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Research Article

BIANCHI TYPE-III MASSIVE STRINGCOSMOLOGICAL MODEL WITH BULK VISCOSITY UNDER ELECTROMAGNETIC FIELD INGENERAL RELATIVITY

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ABSTRACT
In this paper we have investigated Bulk viscous Bianchi type-III cosmological model viscosity under electromagnetic field in general relativity in perfect fluid. We assume that scalar of expansion is proportional to the shear scalar $\theta \propto \sigma$, the condition leads to $B = C^n$ and $\xi = K_3\theta^l$. Thus, the physical and geometrical aspects of the model are also discussed.

Keywords: *Bianchi Type-III, Viscous Constant, Shearing Non-Rotating, Decelerating Cosmological Constant*

INTRODUCTION

The present day Universe is satisfactorily described by homogeneous and isotropic models given by FRW space-time. The Universe on a smaller scale is neither homogeneous nor isotropic nor do we expect the Universe in its early stages to have had these properties. Homogeneous and anisotropic cosmological models have been widely studied in the frame work of general relativity in the search of a realistic picture of the Universe in its early stages (Kibble, 1976, 1980, Everett, 1981; Vilenkin, 1981; Letelier,1979, 1983; Stachel, 1980; Callum and Ellis, 1970).

Cosmic strings plays important role, as temperature goes below critical temperature, these strings arise because of phase transition after the big bang explosion as studied by various researchers (Kibble 1976, 1980; Zeldovich *et al.*, 1974, 1975; Everett, 1981; Vilenkin, 1981). The cosmic strings effect stress energy and couple to the gravitational field (Zeldovich, 1980).

Various researchers like Letelier, (1979, 1983) and Stachel (1980) studied effect of cosmic strings. Zeldovich (1993), Misner *et al.*, (1973), Asseo and Sol (1987), Pudritz and Silk (1989), Kim *et al.*, (1991), Perley and Taylor (1991), Kron berg (1991), Wolfe *et al.*, (1992), Kulsrud *et al.*, (1997), Barrow (1997), Melvin (1975) and Harrison (1973) studied that magnetic field might have cosmological origin.

The string cosmological models with a magnetic field in Bianchi type I, Bianchi type II, Bianchi type III, Bianchi type VI_0, Bianchi type VIII, Bianchi type IX space-time have been studied by various researchers Banerjee *et al.*, (1990), Bali and Anjali (2006), Bali *et al.*,(2007), Pradhan (2007, 2008, 2009, 2010), Wang (2006), Yadav *et al.*, (2007), Amirhashchi and Zainuddin (2010, 2011), Pradhan *et al.*, (2012).

Recently Bianchi type I massive string and Bianchi type II and Bianchi type V cosmological models have been discussed by Pradhan *et al.*, (2012), Yadav (2012), Yadav *et al.*, (2009, 2011). Many researchers also discussed the effect of bulk viscosity on cosmological models (Misner, 1968; Ellis, 1979; Hu, 1983; Maatens, 1995, Pavon, 1991, Zimdahl, 1996; Santos, 1985; Yadav, 2011; Chakraborty, 2001).

As we are aware bulk viscosity increases with expanding universe. Numerous researchers studied presence of dark matter in Bianchi type-III cosmological model Lorentz (1982).

In this paper, I consider space-time of the Bianchi Type III model in a general form with variable G and Λ. To obtain an explicit solution, I assume that scalar of expansion is proportional to the shear scalar $\theta \propto \sigma$, the condition leads to $B=C^n$ and $\xi = K_3\theta^l$. Thus, the physical and geometrical aspects of the model are also discussed.

Model and Field Equations

We consider the Bianchi type-III metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2\alpha x}dy^2 + C^2dz^2, \qquad (1)$$

Where, A, B and C are the function of cosmic time t alone, and α is a constant.

Einstein's field equations with variables G and Λ in suitable units are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} + \Lambda g_{ij} \qquad (2)$$

Where, we take $G=1$.

The energy momentum tensor for a cloud of string with bulk viscosity is Zeldovich (1980)

$$T_{ij} = \left(\rho + \bar{p}\right) u_i\, u_j - \lambda x_i x_j + \bar{p}\, g_{ij} + E_i^j \quad , \qquad (3)$$

Where $\rho=\rho_p+\lambda$ is the rest energy density of the cloud of strings with particles attached to them, ρ_p is the particle density of the configuration , λ is the string tension density of the cloud of the string, $\theta = u^i_{;i}$ is the scalar of expansion and ξ is the coefficient of bulk viscosity. The u^i is the cloud four velocity vectors and x^i represents a direction of anisotropy, i.e. the direction of string. They satisfy the relations (Wang, 2005).

