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Research Article

IONIZING SHOCK WAVE IN A ROTATING AXISYMMETRIC NON-IDEAL GAS WITH HEAT CONDUCTION AND RADIATION HEAT FLUX

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ABSTRACT
The propagation of a cylindrical ionizing shock wave in a rotational axisymmetric non-ideal gas with heat conduction and radiation heat flux, in presence of an azimuthal magnetic field is investigated. The electrical conductivity of the medium behind the shock is assumed to be negligible, which becomes infinitely large due to the passage of the shock. The ambient medium is assumed to have variable axial and azimuthal velocity components and the initial density and the magnetic field are assumed to obey power laws. The thermal conductivity and the absorption coefficients are assumed to vary with temperature and the total energy of the wave to vary with time. Similarity solutions are obtained and the effects of variation of the initial density exponent and the non-idealness parameter on the flow field are investigated.

Keywords: *Ionizing Shock Wave, Non-Ideal Gas, Axisymmetric Flow, Similarity Solution, Heat Transfer Effects*

INTRODUCTION
The experimental studies and astrophysical observations show that the outer atmosphere of the planets rotates due to rotation of the planets. Macroscopic motion with supersonic speed occurs in an interplanetary atmosphere and shock waves are generated. Thus, the rotation of planets or stars significantly affects the process taking place in their outer layers. Therefore, the questions connected with the explosions in the rotating gas atmospheres are of definite astrophysical interest. Shock wave often arises in nature because of a balance between wave breaking non-linear and wave damping disspative forces (Zel'dovich Yab, Raizer YwP). Chaturani (1970) studied the propagation of cylindrical shock waves through a gas having solid body rotation and obtained the solution by a similarity method introduced by Sakurai (1956). Nath *et al.*, (1999) obtained the similarity solutions for the flow behind a spherical shock wave propagating in a non-uniform rotating interplanetary atmosphere with increasing energy.

Vishwarkarma and Vishwakarma (2007) and Vishwakarma *et al.*, (2007) obtained similarity solutions for magnetogasdynamics cylindrical shock waves propagating in a rotating medium. They have taken the electrical conductivity of the initial medium as well as the medium behind the shock to be infinite. But, in many practical cases the medium may be of low conductivity which becomes highly conducting due to passage of shock. Such a shock wave is called a gas-ionizing shock or simply ionizing shock. The propagation of an ionizing shock has been studied by various authors namely Greenspan (1962), Greigfinger and Cole (1962), Christer (1972), Rangarao and Ramana (1973), Singh (1983) for non-rotating medium. Also, the medium in which the shock is propagating is considered to be ideal gas. But at extreme conditions that prevail in most of the problems associated with shock waves, the assumption that the gas is ideal is no longer valid. In recent years, several studies have been performed concerning the problem of shock waves in non-ideal gas. Anisimov and Spiner (1972), investigated the problem of a point explosion in a non-ideal gas by taking the equation of state in a simplified form. Roberts and Wu (1996, 2003) studied the structure and stability of a spherical implosion by assuming the gas to obey a simplified van der Waals equation of state. Vishwakarma and Singh (2012) investigated the cylindrical ionizing shock wave to a non-ideal gas in presence of radiation heat flux considering the van der Waals equation of state.

Marshak (1958) studied the effects of radiation on the shock propagation by introducing the radiation diffusion approximation. Using the same mode of radiation, Elliott (1960) discussed the conditions leading to self-similarity with a specified functional form of the mean free path of radiation and obtained a solution for self- similar spherical explosions. Gretler and Wehle (1993) studied the propagation of blast waves with exponential heat release by taking internal heat conduction and thermal radiation in a detonating medium. Also, Abdel-Raouf and Gretler (1993) obtained the non-self-similar solution for the blast waves with internal heat transfer effects. Ghoniem *et al.,* (1982) obtained a self-similar solution for spherical explosions taking into account the effects of both conduction and radiation in the two limits of Rosseland radiative diffusion and Plank radiative emission. In these works, where both radiation and heat conduction are considered, the density of the ambient medium is taken to be constant. Also, the effects of presence of the magnetic field are omitted.

Keeping in view of the above, in this paper we are considering the propagation of a gas ionizing shock wave in a rotating axisymmetric gas with heat conduction and radiation heat flux, in presence of an azimuthal magnetic field. The initial density of the gas and the initial azimuthal magnetic field are assumed to vary as some power of distance. The heat transfer fluxes are expressed in terms of Fourier's law for heat conduction and a diffusion radiation mode for an optically thick grey gas, which is typical of large scale explosions. The thermal conductivity and absorption co-efficient of the gas are assumed to be proportional to appropriate powers of temperature and density (Ghoneim *et al.*, 1982). Similarity, solutions are obtained and the effects of variation of the initial density exponent and the non-idealness parameter on the flow field are investigated.

