

## VARIATION OF PHASE SHIFT AND ENERGY GAP IN NUCLEAR MATTER

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### ABSTRACT

In this paper, using a nucleon-nucleon exponential interaction potential  $V(r) = -V_0 \exp\left(-2r/\beta\right)$ , an expression for phase shift,  $\delta_\ell(k_f)$ , has been obtained omitting iso-spin effects. For Fermi momentum  $0 \text{ fm}^{-1}$ ,  $0.1 \text{ fm}^{-1}$ ,  $0.2 \text{ fm}^{-1}$  ... the values of phase shift,  $\delta_\ell(k_f)$ , have been calculated. The values of the phase shift,  $\delta_\ell(k_f)$ , increases linearly with the increase in Fermi momentum,  $k_f$ . The expression for the energy gap,  $\Delta(k_f)$ , and its variation with Fermi momentum,  $k_f$ , is also calculated.

**Keywords:** Phase Shift, Nuclear Matter, Energy Gap, Fermi Momentum

### INTRODUCTION

Most of what we know about nuclear pairing comes from nuclei with a sizable neutron excess where the isospin  $T = 1$  neutron-neutron (nn) and proton-proton (pp) pairing dominate (Dean and Hjorth-Jensen, 2003). The nuclear force has been at the heart of nuclear physics ever since the field was born in 1932 with the discovery of the neutron by Chadwick. In fact, during the first few decades of nuclear physics, the term nuclear force was often used as synonymous for nuclear physics as a whole. The interaction between two nucleons is basic for all of nuclear physics. The traditional goal of nuclear physics is to understand the properties of atomic nuclei in terms of the bare interaction between pairs of nucleons.

The oldest theory of nuclear forces was presented by Yukawa based on the model in which the mesons mediate the NN (pp, pn, nn) interactions.

The interaction between nucleons is characterized by the existence of a strongly repulsive core at short distance with a characteristic radius  $\approx 0.5 - 1 \text{ fm}$ . The interaction obeys several fundamental symmetries such as translational, rotational, spatial-reflection, time-reversal invariance and exchange symmetry. General quantum numbers of a two-nucleon system is a natural starting point for a discussion of pairing found in nuclei (Dean and Hjorth-Jensen, 2003).

It is not easy to precisely construct a nucleon-nucleon interaction. However, a description of the interaction in terms of various meson exchanges is at present the most quantitative representation in the energy regime of nuclear physics.

It can be assumed that the meson exchange is an appropriate picture at low and intermediate energies and that at present it is sufficient to limit attention to the time-honoured configuration space version of the nucleon-nucleon interaction (Elgarøy and Hjorth-Jensen, 1997), including only central,  $l$ , spin-spin,

$\sigma \cdot \sigma$ , tensor,  $S_{12} \hat{r}$ , and spin-orbit,  $L \cdot S$ , terms. When two nuclear particles or two nucleons collide with each other or scatter, there is what is called a scattering length,  $a_0$ , the distance of the nearest approach. Once the particles are in the field of force of each other, they interact and scatter in any direction resulting in a phase shift or phase difference,  $\delta_\ell(k_f)$ . Thus, phase shift,  $\delta_\ell(k_f)$ , depends upon the interaction potential. We consider  $^1S_0$  pairing in infinite neutron matter and nuclear matter and show that in the lowest order approximation, where the pairing interaction is taken to be the bare nucleon-

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nucleon interaction in the  $^1S_0$  channel, the pairing interaction can be determined directly from the  $^1S_0$  phase shifts.

Many attempts have been made to derive an effective formula for calculating the phase shift in large nuclear systems using a nucleon-nucleon interaction in a given shell-model space and bare nucleon-nucleon interaction using many-body perturbation theory (Dean and Hjorth-Jensen, 2003). While some theories appeared to work well for some nuclei, there are several indications that these formulae fail to produce results that can be compared with experimental values (Pudliner *et al.*, 1997; Gregory *et al.*, 2014). A formula is derived that is used to find the values of the energy gap,  $\Delta(k_f)$ , and phase shift,  $\delta_\ell(k_f)$ , also a formula has been derived which correlates phase shift,  $\delta_\ell(k_f)$ , and energy gap using some known nucleon-nucleon interactions.

