International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2016 Vol. 6 (4) October-December, pp. 73-76/Pothanna

Research Article

CONVECTIVE FLOW THROUGH FIXED CIRCULAR PIPE

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ABSTRACT
The convective flow of a thermo-viscous incompressible fluid through fixed circular pipe is considered in this paper. The flow under the action of constant temperature and pressure gradients is assumed. The solutions for the velocity and temperature distributions have been obtained in terms of Modified Bessel functions with appropriate boundary conditions. The Heat transfer coefficient and the Drag force on the boundary have been calculated. The effect of thermal conductivity coefficient and the prandantle number on the Heat transfer coefficient have been discussed and shown in the form of graphs.

Keywords: *Heat Transfer Coefficient, Thermal Conductivity Coefficient, Prandantle Number*

INTRODUCTION
The theory of thermo-viscous fluids which reflect the interaction between thermal and mechanical responses in fluids in motion was first introduced by Koh and Eringin (1963). Nageswara Rao and Pattabhi Ramacharyulu (1979) later studied some steady state problems of certain flows dealing with thermo-viscous fluids. Green and Naghdi (1995) has given a new theory on thermo-viscous fluids. Some more problems of thermo-viscous flows studied by Anuradha (2006) in different flow geometries. Muthuraj and Srinivas (2007) studied the problem of Flow of a Thermo-viscous Fluid through an Annular Tube with Constriction.
According to Koh and Eringin (1963) the stress-tensor 't' and heat flux bivector 'h' are expressed as polynomial functions, viz., the rate of deformation tensor 'd':

$$t = \alpha_1 I + \alpha_3 d + \alpha_5 d^2 + \alpha_6 b^2 + \alpha_8 (db - bd) \text{ and } h = \beta_1 b + \beta_3 (bd + db)$$

with

$$d_{ij} = (u_{i,j} + u_{j,i})/2$$

and thermal by gradient bivector 'b'

$$b_{ij} = \in_{ijk} \theta_k$$

where u_i is the i^{th} component of fluid velocity and θ is the fluid temperature. The constitutive parameters α_i, β_i being polynomials in terms of d and b in which the coefficients depend on fluid density (ρ) and the temperature (θ).
The fluid is called Stokesian fluid if the stress tensor depends on the rate of deformation tensor 'd' and it is called Fourier-heat-conducting fluid when the heat flux bi-vector depends on the temperature gradient, the coefficients α_1 and α_3 may be identified as the fluid pressure and coefficient viscosity coefficient respectively and α_5 as that of cross-viscosity coefficient.
The flow of incompressible thermo-viscous fluids satisfies the usual following equations:
Equation of continuity: $v_{i,i} = 0$

Equation of momentum: $\rho\left[\frac{\partial v_i}{\partial t} + v_k v_{i,k}\right] = \rho F_k + t_{ji,i}$

and the energy equation: $\rho c \dot{\theta} = t_{ij} d_{ij} - q_{i,i} + \rho\gamma$

where F_k, the k^{th} Component of external force per unit mass, c is the Specific heat, γ is thermal energy source per unit mass, q_i is the i^{th} Component of heat flux bi-vector t_{ij} is the components of stress tensor and d_{ij} is the components of rate of deformation tensor.

Mathematical Analysis and Solution

With reference to the cylindrical polar coordinate system (r,θ,z) with the z-axis is along the axis of the pipe and r is the radial distance from the centre of the pipe. The flow is represented by the velocity $(0,0,u(r))$ and the temperature of the fluid $\eta(r)$. This choice of velocity evidently satisfies the continuity equation.

