International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2018 Vol. 8 (1) January-March, pp. 1-17/Kalaimaran

Research Article

A SIMPLE TECHNIQUE TO GENERATE ODD ORDER MAGIC SQUARES FOR ANY SEQUENCE OF CONSECUTIVE INTEGERS

***Ara. Kalaimaran**
Civil Engineering Division, CSIR-Central Leather Research Institute,
Chennai-20, Tamil Nadu, India
**Author for Correspondence: klmaran@yahoo.com*

ABSTRACT

A magic square of order $n \times n$ is a square array of numbers consisting of the distinct positive integers {1, 2, 3, ..., n^2} arranged such that the sum of the 'n' numbers in any horizontal, vertical, *main* diagonal and anti-diagonal line is always the same number.

The unique normal square of order three was known to the ancient Chinese, who called it the Lo Shu. A version of the order-4 magic square with the numbers 15 and 14 in adjacent middle columns in the bottom row is called Dürer's magic square.

INTRODUCTION

Magic squares (Weisstein Eric, 2003) have a long history, dating back to at least 650 BC in China. At various times they have acquired magical or mythical significance, and have appeared as symbols in works of art. In modern times they have been generalized a number of ways, including using extra or different constraints, multiplying instead of adding cells, using alternate shapes or more than two dimensions, and replacing numbers with shapes and addition with geometric operations.

There are many ways to construct magic squares, but the standard (and most simple) way is to follow certain configurations/formulas which generate regular patterns. Magic squares exist for all values of *n*, with only one exception: it is impossible to construct a magic square of order 2.

If all the diagonals including those obtained by "wrapping around" the edges-of a magic square sum to the same *magic constant*, the square is said to be a panmagic square (Stephen Wolfram, no date).

Order of Magic square	Magic Constant (m) for magic square starting with unity	No. of possible magic square
(3×3)	15	1
(4×4)	34	880
(5×5)	65	(2,42,000) 275305224
(6×6)	111	$\approx (1.7745 \pm 0.0016) \times 10^{19}$
(7×7)	175	unknown
--	--	--
--	--	--
$(n \times n)$	$\frac{n(n^2+1)}{2}$	$\frac{n^2(n-1)^4}{2}$

In case the numbers starts from 'a' instead of unity, Value of magic constant (m) $= \frac{n^3 + n(2a-1)}{2}$

The 880 squares of order four were enumerated by Frénicle de Bessy in 1693, and are illustrated in Berlekamp *et al.* (1982, pp. 778-783). The number of 5 x 5 magic squares was computed by R. Schroeppel in 1973. The number of 6 x 6 squares is not known, but Pinn and Wieczerkowski (1998) estimated it to be $(1.7745 \pm 0.0016) \times 10^{19}$ using Monte Carlo simulation and methods from statistical mechanics. Methods for enumerating magic squares are discussed by Berlekamp *et al.* (1982) and on the Math Pages website.

Magic squares were known to Islamic mathematicians in Arabia as early as the seventh century. They may have learned about them when the Arabs came into contact with Indian culture and learned Indian astronomy and mathematics.

The 3×3 magic square has been a part of rituals in India since Vedic times, and still is today. The Ganesh yantra is a 3×3 magic square. There is a well-known 10th-century 4×4 magic square on display in the Parshvanath temple in Khajuraho,

Greek Byzantine scholar Manuel Moschopoulos wrote a mathematical treatise on the subject of magic squares, leaving out the mysticism of his predecessors. Moschopoulos was essentially unknown to the Latin west. He was not, either, the first Westerner to have written on magic squares.

There are many ways to construct magic squares, but the standard (and most simple) way is to follow certain configurations/formulas which generate regular patterns. Magic squares exist for all values of n, with only one exception: it is impossible to construct a magic square of order 2. Magic squares can be classified into three types: odd, doubly even (n divisible by four) and singly even (n even, but not divisible by four). Odd and doubly even magic squares are easy to generate; the construction of singly even magic squares is more difficult but several methods exist, including the LUX method for magic squares (due to John Horton Conway) and the Strachey method for magic squares.

