\rho+\lambda)}{\phi(3+2\omega)}. \tag{11}$$

Also the equations of motion $T^{ij}_{;j}=0$ are consequences of the field equations (2) and (3) which take the simplified form for the metric (1) as

$$\dot{\rho}+n\rho\frac{\dot{R}}{R}+(\rho-\lambda)\frac{\dot{A}}{A}=0, \tag{12}$$

where an overhead dot $\left(\,^{\bullet}\,\right)$ denotes differentiation with respect to t.

Cosmic String Model

Here we have four independent field equations (8-11) connecting five unknown quantities R, A, λ, ρ and ϕ. Hence to get a determinate solution one has to assume physical or mathematical conditions. In the literature (Letelier, 1983; Reddy, 2003a,b; Reddy and Rao, 2006a,b), we have equations of state for string models.

$$\begin{aligned}
&\rho=\lambda && \text{(geometric string or Nambu string),}\\
&\rho=(1+\omega)\lambda && (p-\text{string}),\\
&\rho+\lambda=0 && \text{(Reddy string),}
\end{aligned} \tag{13}$$

Since the field equations are highly non-linear, we also assume an analytic relation between the metric coefficients (scale factor)

$$A=\mu R^n \quad \text{(where } \mu \text{ and } n \text{ are constants)} \tag{14}$$

to get determinate solution (Chakraborty and Chakraborty, 1992)

Here we obtain a string model in (n+2) dimensions corresponding to $\rho+\lambda$ (trace of energy momentum tensor of string source, $T=0$ i.e. the sum of the rest energy density and tension density for a cloud of string vanishes.) However, the string models corresponding to $\rho=\lambda$ and $\rho=(1+\omega)\lambda$ could not be obtained due to the highly non-linear character of the field equations.

Using (13) and (14) the field equations (8-11) yield an exact solution given by

$$R=\left[l\left(C_3t+C_4\right)\right]^{\frac{1}{l}}, \tag{15}$$

$$A=\left[l\left(C_3t+C_4\right)\right]^{\frac{m}{l}}, \tag{16}$$

$$\phi=\left[l\left(C_3t+C_4\right)\right]^{\frac{k}{l}}, \tag{17}$$

$$8\pi\lambda=\left[l\left(C_3t+C_4\right)\right]^{\frac{k}{l}}\left(C_3t+C_4\right)^{-2}\left\{\frac{n(n+1)}{2}\frac{C_3^2}{l^2}-\frac{nC_3^2}{l}+\frac{\omega}{2}\frac{k^2C_3^2}{l^2}+\frac{kmC_3^2}{l^2}\right\}, \tag{18}$$

where $l = \dfrac{C_2 + (n+m)C_3}{C_3}$, $\quad k = \dfrac{C_2}{C_3}$ (19)

and C_i, $i = 1,2,3,4$ are constants of integration.

Thus (n+2) dimensional string cosmological model corresponding to the above solution can be written as

$$ds^2 = -dt^2 + [l(C_3 t + C_4)]^{2/l} \sum_{i=1}^{n} dx^{i\,2} + [l(C_3 t + C_4)]^{2m/l} d\psi^2 . \tag{20}$$

where the scalar field rest energy density and the tension density of cosmic string in the model are given by (17) and (18).

Some Physical Properties

The model given by (20) represents an exact cosmological model in (n+2) dimensions in the framework of Brans-Dicke theory of gravitation, when the sum of the tension density λ and the rest energy density ρ of the cosmic string vanishes. The model has no singularity.

I order to gain a further insight into the behavior of the model, physical and kinematical variables and to have relatively simple picture of their explicit expressions we choose the constants of integration as

$C_1 = C_2 = C_3 = 1 \qquad$ and $\qquad C_4 = 0$

which in turn give from (19).

$l = m + n + 1, \qquad K = 1$.

With this choice the explicit expressions are

Spatial volume:

$$V = [(m+n+1)t]^{\frac{m+n}{m+n+1}} . \tag{21}$$

Expansion scalar:

$$\theta = (n+1)H = \frac{m+n}{(m+n+1)t} . \tag{22}$$

Shear scalar:

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{n(m-1)^2}{2(n+1)[(m+n+1)^2]} . \tag{23}$$

The deceleration parameter (Feinstein *et al.*, 1995)

$$q = \left[\frac{(n^2 + m + 2n + mn + 1)}{(m+n)}\right] - 1. \tag{24}$$

Energy density

$$8\pi\rho = -8\pi\lambda = \left(\frac{n^2 + 2m + 2mn - \omega + n}{2}\right)[(m+n+1)t]^{\frac{-(2m+2n+1)}{(m+n+1)}} . \tag{25}$$

And $\quad 8\pi\rho_p = (n^2 + 2m + 2mn - \omega + n)[(m+n+1)t]^{\frac{-(2m+2n+1)}{(m+n+1)}}$. (26)

The scalar field in the model takes form

$$\phi = [(m+n+1)t]^{\frac{1}{(m+n+1)}} . \tag{27}$$

From equation (20), the spatial volume $V \to \infty$ for $t \to \infty$. The model is an exact string cosmological where the spatial volume increases with increase in time representing the model is expanding. The expansion scalar θ and shear scalar σ diverge for large values of t. The scalar field in the model is free from initial singularity. The rest energy density ρ, string tension density λ and particle density ρ_p tends to zero as time increases indefinitely and possess singularities at $t = 0$.

Conclusion

(n+2) dimensional Kaluza-Klein string cosmological model in scalar tensor theory of gravitation proposed by Brans and Dicke (1961) has been obtained. An exact solution is obtained using the equation of state for Reddy string given by $\rho+\lambda=0$ (i.e. the sum of rest energy density and tension density for a cloud of string vanishes) by assuming trace of energy momentum tensor of string source $T=0$. It is verified that the model could not exist for the case when $\rho=\lambda$ (i.e., geometric string or Nambu string) and $\rho=(1+\omega)\lambda$ (i.e. p-string). The model obtained represents (n+2) dimensional Reddy string in Brans Dicke theory.

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