$$u^i\, u_i = -x^i x_i = -1, \quad u^i\, x_j = 0 \qquad (4)$$

The electromagnetic field of equation (3) and $\bar{p}$ is given as

$$E_i^j = \frac{1}{4\pi}\left[g_{lm}F_{il}F_{jm} - \frac{1}{4}F_{lm}F^{lm}g_{ij}\right] \qquad (5)$$

We assume that magnetic field is in xy-plane, thus, along z-axis current flows. Electromagnetic field tensor F_{ij} only non-vanishing component is F_{12}(Magnetic field depends on space x only). Thus, Maxwell equations states

$$F_{ik,l} + F_{kl,i} + F_{li.k} = 0 \text{ and} \left[F^{ik}\left(-g\right)^{1/2}\right]_{,k} = 0 \qquad (6)$$

Leads to

$$F_{12} = Ke^{-ax} \quad . \quad F_{14}=0 \qquad (7)$$

Where K is constant.

$$\bar{p} = \text{p} - \xi \text{v}^j_{;i} \qquad (8)$$

Where ξ is the coefficient of bulk viscosity and p is isotropic pressure.

The expressions for scalar of expansion θ and shear scalar σ are

$$\theta = u^i_{;\text{i}} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 3H, \qquad (9)$$

The non-vanishing component of shear tensor σ_{ij} defined by $\sigma_{ij} = u_{i;j} + u_{j;i} - \frac{2}{3}g_{ij}u;^k_k$ are obtained as

$$\sigma_{11} = \frac{A^2}{3}\left(\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) \qquad (10)$$

$$\sigma_{22} = \frac{B^2 e^{-2\alpha x}}{3} \left(\frac{2\dot{B}}{B} - \frac{\dot{C}}{C} - \frac{\dot{A}}{A} \right) \quad (11)$$

$$\sigma_{33} = \frac{C^2}{3} \left(\frac{2\dot{C}}{C} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) \quad (12)$$

$$\sigma_{44} = 0 \quad (13)$$

Thus the shear scalar σ is obtained as

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA} \right), \quad (14)$$

Where, H is the Hubble parameter.
The average anisotropy parameter A_m as

$$A_m = \frac{1}{3} \cdot \sum_{i=1}^{3} \left(\frac{\Delta H_i}{H} \right)^2, where \Delta H_i = H_i - H \quad (15)$$

Where i=1--3
For the metric (1) Einstein's field equation can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -8\pi \left(\bar{p} - \lambda \right) + \frac{K^2}{A^2 B^2} \quad (16)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi \bar{p} - \frac{K^2}{A^2 B^2} \quad (17)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi \bar{p} - \frac{K^2}{A^2 B^2} \quad (18)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = 8\pi\rho + \frac{K^2}{A^2 B^2} \quad (19)$$

$$\alpha \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = 0, \quad (20)$$

which leads to

$$A = m B \quad (21)$$

Where m is a constant of integration.

Solution of the Field Equations

There are only five independent equations (16)-(20) in the seven unknowns A, B, C, $p, \varepsilon, \rho and \lambda$, an extra equation is needed to solve the system completely. We assume that scalar of expansion is proportional to the shear scalar $\theta \propto \sigma,$ which leads to a relation between metric potential

$$B = C^n \quad (22)$$

On solving equations. (21) and (22), we get

$$C = \left[\frac{n+1}{n} \right]^{1/n+1} \left[K_1 t + K_2 \right]^{1/n+1} \quad (23)$$

On substituting equation (23) in (21), we get

$$B=\left[\frac{n+1}{n}\right]^{n/n+1}\left[K_1t+K_2\right]^{n/n+1} \quad (24)$$

$$A=m\left[\frac{n+1}{n}\right]^{n/n+1}\left[K_1t+K_2\right]^{n/n+1} \quad (25)$$

By using equation (23), (24) and (25) in the metric (1) we get

$$ds^2=-dt^2+\left[\left[\frac{n+1}{n}\right]^{n/n+1}\left[K_1t+K_2\right]^{n/n+1}\right]^2\left[m^2dx^2+e^{-2\alpha x}dy^2\right]+\left[\left[\frac{n+1}{n}\right]^{1/n+1}\left[K_1t+K_2\right]^{1/n+1}\right]^2dz^2,$$

(26)

By using equation (23), (24) and (25) in (17), we get

$$8\pi\bar{p}=\frac{-K^2}{m^2.\left(\frac{n+1}{n}\right)^{4n/n+1}.\left(K_1t+K_2\right)^{4n/n+1}}+\frac{nK_1^2}{(n+1)^2}.\frac{1}{\left(K_1t+K_2\right)^2} \quad (27)$$