In the 19th century, Édouard Lucas devised the general formula for order 3 magic squares.

A method for constructing magic squares of odd order was published by the French diplomat de la Loubère in his book, A new historical relation of the kingdom of Siam (Du Royaume de Siam, 1693), in the chapter entitled. The problem of the magical square according to the Indians.

A magic square can be constructed using genetic algorithms.

Algorithms tend to only generate magic squares of a certain type or classification, making counting all possible magic squares quite difficult. Traditional counting methods have proven unsuccessful, statistical analysis using the Monte Carlo method has been applied. The basic principle applied to magic squares is to randomly generate $n \times n$ matrices of elements 1 to n^2 and check if the result is a magic square. The probability that a randomly generated matrix of numbers is a magic square is then used to approximate the number of magic squares.

On October 9, 2014 the post office of Macao in the People's Republic of China issued a series of stamps based on magic squares.

In this research article the simple techniques to develop magic squares for order $n \times n$ of odd numbers is explained in detail.

TYPES OF MAGIC SQUARES

1. Addition-multiplication magic square

An addition-multiplication square is a square of integers that is simultaneously a magic square and multiplication magic square.

2. Alphamagic square

A magic square for which the number of letters in the word for each number generates another magic square.

3. Antimagic square

An antimagic square is an $n \times n$ array of integers from 1 to n^2 such that each row, column, and main diagonal produces a different sum such that these sums form a sequence of consecutive integers. It is therefore a special case of a heterosquare. It was defined by Lindon (1962) and appeared in Madachy's collection of puzzles (Madachy 1979, p. 103), originally published in 1966

4. Associative magic square

An associative magic square (Stephen Wolfram, No date) is a magic square for which every pair of numbers symmetrically opposite to the centre sum up to the same value.

5. Bimagic square

If replacing each number by its square in a magic square produces another magic square, the square is said to be a bimagic square (Stephen Wolfram, No date). Bimagic squares are also called doubly magic squares, and are 2-multimagic squares. Lucas (1891) and later Hendricks (1998).

6. Complete magic square

These are Magic squares (Onze-Lieve-Vrouw-Presentatie Humaniora, No date) in which the sum of the numbers on the pan-diagonals is also the same as the sum of the numbers on each line, column or diagonal. Pan-diagonals are lines parallel to the diagonal.

7. Ideal Magic square

An ideal Magic square $n \times n$ formed from a sequence of consecutive integers in which the sum of each row, each column, two diagonals and several possible symmetrically added 'n' elements is equal to the magic constant.

8. Most Perfect Magic Square

A most-perfect magic square (Harvey D. Heinz, 2010) of order n is a magic square containing the numbers 1 to n^2 with two additional properties:

1. Each 2×2 sub-square sums to $2s$, where $s = n^2 + 1$.
2. All pairs of integer's distant $n/2$ along a (major) diagonal sum to s.

9. Multimagic square (Stephen Wolfram, No date)

A magic square is said to be p-multimagic if the square formed by replacing each element by its kth power for $k = 1, 2, \ldots, p$ is also magic. A 2-multimagic square is called bimagic, a 3-multimagic square is called trimagic, a 4-multimagic square is called tetramagic, a 5-multimagic square is called pentamagic, and so on.

10. Multiplication magic square

A square which is magic under multiplication instead of addition (the operation used to define a conventional magic square) is called a multiplication magic square (Stephen Wolfram, No date). Unlike (normal) magic squares, the n^2entries for an nth order multiplicative magic square are not required to be consecutive.

11. Normal/Ordinary/Diagonally magic square

The sum of the 'n' numbers in any horizontal, vertical, or *main* diagonal line is always the same magic constant.

12. Panmagic square

A Pan diagonal magic square or panmagic square (Stephen Wolfram, No date) (also diabolic square, diabolical square or diabolical magic square) is a magic square with the additional property that the broken diagonals, i.e. the diagonals that wrap round at the edges of the square, also add up to the magic constant.