Equations of Motion and Boundary Conditions

The fundamental equations governing the unsteady adiabatic axisymmetric rotational flow of a perfectly conducting non-ideal gas in which heat conduction and radiation heat flux are taken into account in presence of an azimuthal magnetic field , may be expressed as (Christer and Helliwell, 1969; Gretler and Whele, 1993; Levin and Skopina, 2004; Singh and Nath, 2013)

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left(\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right) - \frac{v^2}{r} = 0 \tag{2}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} = 0 \tag{3}$$

$$\frac{\partial \upsilon}{\partial t} + u\frac{\partial v}{\partial r} + \frac{uv}{r} = 0 \tag{4}$$

$$\frac{\partial w}{\partial r} + u\frac{\partial w}{\partial r} = 0 \tag{5}$$

$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial r} - \frac{p}{\rho^2}\left[\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right] + \frac{1}{\rho r}\frac{\partial}{\partial r}\left(Fr\right) = 0 \tag{6}$$

Where, r and t are independent space and time co-ordinates, respectively, ρ the density, p the pressure, h the azimuthal magnetic field, u, v and w are the radial, azimuthal and axial components of the fluid velocity $\vec{q}$ in the cylindrical coordinates (r, θ, z), μ is the magnetic permeability, F is the heat flux and e is the internal energy per unit mass.

The total heat flux F, which appears in the energy equation may be decomposed as

$$F = F_C + F_R \tag{7}$$

Where, F_C is the conduction heat flux and F_R is the radiation heat flux.

According to the Fourier's law of heat conduction $F_C = -K\frac{\partial T}{\partial r}$ (8)

Where, K is the coefficient of thermal conductivity of the gas and T is the absolute temperature.

Assuming local thermo dynamical equilibrium and using the radiative diffusion model for an optically thick grey gas (Pomraning, 1973) the term F_R, which represents the radiative heat flux, may be obtained from the differential approximation of the radiation-transport equation in the diffusion limit as

$$F_R = -\frac{4}{3}\left(\frac{\sigma}{\alpha_R}\right)\frac{\partial T^4}{\partial r}, \tag{9}$$

Where, σ is the Stefan-Boltzman constant and α_R is the Rosseland mean absorption co-efficient.

Also, $v = Ar$ (10)

Where, A is the angular velocity of the medium at a radial distance r from the axis of symmetry. In this case the vorticity vector $\vec{\zeta} = \frac{1}{2}\text{curl}\,\vec{q}$ has the components

$$\zeta_r = 0, \quad \zeta_\theta = -\frac{1}{2}\frac{\partial w}{\partial r}, \; \zeta_z = \frac{1}{2r}\frac{\partial}{\partial r}(r\,v). \tag{11}$$

The above system of equations should be supplemented with an equation of state. To discover how deviation from the ideal gas can affect the solutions, we adopt a simple model. We assume that the gas obeys a simplified van der Waals equation of state of the form (Roberts and Wu, 1996, 2003; Nath, 2007; Singh and Nath, 2011)

$$p = \frac{\Gamma\rho T}{1-b\rho}, \; e = C_V T = \frac{p(1-b\rho)}{\rho(\gamma-1)} \tag{12}$$

Where, Γ is the gas constant, $C_V = \frac{\Gamma}{\gamma-1}$ is the specific heat at constant volume and γ is the ratio of specific heats. The constant b is the van der Waals excluded volume; it places a limit $\rho_{max} = \frac{1}{b}$, on the density of the gas.

The thermal conductivity K and the absorption coefficient α_R are assumed to vary with temperature and density. These can be written in the form of power laws, namely (Ghonien *et al.*, 1982).

$$K = K_0\left(\frac{T}{T_0}\right)^{\beta_C}\left(\frac{\rho}{\rho_0}\right)^{\delta_C}, \; \alpha_R = \alpha_{R_0}\left(\frac{T}{T_0}\right)^{\beta_R}\left(\frac{\rho}{\rho_0}\right)^{\delta_R} \tag{13}$$

Where, subscript '0' denotes a reference state. The exponents in above equations should satisfy the similarity requirements if a self-similar solution is sought.

We assume that a cylindrical shock wave is propagating outwards from the axis of symmetry in a rotating non-ideal gas with variable density which has a zero radial velocity, a variable azimuthal and axial velocity and negligible electrical conductivity in presence of an azimuthal magnetic field. The flow variables immediately ahead of the shock front are

$u_1 = 0$, $\rho_1 = DR^d$, $h = h_0 R^m$, $m < 0$, $v_1 = BR^\lambda$, $w_1 = GR^a$, $F_1 = 0$ (Laumbach and Probstein, 1970),

$$p_1 = \frac{B^2 D R^{2\lambda+d}}{2\lambda+d} - \frac{(m+1)\mu h_0^2 R^{2m}}{2m}, \text{ where } 2\lambda + d = 2m, \; m \neq 0 \tag{14}$$

Where, R is the shock radius, a, d, m, h_0, D, B, G are constants.