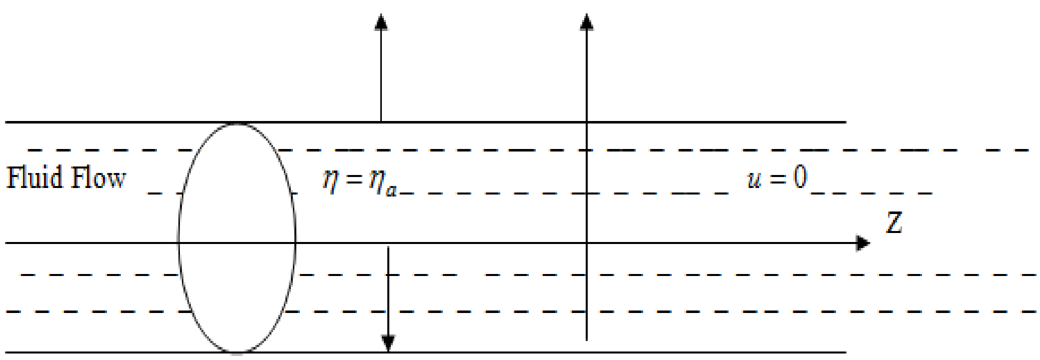

Figure 1: Flow Configuration

The flow is assumed under the action of constant temperature and pressure gradients. It is also assumed that the pipe is fixed and it has the velocity $u = 0$. The leads the basic governing equations under these assumptions reduce to:

The momentum equation in z-direction reduces to

$$-c_1 = \mu\nabla^2 u - \alpha_6 c_2 \nabla^2 \eta \tag{1}$$

and the energy equation reduces to

$$\rho c c_2 u = k\nabla^2 \eta + \beta_3 c_2 \nabla^2 u \tag{2}$$

The equations (1) and (2) represent the coupled equations in u and η.

Here, c_1, c_2 are the constant pressure and temperature gradients, α_6 is the thermo-stress coefficient and β_3 is called thermal conductivity coefficient.

The boundary conditions of the problem for the flow around the moving circular pipe are

$$u(r=0) = finite\,,\; u(r=a) = 0 \tag{3}$$

and

$$\eta(r=0) = finite\,,\; \eta(r=a) = \eta_a \tag{4}$$

The following non-dimensional quantities are introduced to covert the above equations in non-dimensional form:

$$r = aR,\; u = \frac{\mu}{2\rho a}U, \quad \eta = \eta_a T$$

Now, the equations of momentum and energy reduces to

$$\nabla^2 U - m^2 U = -d_1 \tag{5}$$

and

$$\nabla^2 T = m_1(U + b_3) = 0 \tag{6}$$

where $d_1 = \dfrac{1}{1+a_6 b_3 p_r}$, $m^2 = \dfrac{a_6 p_r}{1+a_6 b_3 p_r}$, $m_1 = \dfrac{d_2 p_r}{1+a_6 b_3 p_r}$, $d_2 = \dfrac{\rho c_1 c_2 a^4}{\mu^2 \eta_a}$,

$a_6 = \dfrac{\alpha_6 \rho a^2}{\mu^2} c_2{}^2$, $b_3 = \dfrac{\beta_3}{\rho c a^2}$, $p_r = \dfrac{\mu c}{k}$

The boundary conditions in non-dimensional form are reduced to

$$U(R=0) = finite\,,\ U(R=a) = 0, \quad T(R=0) = finite,\ T(R=1) = 1 \tag{7}$$

Solving the equations (5), (6) and (7), we get the velocity distribution as

$$U(R) = \frac{d_1}{m^2}\left\{1 - \frac{I_0(mR)}{I_0(m)}\right\}$$

and the temperature fields as

$$T(R) = 1 - \frac{d_1 m_1}{m^4}\left\{\frac{I_0(mR)}{I_0(m)} - 1\right\} + \frac{m_1}{4}\left\{\frac{d_1}{m^2} + b_3\right\}(R^2 - 1)$$

Heat Transfer coefficient (Nussult number) on the boundary:

$$\text{Nu} = \frac{\partial T}{\partial R}\Big|_{R=1} = m_1\left\{-\frac{d_1}{m^3}\frac{I_1(m)}{I_0(m)} + \frac{1}{2}\left\{\frac{d_1}{m^2} + b_3\right\}\right\}$$