### **Pairing in Nuclear Matter**

There has been renewed interest in the pairing problem in neutron matter and neutron-rich nuclei. The pairing gap is determined by the attractive part of the NN interaction. For the  $^1S_0$  channel in nuclear physics, depending upon the number of neutrons, when there is a nucleon-nucleon interaction such that their spins are aligned opposite, the angular momentum is zero. The channel for such neutron excess will be  $^1S_0$ . There are two weak coupling limits. One is when the potential is weak and attractive for large inter-particle spacing and second when the potential becomes repulsive at  $r \approx 0.6$  fm. The potential has a value of some few *Mev*. In the strong coupling limit, the nucleon-nucleon potential is large and attractive; its value reaches a maximum of around 100 *Mev* at  $r \approx 1$  fm.

In the  $^1S_0$  channel the potential is attractive for momenta  $k \leq 1.74 \text{ fm}^{-1}$  (or for interparticle distances  $r \geq 0.6$  fm). In the weak coupling regime, where the interaction is weak, attractive and the coherence length is larger than the interparticle spacing a gas of fermions may undergo a superconducting (or superfluid) instability at low temperatures, and a gas of Cooper pairs is formed. This gas of Cooper pairs will be surrounded by unpaired fermions and the typical coherence length is large compared with the interparticle spacing, and the bound pairs overlap.

In the strong-coupling limit, the formed bound pairs have only a small overlap, the coherence length is small, and the bound pairs can be treated as a gas of point bosons. One expects then the system to undergo a Bose-Einstein condensation into a single quantum state with total momentum  $k = 0$ , called the zero-momentum state (Nozieres and Schmitt-Rink, 1985). For the  $^1S_0$  channel in nuclear physics, we may actually expect to have two weak-coupling limits, namely when the potential is weak and attractive for large interparticle spacings and when the potential becomes repulsive at  $r \approx 0.6$  fm. In these regimes, the potential has values of the order of some few *Mev*.

One may also loosely speak of a strong-coupling limit where the *NN* potential is large and attractive. This takes place where the *NN* potential reaches its maximum, with an absolute value of typically  $\sim 100$  *Mev*, at roughly  $\sim 1$  fm. We note that fermion pairs in the  $^1S_0$  wave in neutron and nuclear matter will not undergo the above-mentioned Bose-Einstein condensation, since, even though the *NN* potential is large and attractive for certain Fermi momenta, the coherence length will always be larger than the interparticle spacing, as demonstrated by De Balsio *et al.*, (1992).

The inclusion of in-medium effects, such as screening terms, are expected to further reduce the pairing gap and thereby enhance further the coherence length. This does not imply that such a transition is not possible in nuclear matter. A recent analysis (Lombardo *et al.*, 2001) of triplet  $^3S_1$  pairing in low-density symmetric and asymmetric nuclear matter indicates that such a transition is indeed possible.

It is found that the neutron pairing gap  $\Delta_{F_n}$  is strongly dependent on the Fermi momentum, or equivalently, the nuclear matter density. Energy gap,  $\Delta_{F_n}$ , increases as the Fermi momentum (or density) goes down, reaches a maximum at  $k_{F_n} \approx 0.8 \text{ fm}^{-1}$  in symmetric nuclear matter or  $k_{F_n} \approx 0.9 \text{ fm}^{-1}$  in pure

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neutron matter, and then rapidly drops to zero. A systematical enhancement of about  $0.3 \text{ MeV}$  for  $\Delta_{F_n}$  around  $k_{F_n} = 0.8 \text{ fm}^{-1}$  is revealed in pure neutron matter compared with those in symmetric nuclear matter for all of the adopted pairing interactions (Bao and Wei, 2013).

The energy gap is to a remarkable extent determined by the available  $^1S_0$  phase shifts. Thus, the quantitative features of  $^1S_0$  pairing in neutron matter can be obtained directly from the  $^1S_0$  phase shifts. This happens because the  $NN$  interaction is very nearly rank-one separable in this channel due to the presence of a bound state at zero energy, even for densities as high as  $k_F = 1.4 \text{ fm}^{-1}$ . This explains why all bare  $NN$  interactions give nearly identical results for the  $^1S_0$  energy gap in lowest-order  $BCS$  calculations.

The results of different groups are in close agreement on the  $^1S_0$  energy gap values and on its density dependence, which shows a peak value of about  $3 \text{ MeV}$  at a Fermi momentum close to  $k_F \approx 0.8 \text{ fm}^{-1}$  (Elgarøy and Hjorth-Jensen, 1997). All these calculations adopt the bare  $NN$  interaction as the pairing force, and it has been pointed out that the screening by the medium of the interaction could strongly reduce the pairing strength in this channel (Chen *et al.*, 2008). The issue of the many-body calculation of the pairing effective interaction is a complex one and still far from a satisfactory solution.