By using equation (23), (24) and (25) in (19), we get

$$8\pi\rho=\frac{-1}{m^2.\left(\frac{n+1}{n}\right)^{2n/n+1}.\left(K_1t+K_2\right)^{2n/n+1}}.\left[\alpha^2+\frac{K^2}{\left(\frac{n+1}{n}\right)^{2n/n+1}.\left(K_1t+K_2\right)^{2n/n+1}}\right]+\frac{n(2+n)K_1^2}{(n+1)^2}.\frac{1}{\left(K_1t+K_2\right)^2}$$

(28)

By using equation (23), (24) and (25) in (16), we get

$$8\pi\lambda=\frac{-1}{m^2.\left(\frac{n+1}{n}\right)^{2n/n+1}.\left(K_1t+K_2\right)^{2n/n+1}}.\left[\alpha^2+\frac{K^2}{\left(\frac{n+1}{n}\right)^{2n/n+1}.\left(K_1t+K_2\right)^{2n/n+1}}\right]-\frac{n(1-n)K_1^2}{(n+1)^2}.\frac{1}{\left(K_1t+K_2\right)^2}$$

(29)

.On solving equations $\rho=\rho_p+\lambda$, we get

$$8\pi\rho_p=\frac{K^2}{m^2.\left(\frac{n+1}{n}\right)^{4n/n+1}.\left(K_1t+K_2\right)^{4n/n+1}}-\frac{3nK_1^2}{(n+1)^2}.\frac{1}{\left(K_1t+K_2\right)^2} \quad (30)$$

By using equation (23), (24) and (25) in (9), we get

$$\theta=\frac{(2n+1)K_1}{n+1}.\frac{1}{\left(K_1t+K_2\right)} \quad (31)$$

We assume

$$\xi=K_3\theta^l \quad (32)$$

Where K_3 and l are constants.

By using equation (27), (31) and (32) in (8), we get

$$8\pi p=\frac{-K^2}{m^2.\left(\frac{n+1}{n}\right)^{4n/n+1}.\left(K_1t+K_2\right)^{4n/n+1}}+\frac{nK_1^2}{(n+1)^2}.\frac{1}{\left(K_1t+K_2\right)^2}+8\pi K_3.\left[\frac{(2n+1)K_1}{n+1}\right]^{l+1}.\frac{1}{\left(K_1t+K_2\right)^{l+1}}$$

(33)

By using equation (31) in (32), we get

$$\xi = K_3.\left[\frac{(2n+1)K_1}{n+1}\right]^l.\frac{1}{\left(K_1t+K_2\right)^l} \quad (34)$$

The volume V of the model is given by

$$V\ V = K\left[\frac{(n+1)}{n}\right]^{2n+1/n+1}.\ \left[K_1t+K_2\right]^{2n+1/n+1}.e^{-\alpha x} \quad (35)$$

$$\frac{\sigma}{\theta}=\frac{(n^2-2n+1)^{1/2}}{(2n+1)(3)^{1/2}} \quad (36)$$

The deceleration parameter is given as

$$q=\frac{n+2}{2n+1} \quad (37)$$

The average anisotropy parameter A_m as

$$A_m=\frac{2\left(3n^2-n+1\right)}{\left(2n+1\right)^2} \quad (38)$$

The model has singularity at

$$t=-\frac{K_2}{K_1}=t_0(say) \quad (39)$$

Conclusion

In summary, we have obtained exact solutions of the field equations for Bianchi type-III massive string cosmological model with bulk viscosity under electromagnetic field in general relativity in perfect fluid. We assume that scalar of expansion is proportional to the shear scalar $\theta \propto \sigma$, the condition leads to $B=C^n$ and $\xi = K_3\theta^l$. Thus, the physical and geometrical aspects of the model are also discussed. When $t=t_0 \to 0$, the spatial volume and scale factor A, B, C is zero and the expansion scalar, $\rho,\sigma,p,\xi,\theta,\lambda,\bar{p},\rho_p$ and Hubble parameter is infinite which implies that the big-bang starts evolving. The density, the coefficient of shear scalar and the bulk viscosity diverges at the initial singularity. Hence, the model has a "point type singularity" at the initial epoch. Thus, rate of expansion slows down with increase in time. When $t=t_0 \to \infty$, the spatial volume and scale factor A, B, C is infinity and the expansion scalar, $\rho,\sigma,p,\xi,\theta,\lambda,\bar{p},\rho_p$ and Hubble parameter is zero thus tend to isotropic. When $t=t_0$ increases then spatial volume also increases. As we know $\lim_{t\to\infty}\frac{\sigma}{\theta}$ = constant, the model does not approach isotropy for large value of t. The model describes a shearing non-rotating continuously expanding universe with a big-bang start. The model is decelerating i.e. q>0 for n<-1/2 and accelerating when q<0 for n>-1/2. The model is accelerating and decelerating due to combined effect of gravitational constant and cosmological constant. Our model is in consistent with above observations made by researchers.

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