From equations (10) and (14), we find the initial angular velocity varies as $A_1 = BR^{\lambda-1}$, it decreases as the distance from the axis increases if $\lambda < 1$

Ahead of the shock, the components of the vorticity vector, therefore vary as

$$\zeta_{r_1} = 0, \quad \zeta_{\theta_1} = -\frac{GaR^{a-1}}{2}, \quad \zeta_{z_1} = \frac{B(\lambda+1)R^{\lambda-1}}{2}.$$

Due to passage of the shock, the gas is highly ionized and its electrical conductivity becomes infinitely large. The conditions across such a gas ionizing shock are (Singh & Srivastava, 1991; Viswakarma & Pandey, 2006)

$$\left.\begin{aligned} &\rho_2(V-u_2)=\rho_1 V=m_s \\ &p_2-p_1=m_s u_2 \\ &e_2+\frac{p_2}{\rho_2}+\frac{1}{2}(V-u_2)^2-\frac{F_2}{\rho_1 v}=e_1+\frac{p_1}{\rho_1}+\frac{1}{2}V^2 \\ &h_1=h_2,\ v_1=v_2, w_1=w_2, T_1=T_2 \end{aligned}\right\} \qquad (15)$$

where, the subscripts '1' and '2' refer to the values just ahead and just behind the shock respectively and V denotes the shock velocity.

From equations (15) we get

$$\left.\begin{aligned} &\rho_2=\frac{\rho_1}{\beta} \\ &u_2=(1-\beta)V \\ &p_2=\left[\frac{1}{\gamma M^2}+(1-\beta)\right]\rho_1 V^2 \\ &F_2=\frac{\rho_1 V^3(\gamma+1)}{2(\gamma-1)}\left[\frac{2\beta}{\gamma+1}\left(\bar{b}+\frac{1}{M^2}+\gamma\right)-\beta^2-\frac{2\bar{b}}{\gamma+1}-\frac{2}{M^2(\gamma+1)}-\frac{\gamma-1}{\gamma+1}\right] \end{aligned}\right\} \qquad (16)$$

where $\beta=\bar{b}+\dfrac{1}{\gamma M^2}$, $\beta(0<\beta<1)$ and $\bar{b}=\rho_1 b$ is the parameter of non-idealness of the gas. Here, M is the shock-Mach number referred to the frozen speed of sound $\left(\dfrac{\gamma p_1}{\rho_1}\right)^{1/2}$

Following Levin and Skopina (2004) and Nath (2011) we obtain the jump conditions for the components of the vorticity vector across the shock as

$$\zeta_{\theta_2}=\frac{1}{\beta}\zeta_{\theta_1}, \qquad \zeta_{z_2}=\frac{1}{\beta}\zeta_{z_1}$$

The total energy of the flow field behind the shock is not constant, but assumed to be time dependent and varying as (Rogers, 1958; Freeman, 1968).

$$E=E_0 t^s \qquad s\geq 0$$

where, E_0 and s are constants. The positive values of $'s'$ correspond to the class in which the total energy increases with time. This increase can be achieved by the pressure exerted on the fluid by an inner expanding surface (a contact surface or a piston).

Similarity Solutions

We introduce the following similarity transformations to reduce the equations of motion into ordinary differential equations

$$u=VU(\eta),\ v=V\phi(\eta),\ w=VW(\eta),\ \rho=\rho_1 g(\eta),\ p=\rho_1 V^2 P(\eta),\ F=\rho_1 V^3 Q(\eta),\ \sqrt{\mu}h=\sqrt{\rho_1}VH(\eta) \qquad (17)$$

Where, U, ϕ, W, g, P, H, Q are functions of the non-dimensional parameter η only and

$$\eta=\frac{r}{R},\quad R=R(t)$$

Evidently $\eta=\eta_p=\dfrac{r_p}{R}$ at the inner expanding surface and at the shock front $\eta=1$

The total energy of the gas behind the shock is given by

$$E = 2\pi\int_{r_p}^{R}\left[\frac{1}{2}\rho\left(u^2+v^2+w^2\right)+\frac{p\left(1-b\rho\right)}{\gamma-1}+\frac{\mu h^2}{2}\right]rdr = E_0 t^s \tag{18}$$

Where, r_p is the radius of the inner expanding surface. Applying similarity transformations (17), the relation (18) becomes

$$\frac{E_0 t^s}{2\pi J} = \rho_1 R^2 v^2 . \tag{19}$$

where $J = 2\pi\int_{\eta_p}^{1}\left[\frac{1}{2}g\left(U^2+\phi^2+W^2\right)+\frac{P\left(1-\bar{b}g\right)}{\gamma-1}+\frac{H^2}{2}\right]\eta d\eta$

For similarity transformations, the shock-Mach number and the Alfven-Mach number M_A must be constant. Therefore we have

$$2m = 2\omega + d, \tag{20}$$

$$V = LR^{\omega} \tag{21}$$

Where, L is a constant.
From equation (21) we get

$$R = \left\{\left(1-\omega\right)L\right\}^{1/(1-\omega)} t^{1/(1-\omega)} \tag{22}$$

Also from equation (19) we get

$$R = \left(\frac{E_0}{2\pi J}\right)^{\frac{1}{2\omega+d+2}}\left(\frac{1}{L^2 D}\right)^{\frac{1}{2\omega+d+2}} t^{\frac{s}{2\omega+d+2}} \tag{23}$$

Comparing equations (22) and (23) and using relation (20) we obtain

$$s = \frac{2\left(1+m\right)}{1-\omega}$$

This show that, for $s > 0,\ \omega < 1,\ -1 < m < 0$

From equation $\frac{B}{L} = \sqrt{\frac{2m}{\gamma M^2}+\frac{1+m}{{M_A}^2}}$

Where, $M_A = \frac{V}{\sqrt{\frac{\mu {h_1}^2}{\rho_1}}}$ is the Alfven-Mach number