The calculation of the  $^1S_0$  gap in symmetric nuclear matter is closely related to the one for neutron matter. Even with modern charge-dependent interactions, the resulting pairing gaps for this partial wave are fairly similar (Elgarøy and Hjorth-Jensen, 1997).

The size of the neutron-proton (np)  $^3S_1 - ^3D_1$  energy gap in symmetric or asymmetric nuclear matter has, however, been a much debated issue since the first calculations of this quantity appeared. While solutions of the  $BCS$  equations with bare nucleon-nucleon  $NN$  forces give a large energy gap of several  $\text{MeVs}$  at the saturation density  $k_F = 1.36 \text{ fm}^{-1}$  (Akmal *et al.*, 1998; Takatsuka and Tamagaki, 1997; Vonderfecht *et al.*, 1993), there is little empirical evidence from finite nuclei for such strong np pairing correlations, except possibly for isospin  $T = 0$  and  $N = Z$ .

Since the pairing correlations are largest for small densities, i.e., at the surface of the nucleus, this result agrees with the usual observation that pairing in nuclei is a surface phenomenon. Moreover, the agreement between the results obtained with bare relativistic forces and those obtained with the Gogny force for the pairing properties of the symmetric nuclear matter is a particularly interesting outcome as in that using this density dependence of the gap parameter in semi classical calculations the average pairing properties of finite nuclei can be reproduced rather well (Serra *et al.*, 2001).

Following the discovery of neutron stars, (Hoffberg *et al.*, 1970) calculated gaps for neutron matter within the  $BCS$  theory using a separable nucleon-nucleon interaction that had been fitted to two-nucleon scattering data. Their calculations predicted a pairing gap for neutrons in the  $^1S_0$  state that first rose with increasing density, reached a maximum of roughly  $3 \text{ MeV}$  at a density of about  $n_s/10$ , where  $n_s \approx 0.16 \text{ fm}^{-3}$  is the saturation density of nuclear matter with equal numbers of neutrons and protons, a density typical of the interiors of heavy nuclei. With further increase in density, the gap dropped and vanished at a density somewhat below  $n_s$ .

The qualitative behavior of the gaps may be understood in terms of the measured phase shifts for nucleon-nucleon interactions. A positive phase shift corresponds to an attractive interaction between neutrons, and therefore at low  $k$ , which corresponds to low Fermi momentum and low density, the most attractive channel is  $^1S_0$ , (Dean and Hjorth-Jensen, 2003; Gezerlis *et al.*, 2014).

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The main objective of this paper is to establish the correlation between the Phase Shift,  $\delta_\ell(k_f)$ , and Fermi momentum,  $k_f$ , using the exponential potential in Nuclear Systems.

**Theoretical Formulations**

At a low density limit a general two-body Hamiltonian for an assembly in the ground state can be written as (Dean and Hjorth-Jensen, 2003),

$$\hat{H} = \hat{H}_1 + \hat{H}_2 = \sum_{\alpha} \epsilon_{\alpha} a_{\alpha}^{+} a_{\alpha} + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} a_{\alpha}^{+} a_{\beta}^{+} a_{\delta} a_{\gamma} \quad [1]$$

Where,  $\hat{H}_1$  represents the kinetic energy of the system,  $\hat{H}_2$  represents the potential energy of the system,  $a^{+}$  is the fermion creation operator,  $a$  is the fermion annihilation operator,  $V_{\alpha\beta\gamma\delta}$  represents the coupled matrix elements of the two-body interaction  $V(r)$ . The sums run over all possible single-particle quantum numbers. The pairing gap is determined by the attractive part of the nucleon-nucleon interaction.

Within the meson-exchange models, we may have exchange of  $\pi$ ,  $\rho$ ,  $\omega$ ,  $\sigma$ ,  $\eta$  and  $\delta$  mesons. As an example, for a potential due to the exchange of a  $\pi$ -meson, the co-efficient  $C_{\sigma} = C_T = (g_{\pi NN}^2/4\pi)$  ( $m_{\pi}^3/12M^2$ ); and  $C_c^0 = C_c^1 = C_{SL} = 0$  with experimental value for  $g_{\pi NN}^2 \approx 13-14$ ,  $m_{\pi}$  is the mass of the  $\pi$ -meson,  $M$  is the mass of the nucleon (Machleidt et al., 1996; Stoks et al., 1995).