Drag force on the boundary:

$$Drag\,|_{R=1} = -2\pi\left\{\left[m - \frac{a_6 c_2 \eta_a}{c_1 a^2}\frac{m_1}{m}\right]\frac{d_1}{m^2}\frac{I_1(m)}{I_0(m)} - \frac{a_6 c_2 \eta_a}{2 c_1 a^2} m_1\left[\frac{d_1}{m^2} + b_3\right]\right\}$$

Graphs

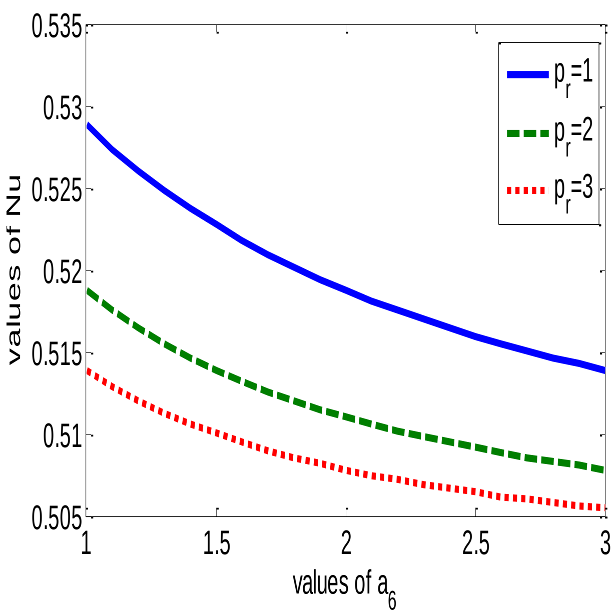

Figure 2: Nu Versus p_r and $b_3 = 1$

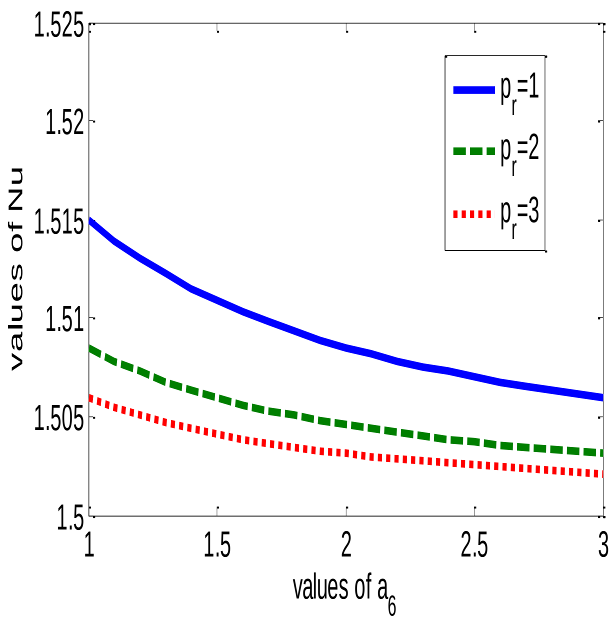

Figure 3: Nu Versus p_r and $b_3 = 3$

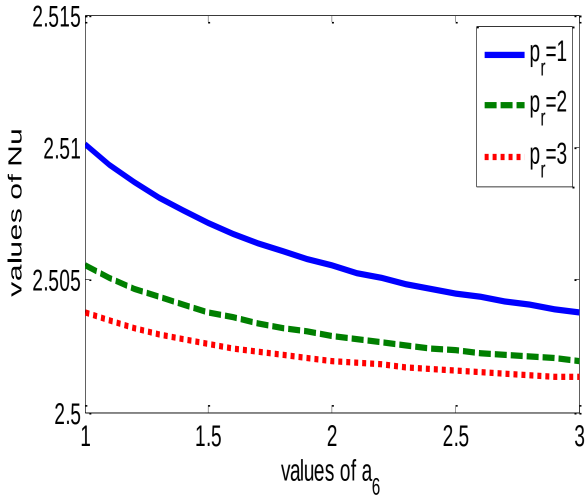

Figure 4: Nu versus p_r and $b_3 = 5$

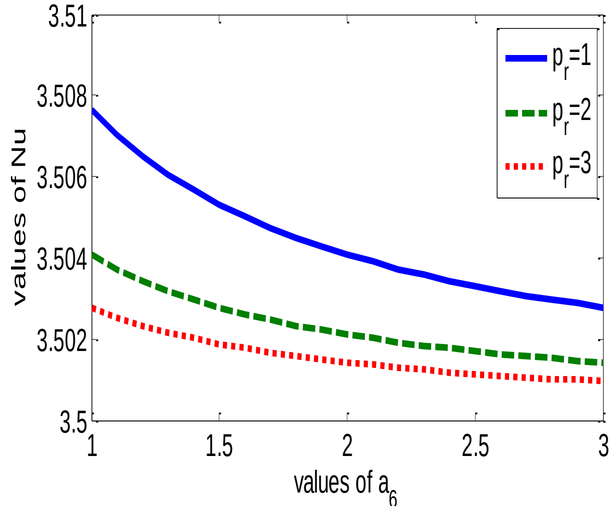

Figure 5: Nu Versus p_r and $b_3 = 7$

Conclusion

The convective steady flow of a thermo-viscous incompressible fluid through fixed circular pipe is considered in this paper. The flow under the action of constant pressure and constant temperature gradients is assumed. The following are the conclusions drawn from the above graphs:

1. As increasing the value of prandantle number, the variations of heat transfer coefficient is decreasing exponentially and the parabolic profiles are realized.
2. It is noticed that, the variations of heat transfer coefficient is increasing with the increasing value of thermo conductivity coefficient.

REFERENCES

Anuradha K (2006). On steady and unsteady flows of thermo-viscous Fluids. Ph. D thesis, J.N.T.U. Hyderabad.

Green AE and Naghdi PM (1995). A new thermo-viscous theory for fluids. *Journal of Non-Newtonian Fluid Mechanics* **56**(3) 89-306.

Koh SL and Eringin AC (1963). On the foundations of non-linear thermo-elastic fluids. *International Journal of Engineering and Science* **1** 199 -229.