Then the shock conditions (17) are transformed into

$$U(1) = (1-\beta),\ g(1) = \frac{1}{\beta},\ \phi(1) = \sqrt{\frac{2m}{\gamma M^2}+\frac{1+m}{{M_A}^2}},\ W(1) = \frac{G}{L},\ H(1) = {M_A}^{-1},\ \ P(1) = \frac{1}{\gamma M^2}+(1-\beta),$$

$$Q\left(1\right) = \frac{\left(\gamma+1\right)}{2\left(\gamma-1\right)}\left[\frac{2\beta}{\gamma+1}\left(\bar{b}+\frac{1}{M^2}+\gamma\right)-\beta^2-\frac{2\bar{b}}{\gamma+1}-\frac{2}{M^2\left(\gamma+1\right)}-\frac{\gamma-1}{\gamma+1}\right] \tag{24}$$

Where, $a = \omega$ is taken in order to make the shock conditions consistent for similarity solutions.
In addition to the above mentioned shock conditions, the condition to be satisfied at the inner boundary surface is that the velocity of the fluid is equal to the velocity of the inner boundary itself. This kinematic condition can be written as

$$U\left(\eta_p\right) = \eta_p$$

Using transformations (17), the equations of motion (1) to (6) takes the form

$$(U-\eta)\frac{dg}{d\eta}+g\frac{dU}{d\eta}+g\left(d+\frac{U}{\eta}\right)=0 \tag{25}$$

$$(U-\eta)\frac{dU}{d\eta}+\frac{1}{g}\frac{dP}{d\eta}+\frac{H}{g}\frac{dH}{d\eta}+bU+\frac{H^2}{\eta g}-\frac{\phi^2}{\eta}=0 \tag{26}$$

$$(U-\eta)\frac{dH}{d\eta}+H\frac{dU}{d\eta}+\left(\frac{d+2\omega}{2}\right)H=0 \tag{27}$$

$$(U-\eta)\frac{d\phi}{d\eta}+\frac{\phi U}{\eta}+b\phi=0 \tag{28}$$

$$(U-\eta)\frac{dW}{d\eta}+bW=0 \tag{29}$$

$$(U-\eta)\frac{dP}{d\eta}-(U-\eta)\frac{\gamma P}{g\left(1-\bar{b}g\right)}\frac{dg}{d\eta}+\left(\frac{(\gamma-1)}{1-\bar{b}g}\right)\frac{dQ}{d\eta}+\rho\left[d+2\omega-\frac{\gamma d}{1-\bar{b}g}\right]+\frac{Q(\gamma-1)}{\eta\left(1-\bar{b}g\right)}=0 \tag{30}$$

Using equations (8), (9) and (17) in (7), we get

$$F=\left[\frac{-K_0}{T_0^{\beta_c}\rho_0^{\delta_c}}T^{\beta_c}\rho^{\delta_c}+\frac{16\sigma}{3\alpha_{R_0}}T_0^{\beta_R}\rho_0^{\delta_c}T^{3-\beta_R}\rho^{-\delta_R}\right]\frac{\partial T}{\partial r} \tag{31}$$

Using equations (13) and (17) in the equation (31) we get

$$Q=-\left[\begin{array}{l}\dfrac{K_0 g^{\delta_c-\beta_c}\left\{P\left(1-\bar{b}g\right)\right\}^{\beta_c}D^{\frac{d}{\omega}(\delta_c-1)}}{T_0^{\beta_c}\rho_0^{\delta_c}\Gamma^{\beta_c+1}L^{\frac{1}{\omega}\{d(\delta_c-1)+1\}}}V^{2\beta_c-1+\frac{d}{\omega}(\delta_c-1)}+\\ \dfrac{16T_0^{\beta_c}\rho_0^{\delta_c}g^{\beta_R-\delta_R-3}\left\{P\left(1-\bar{b}g\right)\right\}^{3-\beta_R}D^{-\frac{d\delta_R}{\omega}}}{3\alpha_{R_0}\Gamma^{4-\beta_R}L^{\frac{1}{\omega}(1-d\delta_R)}}V^{5-2\beta_R-\frac{1}{\omega}(\delta_R d+1)}\end{array}\right]\times\left[\frac{\left(1-\bar{b}g\right)}{g}\frac{dP}{d\eta}-\frac{P}{g^2}\frac{dg}{d\eta}\right] \tag{32}$$

which shows that the similarity solution of the present problem exists only when

$$\beta_c=\frac{1}{2}-\frac{d}{2\omega}(\delta_c-1), \qquad \beta_R=5-\frac{1}{\omega}(\delta_R d+1)$$

Therefore, equation (32) becomes

$$Q=-X\left[\frac{1-\bar{b}g}{g}\frac{dp}{d\eta}-\frac{p}{g^2}\frac{dg}{d\eta}\right] \tag{33}$$

Where, $X=\Gamma_c g^{\delta_c-\frac{1}{2}+\frac{d}{2\omega}(\delta_c-1)}\left[p\left(1-\bar{b}g\right)\right]^{\frac{1}{2}-\frac{d}{2\omega}(\delta_c-1)}+\Gamma_R g^{2-\delta_R\left(\frac{d}{\omega}+1\right)-\frac{1}{\omega}}\left[p\left(1-\bar{b}g\right)\right]^{\frac{1}{\omega}(\delta_R d+1)-2}$