13. Semi magic square

A semi magic square (Stephen Wolfram, No date) is a square that fails to be a magic square only because one or both of the main diagonal sums do not equal the magic constant (Kraitchik 1942, p. 143).

14. Trimagic square

If replacing each number by its square or cube in a magic square produces another magic square, the square is said to be a tri-magic square (Stephen Wolfram, No date). Tri-magic squares are also called trebly magic squares and are 3-multimagic squares.

15. Ultra-super Magic squares

An ultra-super Magic square (Onze-Lieve-Vrouw-Presentatie Humaniora, No date) is a Magic square that is complete and symmetrical. Ultra-super Magic squares of order 4 don' t exist. There are only 16 different ultra-super Magic squares of order 5.

International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2018 Vol. 8 (1) January-March, pp. 1-17/Kalaimaran
Research Article

METHODOLOGY

A New Techniques for Generating Magic Squares of order Odd Numbers

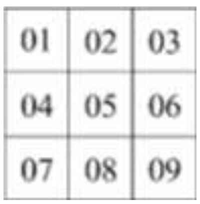
We can categorize into two viz. (i) Magic squares with order of odd number and (ii) Magic squares with order of even number.

In this paper the techniques to generate Magic squares of order of odd number from a sequence of consecutive integers is only described.

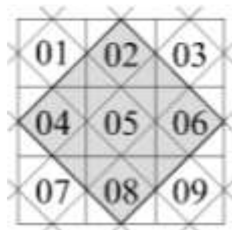
<u>Method to generate magic squares of order of odd numbers</u>

Magic square of order 3 x 3

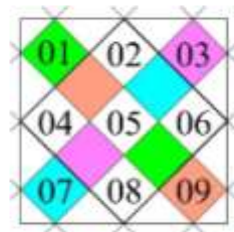
To form an Ideal Magic square of order 3×3 from a sequence of consecutive integers i.e. numbers taken from 1 to 9. Referring figures 1 to 6 of 3×3 squares, we can understand the method of forming ideal magic square.



1. Simple number square

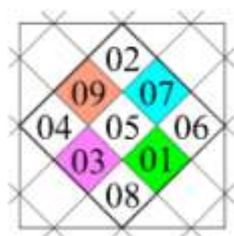


2. Diamond core



3. Voided Diamond core

4. Consolidating Diamond core



5. Consolidated diamond

02	07	06
09	05	01
04	03	08

6. Magic square

The following are the possible sum of rows/columns/diagonal/symmetrical

02	07	06
09	05	01
04	03	08

$02 + 07 + 06$

02	07	06
09	05	01
04	03	08

$04 + 03 + 08$

02	07	06
09	05	01
04	03	08

$02 + 09 + 04$

02	07	06
09	05	01
04	03	08

$06 + 01 + 08$

02	07	06
09	05	01
04	03	08

$02 + 05 + 08$

02	07	06
09	05	01
04	03	08

$07 + 05 + 03$

02	07	06
09	05	01
04	03	08

$06 + 05 + 04$

02	07	06
09	05	01
04	03	08

$09 + 05 + 01$

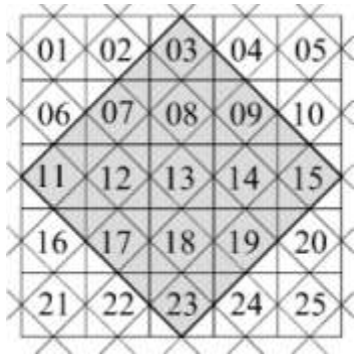
In 3 x 3 magic square, maximum possible way to get sum of rows/columns/diagonals/any other symmetrical to get the magic constant is 8. There is only one magic square.