Muthuraj R and Srinivas S (2007). Flow of a Thermo-viscous Fluid through an Annular Tube with Constriction. *Defence Science Journal* **57**(5) 653-659.

Nageswara Rao P (1979). Some problems in thermo-viscous fluid Dynamics. Ph. D thesis, K.U. Warangal.

Nageswara Rao P and Pattabhi Ramacharyulu NCH (1979). Steady flow of a thermo-viscous fluid through straight tubes. *Journal of Indian Institute of Science* **61**(B) 89-102.

Nageswara Rao P and Pattabhi Ramacharyulu NCH (1980). A note on steady slow motion of thermo-viscous fluid through a circular tube. *Indian Journal of Pure and Applied Mathematics* **11**(4) 487-491.

Pothanna N, Nageswara Rao P and Pattabhi Ramacharyulu NCH (2015). Flow of slightly thermo-viscous fluid in a porous slab bounded between two permeable parallel plates. *International Journal of Advances in Applied Mathematics and Mechanics* **2**(3) 1 – 9.

Pothanna N, Srinivas J, Nageswara Rao P and Pattabhi Ramacharyulu NCH (2014). Linearization of thermo-viscous fluid in a porous slab bounded between two fixed permeable horizontal parallel plates in the absence of thermo-mechanical interaction coefficient. *International Journal of Modern Trends in Engineering and Research* **1**(5) 412-424.

Pothanna N, Aparna P and Srinivas J (2016). Unsteady Forced Oscillations of a fluid bounded by rigid bottom. *International Journal of Control Theory and Applications* **9**(19) 9049-9054**.**