Here, Γ_C and Γ_R are the conductive and radiative non-dimensional heat transfer parameters respectively. The parameters Γ_C and Γ_R depend on the thermal conductivity K and the mean free path of the radiation $1/\alpha_R$ respectively and also on the exponents d and ω and are given by

$$\Gamma_C=\frac{K_0 D^{(\delta_c-1)\frac{d}{\omega}}L^{-\frac{1}{\omega}[d(\delta_c-1)+1]}}{T_0^{\frac{1}{2}-\frac{d}{2\omega}(\delta_c-1)}\rho_0^{\delta_c}\Gamma^{\frac{3}{2}-\frac{d}{2\omega}(\delta_c-1)}}$$

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$$\Gamma_R = \frac{16T_0^{5-\frac{1}{\omega}(\delta_R d+1)} \rho_0^{\delta_R} D^{-\frac{d\delta_R}{\omega}} L^{-\frac{1}{\omega}[1-\delta_R d]}}{3\alpha_{R_0} T^{\frac{1}{\omega}(\delta_R d+1)-1}}$$

By solving equations (25) to (30) and (33) we have

$$\frac{dg}{d\eta} = -\left(\frac{g}{U-\eta}\right)\left[\frac{dU}{d\eta} + \left(d + \frac{U}{\eta}\right)\right] \tag{34}$$

$$\frac{dH}{d\eta} = -\left(\frac{H}{U-\eta}\right)\left[\frac{dU}{d\eta} + \left(\frac{d+2b}{2}\right)\right] \tag{35}$$

$$\frac{d\phi}{d\eta} = -\left(\frac{\phi}{U-\eta}\right)\left[b + \frac{U}{\eta}\right] \tag{36}$$

$$\frac{dw}{d\eta} = -\left(\frac{bw}{U-\eta}\right) \tag{37}$$

$$\frac{dp}{d\eta} = \left\{\frac{H^2 - g(U-\eta)^2}{(U-\eta)}\right\}\frac{dU}{d\eta} + \frac{H^2}{(U-\eta)}\left(\frac{d+2\omega}{2}\right) - bUg - \frac{H^2}{\eta} + \frac{\phi^2 g}{\eta} \tag{38}$$

$$\frac{dQ}{d\eta} = -\left(\frac{1-\bar{b}g}{\gamma-1}\right)\left[\begin{array}{l}\left\{\frac{\rho\gamma H}{(1-\bar{b}g)g}\right\} + \left\{H^2 - g(U-\eta)^2\right\}\frac{dU}{d\eta} + H^2\left\{\frac{d+2\omega}{2} - \frac{U-\eta}{\eta}\right\} - bUg(U-\eta) \\ -\frac{\gamma PH}{(1-\bar{b}g)g}\left(\frac{d+2\omega}{2}\right) + \frac{\phi^2 g(U-\eta)}{\eta} + P\left\{(d+2\omega) - \frac{\gamma d}{1-\bar{b}g}\right\} + \frac{Q(\gamma-1)}{\eta(1-\bar{b}g)}\end{array}\right] \tag{39}$$

$$\frac{dU}{d\eta} = \frac{g(U-\eta)}{P - \left[H^2 - g(U-\eta)^2\right](1-\bar{b}g)}\left[\begin{array}{l}\frac{H^2(1-\bar{b}g)}{g(U-\eta)}\left\{\frac{d+2\omega}{2} - \frac{(U-\eta)}{\eta}\right\} - bU(1-\bar{b}g) + \\ \frac{\phi^2}{\eta}(1-\bar{b}g) - \frac{P}{g(U-\eta)}\left(d + \frac{U}{\eta}\right) + \frac{Q}{X}\end{array}\right] \tag{40}$$

Also applying the similarity transformations on equation (11), we obtain the non dimensional components of the vorticity vectors $l_r = \frac{\zeta_r}{\frac{V}{R}}$, $l_\theta = \frac{\zeta_\theta}{\frac{V}{R}}$, $l_z = \frac{\zeta_z}{\frac{V}{R}}$, in the flow field behind the shock as

$$l_r = 0, \quad l_\theta = \frac{aW}{2(U-\eta)}, \; l_z = \frac{\phi}{2}\left[\frac{1}{\eta} - \frac{d + \frac{U}{\eta}}{U-\eta}\right]$$

Now the ordinary differential equations (34) to (40) may be integrated numerically with the boundary conditions (24) and appropriate values of the constant parameters to obtain the solutions. For exhibiting the numerical solution it is convenient to write the flow variables in the following form as

$$\frac{u}{u_2} = \frac{U(\eta)}{U(1)}, \; \frac{v}{v_2} = \frac{\phi(\eta)}{\phi(1)}, \; \frac{w}{w_2} = \frac{W(\eta)}{W(1)}, \; \frac{\rho}{\rho_2} = \frac{g(\eta)}{g(1)}, \; \frac{p}{p_2} = \frac{P(\eta)}{P(1)}, \; \frac{h}{h_2} = \frac{H(\eta)}{H(1)}, \; \frac{F}{F_2} = \frac{Q(\eta)}{Q(1)},$$

RESULTS AND DISCUSSION

In order to get physical insight into the problem considered the non-dimensional flow variables $\frac{u}{u_2}$, $\frac{v}{v_2}$, $\frac{w}{w_2}$, $\frac{\rho}{\rho_2}$, $\frac{h}{h_2}$, $\frac{p}{p_2}$, $\frac{F}{F_2}$ are obtained by integrating the equations (34) to (40) numerically with the boundary conditions (24).