Magic square of order 5 x 5

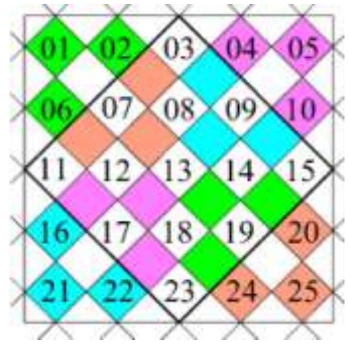
To form an Ideal Magic square of order 5 x 5 from a sequence of consecutive integers i.e. numbers taken from 1 to 25. Referring fig.1 to fig.6, we can understand the method of forming ideal magic square of 5×5.

01	02	03	04	05
06	07	08	09	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

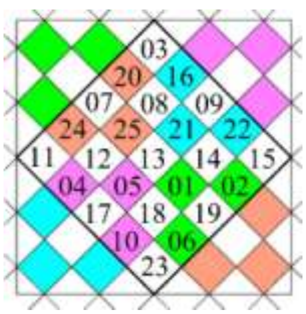
1. Simple number square



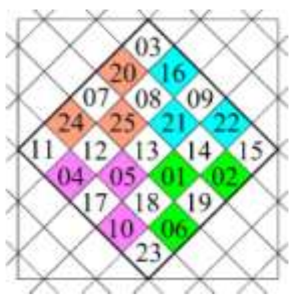
2. Central core



3. Voided core



4. Consolidating core



5. Consolidated core

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

6. Magic square

The following are the possible sum of rows/columns/diagonal/symmetrical

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

$03 + 16 + 09 + 22 + 15$

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

$20 + 08 + 21 + 14 + 02$

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

$24 + 12 + 05 + 18 + 06$

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

$11 + 04 + 17 + 10 + 23$

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

$03 + 20 + 07 + 24 + 11$

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

$16 + 08 + 25 + 12 + 04$

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

22 + 14 + 01 + 18 + 10

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

15 + 02 + 19 + 06 + 23

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

03 + 16 + 13 + 10 + 23

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

03 + 09 + 13 + 17 + 23

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

03 + 22 + 13 + 04 + 23

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

03 + 15 + 13 + 11 + 23

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

03 + 02 + 13 + 24 + 23

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

03 + 07 + 13 + 19 + 23

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

03 + 20 + 13 + 06 + 23

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

03 + 08 + 13 + 18 + 23

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

03 + 21 + 13 + 05 + 23

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

03 + 14 + 13 + 12 + 23

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

03 + 25 + 13 + 01 + 23

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

16 + 09 + 13 + 17 + 10

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

16 + 22 + 13 + 04 + 10

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

16 + 15 + 13 + 11 + 10

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

16 + 02 + 13 + 24 + 10

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

16 + 07 + 13 + 19 + 10

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

16 + 20 + 13 + 06 + 10

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

16 + 08 + 13 + 18 + 10

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

16 + 21 + 13 + 05 + 10

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

16 + 14 + 13 + 12 + 10

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

16 + 25 + 13 + 01 + 10

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

09 + 22 + 13 + 17 + 04

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

09 + 15 + 13 + 17 + 11

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

09 + 02 + 13 + 17 + 24

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

09 + 19 + 13 + 17 + 07

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

09 + 06 + 13 + 20 + 17

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

09 + 18 + 13 + 08 + 17

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

09 + 21 + 13 + 05 + 17

International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2018 Vol. 8 (1) January-March, pp. 1-17/Kalaimaran
Research Article