The complete interaction is written as (Dean and Hjorth-Jensen, 2003),

$$V(r) = \left\{ C_c^0 + C_c^1 + C_{\sigma} \sigma_1 \cdot \sigma_2 + C_T \left[ 1 + \frac{3}{m_{\alpha} r} + \frac{3}{(m_{\alpha} r)^2} \right] S_{12}(\hat{r}) + C_{SL} \left[ \frac{1}{m_{\alpha} r} + \frac{1}{(m_{\alpha} r)^2} \right] L \cdot S \right\} \frac{e^{-m_{\alpha} r}}{m_{\alpha} r} \quad [2]$$

Where,  $m_{\alpha}$  is the mass of the relevant meson and  $S_{12}$  is due to the tensor force term; such that

$$S_{12}(r) = \sigma_1 \cdot \sigma_2 \hat{r}^2 - \sigma_1 \cdot \hat{r} \sigma_2 \cdot \hat{r}$$

Where,  $\sigma$  is the standard operator notation for spin -  $\frac{1}{2}$  particles.

From discussion of the potential due to the exchange of a  $\pi$ -meson, the co-efficient  $C_{\sigma} = C_{SL} = (g_{\pi NN}^2/4\pi)$  ( $m_{\pi}^3/12M^2$ );  $C_c^0 = C_c^1 = C_{SL} = 0$  with experimental value for  $g_{\pi NN}^2 \approx 13-14$ , the interaction potential,  $V(r)$  therefore, reduces to (Dean and Hjorth-Jensen, 2003),

$$V(r) = C_T \left[ 1 + \frac{3}{m_{\alpha} r} + \frac{3}{(m_{\alpha} r)^2} \right] S_{12}(\hat{r}) \frac{e^{-m_{\alpha} r}}{m_{\alpha} r} \quad [3]$$

Other potentials that may be used (Blatt and Weisskoff, 1952) are,

i. Square well potential,  $V(r) = -V_0$ ,

ii. Gaussian well potential,  $V(r) = -V_0 \exp\left(-r^2/\beta^2\right)$

iii. Yukawa well potential,  $V(r) = \frac{-\left[V_0 \exp\left(-r/\beta\right)\right]}{\left(r/\beta\right)}$

iv. Exponential well potential,  $V(r) = -V_0 \exp\left(-2r/\beta\right)$

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Where,  $V(r)$  is the interaction potential,  $r$  is the inter-particle distance,  $\beta$  is the range of nucleon-nucleon force and  $V_0$  is the potential well depth.

Calculations are done to correlate the interaction potential,  $V(r)$ , to the phase shift,  $\delta_\ell(k)$ .

The exponential potential  $V(r) = -V_0 \exp\left(-2r/\beta\right)$  used (Blatt and Weisskoff, 1952) in our derivation of phase shift and energy gap equations, since it is simple to integrate and does not require complex integration methods like the Gaussian well potential and Yukawa well potential and also it has not been explored fully by others.

The Born-approximation expression for phase shifts,  $\delta_\ell(k_f)$ , for scattering from a spherical potential,  $V(r)$ , in 3-D is given by, (Gezerlis et al., 2014)

$$\delta_\ell(k_f) = -\frac{2\mu k_f}{\hbar^2} \int_0^\infty V(r) j_\ell^2(k_f r) r^2 dr \quad [4]$$

Equation [4] is used in calculating the phase shift,  $\delta_\ell(k_f)$ , by using the exponential potential given as

$$V(r) = -V_0 \exp\left(-2r/\beta\right) \text{ and using Bessel Function in the form } j_0^2(k_f r) = \frac{\sin^2(k_f r)}{(k_f r)^2} \text{ for } \ell=0, \text{ ground state, where } k_f r \ll 1. \quad [5]$$

Using derived equations and the values of the constants available, data was then generated and tabulated. Graphs have been drawn to show how the phase shift  $\delta_\ell(k_f)$ , varies with the Fermi momentum,  $k_f$ . This potential is substituted in the Born-approximation phase shifts,  $\delta_\ell(k_f)$ , for scattering from a spherical potential,  $V(r)$ , in 3-D, and to find the values of phase shifts.