Throughout our discussion, the values of the constant parameters are taken as $\gamma = \frac{5}{3}$; $M^{-2} = 0.1$; $M_A^{-2} = 0.05, 0.1$; $\frac{G}{Q} = 0.1$; $m = -0.1$; $d = -0.5, -1.0$; $\delta_C = 1$; $\delta_R = 2$; $\Gamma_C = 10$; and $\bar{b} = 0, 0.05, 0.1$. For a fully ionized gas $\gamma = \frac{5}{3}$, and therefore it is applicable to stellar medium.

The value $\bar{b} = 0$, corresponds to the case of perfect gas. The results obtained are discussed thoroughly in graphs (Figure 1 to 9) and in tabular form.

Figures 1 to 7 show the variation of the flow variables for different values of the parameters and figure 8 and 9 show the variation of non-zero and non-dimensional azimuthal and axial components l_θ and l_z of the vorticity vector.

Figure 1 and 5 shows that as we move towards the inner expanding surface, the non-dimensional radial velocity and the magnetic field increases for the case when $d = -0.5$ and decreases when $d = -1$. It can be observed from figures 2, 3 and 7 that the non-dimensional azimuthal velocity, the axial velocity and the total heat flux decreases as we move inward from the shock front.

Figure 4 shows that the non-dimensional density decreases slightly but increases rapidly near the inner expanding surface.

Figure 6 shows that the non-dimensional pressure increases for the case when $d = -1$ whereas decreases rapidly near the inner expanding surface for the case when $d = -0.5$. Figures 8 and 9 show that the profiles of the non-dimensional vorticity vector increases near the inner expanding surface except for the case when $d = -1$ where l_z exhibits no visible change.

Table 1 shows the variation of η_p for different values of M_A^{-2}, d and $\bar{b}$. Table 2 shows that the variation of the density ratio β for different values of $\bar{b}$.

The Effects of an Increase in the Value of the Non-Idealness Parameter are

(i) to increase the density ratio β i.e. to decrease the shock strength. (See table 2)

(ii) to increase the distance of the inner expanding surface from the shock front. Thus, the non-idealness of the gas has a decaying effect on the shock strength. (See table 1)

(iii) to decrease the reduced radial velocity, pressure, magnetic field where as to increase the axial and azimuthal velocity component and density. To decrease the non-dimensional reduced heat flux for the case when $d = -0.5$ and to increase when $d = -1.2$.

(iv) To decrease the non-dimensional reduced axial component of vorticity vector in the case when $d = -0.5$

The Effects of Increase in the Index for Initial Density are

(i) to increase the distance of the inner expanding surface from the shock front i.e. the gas behind the shock is less compressed. (See table 2)

(ii) to decrease the non-dimensional reduced radial velocity, azimuthal velocity, magnetic field, density, total heat flux where as to increase the non-dimensional reduced axial velocity and pressure.

(iii) to decrease the reduced axial component of the vorticity vector l_z and to increase the reduced azimuthal component of the vorticity vector l_θ.

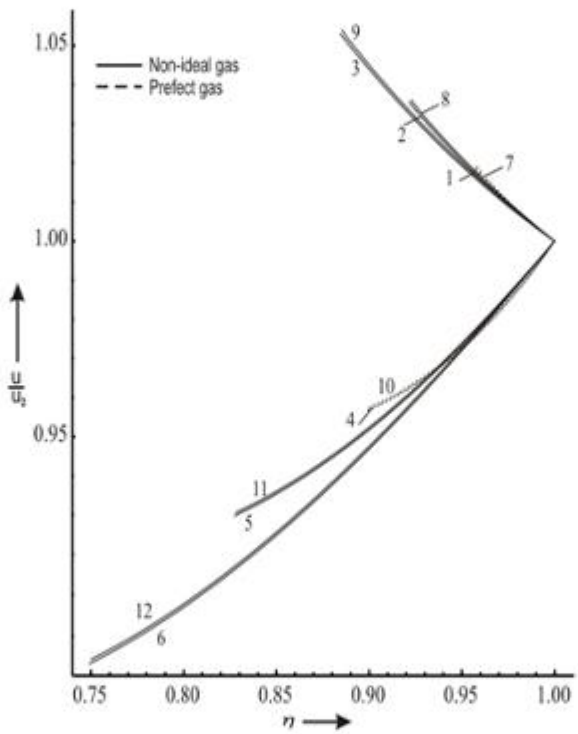

Figure 1: Variation of the Reduced Radial Component of Fluid Velocity in the Flow Field behind the Shock