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

09 + 14 + 13 + 12 + 17

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

09 + 01 + 13 + 25 + 17

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

22 + 15 + 13 + 11 + 04

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

22 + 02 + 13 + 24 + 04

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

22 + 19 + 13 + 07 + 04

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

22 + 06 + 13 + 20 + 04

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

22 + 18 + 13 + 08 + 04

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

22 + 21 + 13 + 05 + 04

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

22 + 14 + 13 + 12 + 04

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

22 + 01 + 13 + 25 + 04

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

15 + 02 + 13 + 24 + 11

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

15 + 19 + 13 + 07 + 11

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

15 + 06 + 13 + 20 + 11

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

15 + 18 + 13 + 08 + 11

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

15 + 21 + 13 + 05 + 11

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

15 + 14 + 13 + 12 + 11

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

15 + 01 + 13 + 25 + 11

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

02 + 09 + 13 + 07 + 24

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

02 + 06 + 13 + 20 + 24

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

02 + 18 + 13 + 08 + 24

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

02 + 21 + 13 + 05 + 24

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

02 + 14 + 13 + 12 + 24

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

02 + 01 + 13 + 25 + 24

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

19 + 06 + 13 + 20 + 07

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

19 + 18 + 13 + 08 + 07

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

19 + 21 + 13 + 05 + 07

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

19 + 14 + 13 + 12 + 07

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

19 + 01 + 13 + 25 + 07

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

06 + 18 + 13 + 08 + 20

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

06 + 21 + 13 + 05 + 20

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

06 + 14 + 13 + 12 + 20

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

06 + 01 + 13 + 25 + 20

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

08 + 21 + 13 + 05 + 18

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

08 + 14 + 13 + 12 + 18

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

08 + 25 + 13 + 01 + 18

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

21 + 14 + 13 + 05 + 12

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

21 + 01 + 13 + 25 + 05

03	16	09	22	15
20	08	21	14	02
07	25	13	01	19
24	12	05	18	06
11	04	17	10	23

14 + 01 + 13 + 25 + 12

Method of developing other Magic squares

Table-1

Initial stage	**Stage-2**	**Stage-3**	**Stage-4**	**Stage-5**	**Stage-6**	**Stage-7**	**Stage-8**
Primary table	Row Interchanging	Column Inter changing	Rotation	Reflection	Choosing $R_n C_n$ as Key number	Selecting entire	Forming Mag. sq. towards
Square of sequence of consecutive integers	Without Interchange of Rows	Without Interchange of Columns	Without rotation	Without Reflection	R_1C_1	Row of Key Number	Forward Direction
- -	Interchanging R_1 & R_2	Interchanging C_1 & C_2	Rotation 90°	Reflection along horizontal axis	R_1C_2	Column of Key Number	Reverse Direction
- -	Interchanging R_1 & R_3	Interchanging C_1 & C_3	Rotation 180°	Reflection along vertical axis	R_1C_3	- -	- -
- -	Interchanging R_1 & R_4	Interchanging C_1 & C_4	Rotation 270°	Reflection along main diagonal	R_1C_4	- -	- -
- -	Interchanging R_1 & R_5	Interchanging C_1 & C_5	- -	Reflection along anti-diagonal	R_1C_5	- -	- -

- -	Interchanging R_2 & R_3	Interchanging C_2 & C_3	- -	- -	R_2C_1	- -	- -
- -	Interchanging R_2 & R_4	Interchanging C_2 & C_4	- -	- -	R_2C_2	- -	- -
- -	Interchanging R_2 & R_5	Interchanging C_2 & C_5	- -	- -	R_2C_3	- -	- -
- -	Interchanging R_3 & R_4	Interchanging C_3 & C_4	- -	- -	R_2C_4	- -	- -
- -	Interchanging R_3 & R_5	Interchanging C_3 & C_5	- -	- -	R_2C_5	- -	- -
- -	Interchanging R_4 & R_5	Interchanging C_4 & C_5	- -	- -	R_3C_1	- -	- -
- -	- -	- -	- -	- -	...	- -	- -
- -	- -	- -	- -	- -	...	- -	- -
- -	- -	- -	- -	- -	...	- -	- -
- -	- -	- -	- -	- -	R_5C_5	- -	- -

Example-1

1	Primary number square of sequence
2	Interchanging R_1 & R_2
3	Interchanging C_1 & C_4
4	Anti-Clockwise Rotation 90°
5	Reflection along main diagonal
6	Choosing R_2C_2 as key number
7	Forming Magic square in Forward Direction
8	Final Magic square

Stage	Description of Process	Square before Process	Square after Process
Initial stage	Primary number square of sequence without any change	--	01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Stage-2	Interchanging R_1 & R_2	01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	06 07 08 09 10 01 02 03 04 05 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