The pairing gap for small values of  $k_f |a_0|$  is (Dean and Hjorth-Jensen, 2003; Gorkov and Melik-Barkhudarov, 1961),

$$\Delta(k_f) = \frac{8}{e^2} \lambda \exp\left(-\pi/2k_f |a_0|\right) \quad [6]$$

Where,  $a_0$  is the scattering length in the  $^1S_0$  channel ( $a_0 = -23.7$  fm),  $\lambda$  is a constant  $\approx 1$ ,  $e = 2.718$ . Here,  $a_0$  is related to the interaction potential between a pair of nucleons. However, at saturation density  $\rho_0 = 0.17 \text{ fm}^{-3}$ ,  $k_f = 1.36 \text{ fm}^{-1}$ .

For low energy scattering, especially in nuclear physics, the phase shift,  $\delta_\ell(k_f)$ , due to scattering is given by the relation (Gezerlis et al., 2014; Elgarøy & Hjorth-Jensen, 1997).

$$k_f \cot \delta_\ell(k_f) = -\frac{1}{a_0} + \frac{1}{2} r_0 k_f^2 \quad [7]$$

Here,  $r_0$  is the effective range of the nuclear force which roughly corresponds to the size of the potential and  $\delta_\ell(k_f)$  is the S-wave scattering phase-shift.

At ground state  $\ell = 0$  therefore Equation [4] becomes,

$$\delta_0(k_f) = -\frac{2\mu k_f}{\hbar^2} \int_0^\infty V(r) j_0^2(k_f r) r^2 dr \quad [8]$$

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Here,  $j_0(k_f r)$  is the spherical Bessel function of zeroth order i.e.  $\ell=0$  such that,

$$j_0^2(k_f r) = \frac{\sin^2(k_f r)}{(k_f r)^2} \quad [9]$$

It is valid to represent the interaction energy of a particle with momentum  $k_i < k_f$ , with all the particles within the Fermi surface. The values for  $k_i$  could be  $0.1\text{fm}^{-1}$ ,  $0.2\text{fm}^{-1}$ ,  $0.3\text{fm}^{-1}$ ... (Khanna & Barhai, 1975) and therefore for  $k_f r \ll 1$ , hence  $\sin^2(k_f r) \cong (k_f r)^2$  therefore equation [16] reduces to,

$$j_0^2(k_f r) = 1 \quad [10]$$

Using the exponential potential, Equation [10] and substituting them in Equation [4], we have,

$$\delta_0(k_f) = \frac{2k_f V_0}{\hbar^2} \int_0^\infty e^{-r/\beta} r^2 dr \quad [11]$$

or

$$\delta_0(k_f) = \frac{\mu k_f V_0 \beta^3}{2\hbar^2} \quad [12]$$

Now for exponential potential the values of,  $s$ , is (Blatt and Weisskoff, 1952)

$$s = 0.17291 \frac{M V_0 \beta^2}{\hbar^2} \quad [13]$$

But  $s \cong 1$  for the bound state of the nuclear matter; and hence  $V_0$  is given by,

$$V_0 = 5.7834 \frac{\hbar^2}{M \beta^2} \quad [14]$$

Substituting Equation [14] in Equation [12] we have,

$$\delta_0(k_f) = 2.7834 \frac{\mu k_f \beta}{M} \quad [15]$$

Now in the exponential potential, the range of the potential well,  $V_0$ , is,  $b$ , and,  $\beta$ , are related by the relation,

$$b = 1.7706\beta = 2.15\text{fm} \quad [16]$$

Therefore,  $\beta = 1.2143\text{fm}$ , for  $^1\text{S}_0$  scattering, and the reduced mass,  $\mu$ , of the two interacting nucleons is given by

$$\mu = \frac{m_p \cdot m_n}{m_p + m_n} \quad [17]$$

Where,  $m_p$  is the proton mass equal to  $938.27\text{Mev}$ , and  $m_n$  is the neutron mass equal to  $939.57\text{Mev}$ .