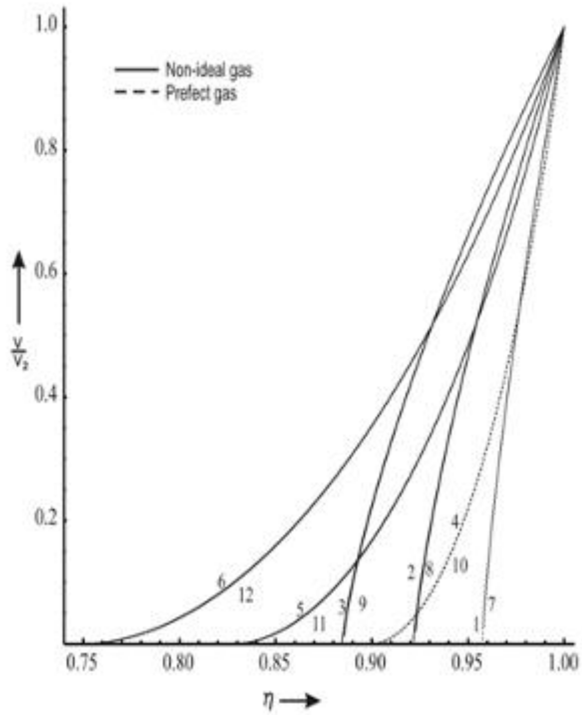

Figure 2: Variation of Reduced Azimuthal Component of Fluid Velocity in the Flow Field behind the Shock

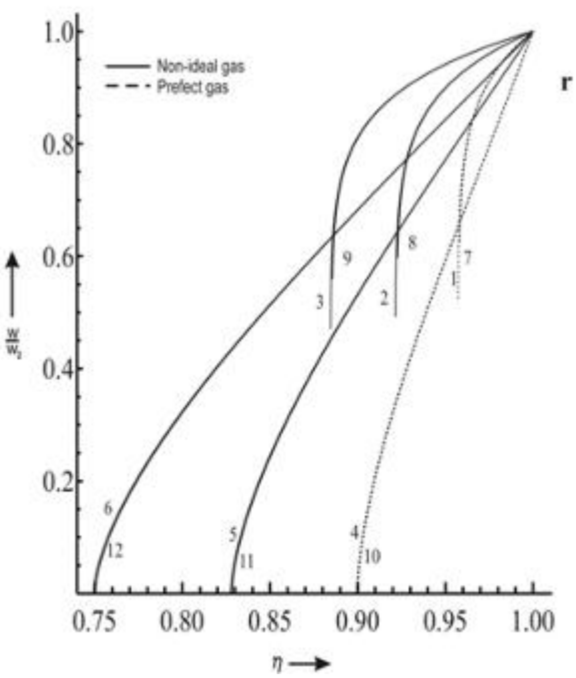

Figure 3: Variation of the Reduced Axial Component of Fluid Velocity in the Flow Field behind the Shock

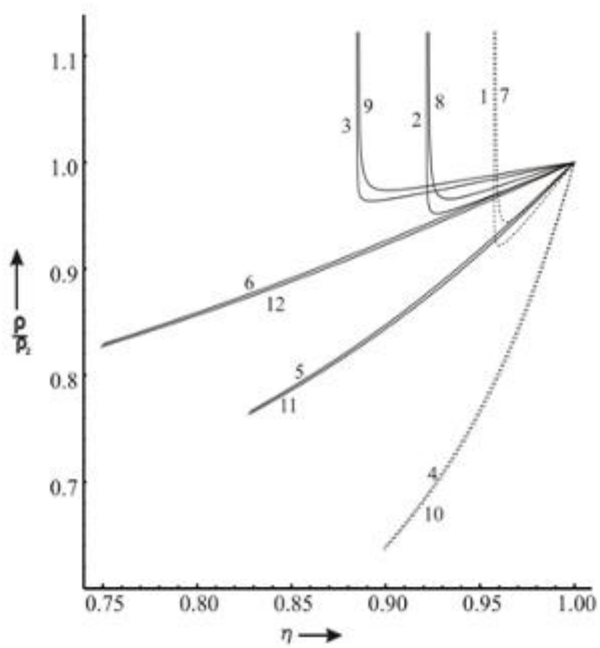

Figure 4: Variation of the Reduced Density in Flow Field behind the Shock

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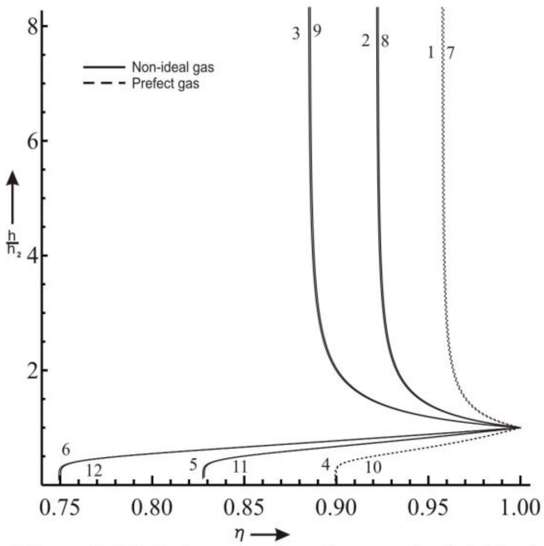