Research Article

Stage-3 Interchanging C_1 & C_4

06	07	08	09	10
01	02	03	04	05
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

9	7	8	6	10
4	2	3	1	5
14	12	13	11	15
19	17	18	16	20
24	22	23	21	25

Stage-4 Rotation 90°

9	7	8	6	10
4	2	3	1	5
14	12	13	11	15
19	17	18	16	20
24	22	23	21	25

10	5	15	20	25
6	1	11	16	21
8	3	13	18	23
7	2	12	17	22
9	4	14	19	24

Stage-5 Reflection along Main diagonal

10	5	15	20	25
6	1	11	16	21
8	3	13	18	23
7	2	12	17	22
9	4	14	19	24

10	6	8	7	9
5	1	3	2	4
15	11	13	12	14
20	16	18	17	19
25	21	23	22	24

Stage-6 Choosing R_1C_2 as key number and forming Magic square by choosing R_1C_2 as key number

10	06	08	07	09
05	01	03	02	04
15	11	13	12	14
20	16	18	17	19
25	21	23	22	24

		06		
		03	10	
		12		09
07		19		
	08	25		

Stage-7 Selecting its Row and filling the box in forward direction (Top to bottom)

		06		
		03	10	
		12		09
07		19		
	08	25		

15		06		
16		03	10	
23		12		09
07		19		
04	08	25		

Stage-7 (Final Stage)	Final result of Magic square

15	02	06	24	18
16	14	03	10	22
23	20	12	01	09
07	21	19	13	05
04	08	25	17	11

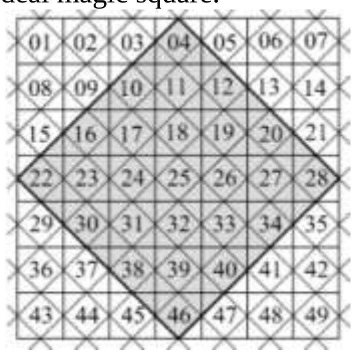
15	02	06	24	18
16	14	03	10	22
23	20	12	01	09
07	21	19	13	05
04	08	25	17	11

Magic square of order 7 x 7

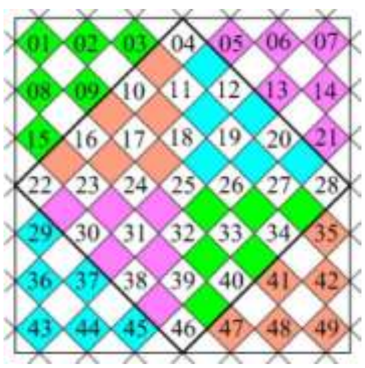
(i) Method to form an Ideal Magic square of 7 x 7 is shown below. Referring figures 1 to 6 of 7×7 squares, we can understand the method of forming ideal magic square.

01	02	03	04	05	06	07
08	09	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

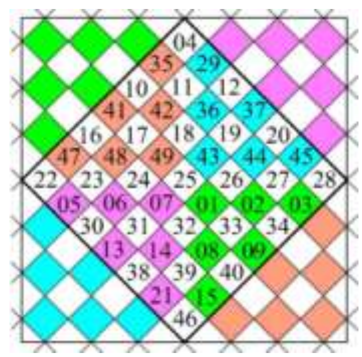
1. Simple number square



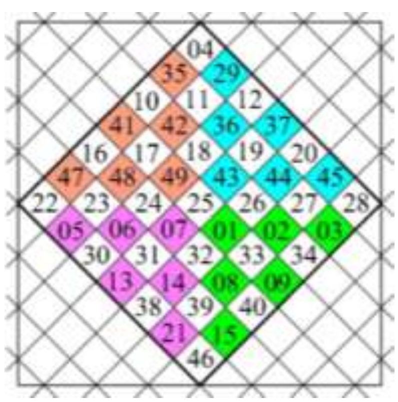
2. Central core



3. Core with voids



4. Consolidating the Core



5. Consolidated core

04	29	12	37	20	45	28
35	11	36	19	44	27	03
10	42	18	43	26	02	34
41	17	49	25	01	33	09
16	48	24	07	32	08	40
47	23	06	31	14	39	15
22	05	30	13	38	21	46

6. Ideal Magic square

Method of developing other Magic squares of 7×7

To form an Ideal Magic square of order 7×7 from a sequence of consecutive integers i.e. numbers taken from 1 to 49. The sum of the row/column/diagonal is 175.