Substituting the values of the masses in Equation [17] we have the reduced mass  $\mu = 469.46\text{Mev}$ . The average mass  $M$  of the proton and neutron is given by the equation,

$$M = \frac{m_p + m_n}{2} \quad [18]$$

Substituting for the values of the masses  $M = 938.92\text{Mev}$ , then Equation [15] becomes

$$\delta_0(k_f) = 1.6899k_f \quad [19]$$

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We will calculate the values of phase shift,  $\delta_0(k_f)$ , for different values of  $k_f$  ranging from  $0.1 \text{ fm}^{-1}$ ,  $0.2 \text{ fm}^{-1}$ ,  $0.3 \text{ fm}^{-1}$  ...  $1.6 \text{ fm}^{-1}$ , and plot a graph of phase shift,  $\delta_0(k_f)$ , against Fermi momentum,  $k_f$ .

For scattering length in the  $^1S_0$  channel,  $a_0 = -23.7 \text{ fm}$ ,  $e = 2.718$  and  $\lambda \approx 1$  Equation [6] for energy gap,  $\Delta(k_f)$ , becomes,

$$\Delta(k_f) = 1.0827 \cdot \exp\left(\frac{-0.0663}{k_f}\right) \quad [20]$$

Equation [20] will give the values of energy gap,  $\Delta(k_f)$ , for different values of the Fermi momentum,  $k_f$ .

**RESULTS AND DISCUSSION**

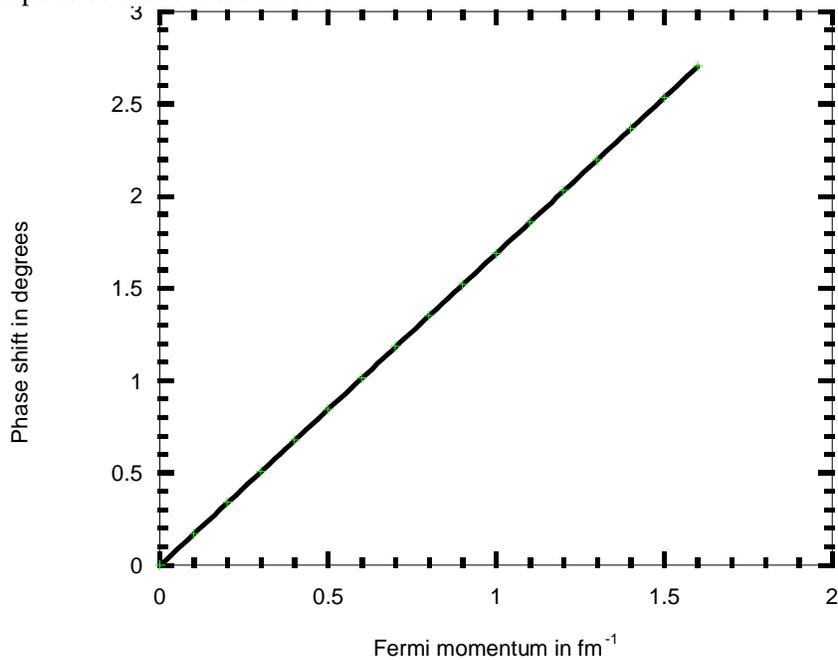
Equation [19] is used to compute the values of phase shift,  $\delta_0(k_f)$ , against changes in the Fermi momentum,  $k_f$ , and this variation, is studied using MathCAD software and data tabulated as shown in table 1.

**Table 1: Values of Fermi Momentum against the Values of Phase Shift**

Fermi Momentum, $k_f$ , in $\text{fm}^{-1}$	Phase Shift, $\delta_0(k_f)$ , in Degrees
0	0
0.1	0.169
0.2	0.338
0.3	0.507
0.4	0.676
0.5	0.845
0.6	1.014
0.7	1.183
0.8	1.352
0.9	1.521
1.0	1.690
1.1	1.859
1.2	2.028
1.3	2.197
1.4	2.366
1.5	2.535
1.6	2.704

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The following graph is obtained from table 1.



**Figure 1: A Graph of Phase Shift,  $\delta_0(k_f)$ , against Fermi Momentum,  $k_f$**

When the Fermi momentum,  $k_f$ , of the interacting nucleons is zero the value of phase shift,  $\delta_0(k_f)$ , is equal to zero in the ground state as seen from figure 1. This shows that an increase in Fermi momentum,  $k_f$ , leads to an increase in phase shift,  $\delta_0(k_f)$ .