Figure 5. Variation of reduced magnetic field in the flow field behind the shock

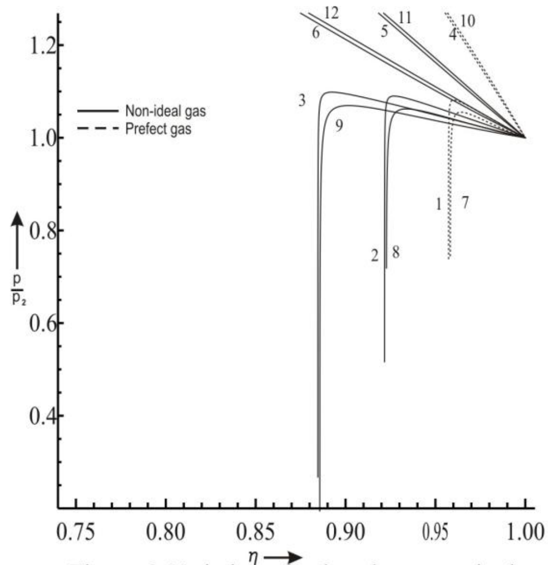

Figure 6. Variation of reduced pressure in the flow field behind the shock

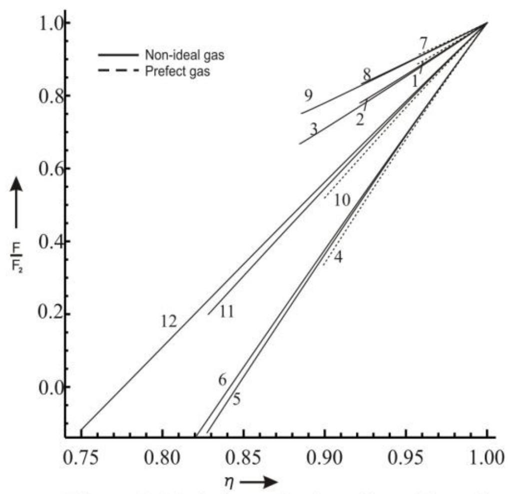

Figure 7. Variation of reduced total heat flux in the flow field behind the shock

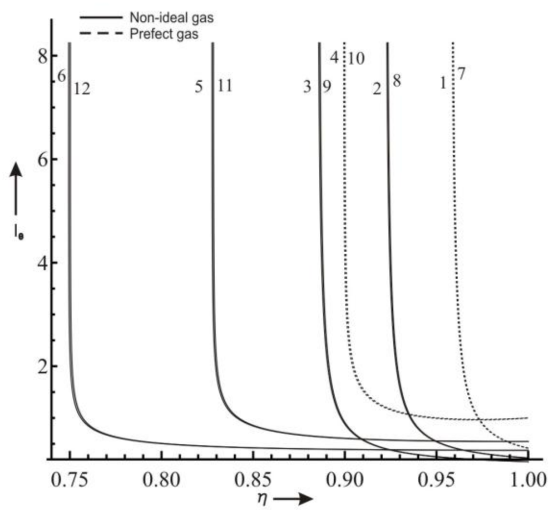

Figure 8. Variation of the non-dimensional azimuthal component of the vorticity vector in the region behind the shock

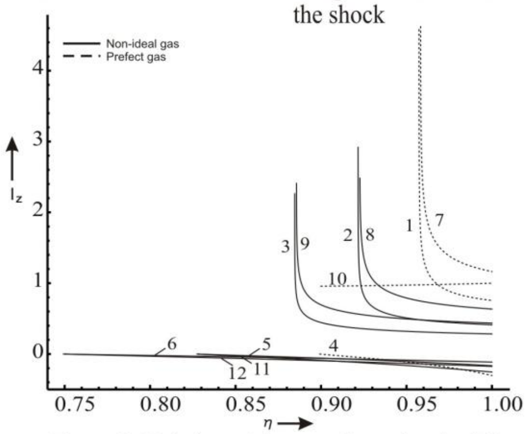

Figure 9. Variation of the non-dimensional axial component of the vorticity vector in the region behind the shock

Table 1: Position of the Inner Expanding Surface η_p for Different Values of M_A^{-2}, d and $\bar{b}$ and $\gamma = \frac{5}{3}$, $M^{-2} = 0.1$, $m = -0.1$, $\Gamma_c = 10$, $\Gamma_R = 100$ and $\frac{G}{Q} = 0.1$

M_A^{-2}	d	$\bar{b}$	η_p
0.05	-0.5	0	0.957343
		0.05	0.921688
		0.1	0.884598
	-1.2	0	0.899297
		0.05	0.827488
		0.1	0.749283
0.1	-0.5	0	0.958364
		0.05	0.922751
		0.1	0.885655
	-1.2	0	0.899943
		0.05	0.828259
		0.1	0.750205

Table 2: Density Ratio β for Different Values of $\bar{b}$ and $\gamma = \frac{5}{3}$, $M^{-2} = 0.1$, $m = -0.1$, $\Gamma_c = 10$, $\Gamma_R = 100$ and $\frac{G}{Q} = 0.1$

$\bar{b}$	β
0	0.06
0.05	0.11
0.1	0.16

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