Stage	Description of Process	Square before Process							Square after Process						
Stage-1	Initial stage	--							01	02	03	04	05	06	07
									08	09	10	11	12	13	14
									15	16	17	18	19	20	21
									22	23	24	25	26	27	28
									29	30	31	32	33	34	35
									36	37	38	39	40	41	42
									43	44	45	46	47	48	49
Stage-2	No interchange of Rows	01	02	03	04	05	06	07	01	02	03	04	05	06	07
		08	09	10	11	12	13	14	08	09	10	11	12	13	14
		15	16	17	18	19	20	21	15	16	17	18	19	20	21
		22	23	24	25	26	27	28	22	23	24	25	26	27	28
		29	30	31	32	33	34	35	29	30	31	32	33	34	35
		36	37	38	39	40	41	42	36	37	38	39	40	41	42
		43	44	45	46	47	48	49	43	44	45	46	47	48	49
Stage-3	No interchange of Columns	01	02	03	04	05	06	07	01	02	03	04	05	06	07
		08	09	10	11	12	13	14	08	09	10	11	12	13	14
		15	16	17	18	19	20	21	15	16	17	18	19	20	21
		22	23	24	25	26	27	28	22	23	24	25	26	27	28
		29	30	31	32	33	34	35	29	30	31	32	33	34	35
		36	37	38	39	40	41	42	36	37	38	39	40	41	42
		43	44	45	46	47	48	49	43	44	45	46	47	48	49
Stage-4	No rotation	01	02	03	04	05	06	07	01	02	03	04	05	06	07
		08	09	10	11	12	13	14	08	09	10	11	12	13	14
		15	16	17	18	19	20	21	15	16	17	18	19	20	21
		22	23	24	25	26	27	28	22	23	24	25	26	27	28
		29	30	31	32	33	34	35	29	30	31	32	33	34	35
		36	37	38	39	40	41	42	36	37	38	39	40	41	42
		43	44	45	46	47	48	49	43	44	45	46	47	48	49

Stage-5 — No reflection

01	02	03	04	05	06	07
08	09	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

01	02	03	04	05	06	07
08	09	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

Stage-6 — Choosing R_3C_1 is the key number to start the magic square

01	02	03	04	05	06	07
08	09	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

01	02	03	04	05	06	07
08	09	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

Stage-7 — Magic square is started with key number as R_1C_4

			15			
			23	08		
			31		01	
			39			43
36			47			
	29		06			
		22	14			

11	45	30	15	07	41	26
19	04	38	23	08	49	34
27	12	46	31	16	01	42
35	20	05	39	24	09	43
36	28	13	47	32	17	02
44	29	21	06	40	25	10
03	37	22	14	48	33	18

Stage-8 (Final Stage) — Completed Magic square

11	45	30	15	07	41	26
19	04	38	23	08	49	34
27	12	46	31	16	01	42
35	20	05	39	24	09	43
36	28	13	47	32	17	02
44	29	21	06	40	25	10
03	37	22	14	48	33	18

--

For $(n \times n)$ Magic square

Similarly, magic square of any square order of n x n of odd numbers can be formed easily.

To form a Magic square of order n x n from a sequence of consecutive integers.