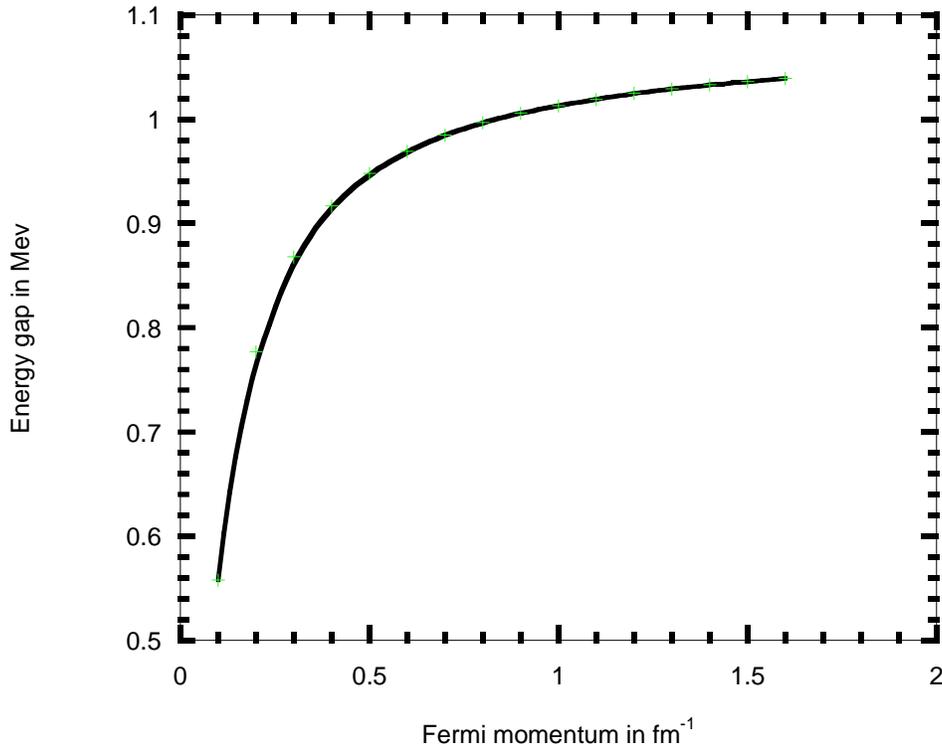
Equation [19] is used to compute the values of energy gap,  $\Delta(k_f)$ , against changes in the Fermi momentum,  $k_f$ , and this variation is studied using MathCAD software and the data tabulated as shown in table 2.

**Table 2: Values of Fermi Momentum and against the Values of Energy Gap**

Fermi Momentum, $k_f$ , in $fm^{-1}$	Energy Gap, $\Delta(k_f)$ , in Mev
0.1	0.558
0.2	0.777
0.3	0.868
0.4	0.917
0.5	0.948
0.6	0.969
0.7	0.985
0.8	0.997
0.9	1.006
1.0	1.013
1.1	1.019
1.2	1.025
1.3	1.029
1.4	1.033
1.5	1.036
1.6	1.039

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The following graph is obtained from table 2.



**Figure 2: A Graph of Energy Gap,  $\Delta(k_f)$ , against Fermi Momentum,  $k_f$**

The energy gap,  $\Delta(k_f)$ , increases steadily and faster for low Fermi momentum,  $k_f$ , upto around  $0.4 fm^{-1}$  and it is more or less constant with a value  $\cong 1.0Mev$ . In literature the values of the energy gap are  $0.2, 0.4, 0.6, 0.8Mev...$  for  $0.2, 0.3, 0.4 fm^{-1}$  respectively which are in agreement.

**Conclusion**

When two nuclear particles or two nucleons collide with each other or scatter, there is what is called a scattering length,  $a_0$ , the distance of the nearest approach. Once the particles are in the field of force of each other, they interact and scatter in any direction resulting in a phase shift or phase difference,  $\delta_\ell(k_f)$ , thus phase shift,  $\delta_\ell(k_f)$ , depends upon the interaction potential,  $V(r)$ . Considering  $^1S_0$  pairing in infinite neutron matter and nuclear matter and the bare nucleon-nucleon exponential interaction in the  $^1S_0$  channel, our calculations shows that the phase shift,  $\delta_\ell(k_f)$ , increases linearly with Fermi momentum,  $k_f$ . The energy gap,  $\Delta(k_f)$ , increases steadily and faster for low Fermi momentum,  $k_f$ , and become constant at higher values. It will be interesting to see if other potentials are used, how phase shift,  $\delta_\ell(k_f)$ , and energy gap,  $\Delta(k_f)$ , varies with Fermi momentum,  $k_f$ .

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