$n \times n$	No. of Row Interchanging	No. of Column interchanging	No. of Rotations	No. of Reflection	No. of selection	Possible No. of Magic square
3×3	3	3	4	5	5	1
5×5	11	11	4	5	9	$11 \times 11 \times 4 \times 5 \times 9 = 21{,}780$
7×7	22	22	4	5	13	$22 \times 22 \times 4 \times 5 \times 13 = 1{,}25{,}840$
...	...	...	...	...	...	...
...	...	...	...	...	...	...
...	...	...	...	...	...	...
$n \times n$	$\frac{n(n-1)}{2}+1$	$\frac{n(n-1)}{2}+1$	4	5	$2n-1$	$20(2n-1)\left(\frac{n(n-1)}{2}+1\right)^2$

CONCLUSION

a square array of rows of number sequence of consecutive integers arranged so that the sum of the integers is the same when taken vertically, horizontally, or diagonally. Simple technique to generate magic squares for order $n \times n$ of odd numbers have been explained in detail. The magic of Ideal magic square has been defined with examples.

ACKNOWLEDGEMENT

I am very thankful to Dr. B. Chandrasekaran, Director, CSIR-Central Leather Research Institute and Dr. V.Subramanian, Chief Scientist cum Head- Engineering services, CLRI, Chennai for avail this opportunity for publication of my paper in the journal.

REFERENCES

Weisstein Eric W (2003). CRC Concise Encyclopaedia of Mathematics, **2nd edition** (New York: CRC Press of Wolfram Research Inc.) p.1838.

Web Sites

Harvey D. Heinz (19.10.2010). Most-perfect Magic Squares (online), Magic-Squares
Available: http://www.magic-squares.net/most-perfect.htm [18.02.2018]

Onze-Lieve-Vrouw-Presentatie Humaniora, Liceo Ginnasio Marcantonio Flaminio, Greåker

Stephen Wolfram (No date). Addition-Multiplication magic Square (online), Wolfram Mathworld
Available: http://mathworld.wolfram.com/Addition-MultiplicationMagicSquare.html [*Accessed on 13.12.2017*]

Stephen Wolfram (No date). Alphamagic Square (online), Wolfram Mathworld
Available: http://mathworld.wolfram.com/AlphamagicSquare.html [*Accessed on 13.12.2017*]

Stephen Wolfram (No date). Antimagic Square (online), Wolfram Mathworld
Available: http://mathworld.wolfram.com/AntimagicSquare.html [*Accessed on 13.12.2017*]

Stephen Wolfram (No date). Associative magic Square (online), Wolfram Mathworld
Available: http://mathworld.wolfram.com/AssociativemagicSquare.html [*Accessed on 13.12.2017*]

Stephen Wolfram (No date). Bimagic Square (online), Wolfram Mathworld
Available: http://mathworld.wolfram.com/BimagicSquare.html [*Accessed on 13.12.2017*]

Stephen Wolfram (No date). Multiplication magic Square (online), Wolfram Mathworld
Available: http://mathworld.wolfram.com/MultiplicationMagicSquare.html [*Accessed on 13.12.2017*]

Stephen Wolfram (No date). Multimagic Square (online), Wolfram Mathworld
Available: http://mathworld.wolfram.com/MultimagicSquare.html [*Accessed on 13.12.2017*]

Stephen Wolfram (No date). pandiagonal magic Square (online), Wolfram Mathworld
Available: http://mathworld.wolfram.com/PandiagonalMagicSquare.html [*Accessed on 13.12.2017*]

Stephen Wolfram (No date). Panmagic Square (online), Wolfram Mathworld
Available: http://mathworld.wolfram.com/PanmagicSquare.html [*Accessed on 13.12.2017*]

Stephen Wolfram (No date). Semi magic Square (online), Wolfram Mathworld
Available: http://mathworld.wolfram.com/SemimagicSquare.html [*Accessed on 13.12.2017*]

Stephen Wolfram (No date). Trimagic Square (online), Wolfram Mathworld
Available: http://mathworld.wolfram.com/TrimagicSquare.html [*Accessed on 13.12.2017*]

Videregående Skole and Mikkelin Yhteiskoulun Lukio (No date). Magic Squares (online), Maths for Europe Available: http://mathsforeurope.digibel.be/Magic2.htm [*Accessed on 13.12.2017*]