**INNER PRODUCT SPACE**

\[ A = \{ a_0 + a_1 x_1 + \cdots + a_{n-1} x_{n-1} + a_n x_n / a_i \in F \ and \ n \in N \} \]

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**ABSTRACT**

This piece of work consist of \( \left( A, \oplus, \odot \right) \) is an abelian group, \( \left( A, \oplus, \otimes \right) \) is a vector space, \( A = \{ a_0 + a_1 x_1 + \cdots + a_{n-1} x_{n-1} + a_n x_n / a_i \in F \ and \ n \in N \} \), is a modified inner product space.

**Keywords:** Binary Operation, Abelian Group, Vector Space, Inner Product, Field.

**INTRODUCTION**

Herstein cites in (1)

**Definition:** A nonempty set of elements \( G \) is said to form a group if in \( G \) there is defined a binary operation, called the product and defined by *, such that

1. \( a, b \in G \) implies that \( a*b \in G \)
2. \( a, b, c \in G \) implies that \( (a*b)*c = a * (b*c) \)
3. There exist an element \( e \in G \) such that \( a*e = e*a = a \) for all \( a \in G \)
4. For every \( a \in G \) there exist an element \( a^{-1} \in G \) such that \( a * a^{-1} = a^{-1} * a = e \)

**Definition:** A group \( G \) is said to be abelian (or Commutative) if for every \( a, b \in G \),

\[ a * b = b * a. \]

**Definition:** A nonempty set \( V \) is said to be vector space over a field \( F \) if \( V \) is an abelian group under an operation which we denote by +, and if for every \( a \in F, \, v \in V \); there is defined an element, written as \( av \), in \( V \) subject to

1. \( a (v+w) = av + aw \);
2. \( (a + b)v = av + bv \);
3. \( a (bv) = (ab)v \);
4. \( 1v = v \);

For all \( a, b \in F; \, v, w \in V \) Where the 1 represent the unit element of \( F \) under multiplication.

**Definition:** The Vector Space \( V \) over \( F \) is said to be an inner product space if there is defined for any two vectors \( x, y \in V \) an element \( (x, y) \) in \( F \) such that

1. \( (x, y) = (y, x)^\top, \forall x, y \in V \)
2. \( (x, x) \geq 0 \) and \( (x, x) = 0 \ if \ f x = 0 \)
3. \( (c_1 x + c_2 y, z) = c_1 (x, z) + c_2 (y, z), \forall c_1, c_2 \in F \ & \ x, y, z \in V \)

**DISCUSSION**

Let \( A = \{ a_0 + a_1 x_1 + \cdots + a_{n-1} x_{n-1} + a_n x_n / a_i \in F \ and \ n \in N \} \) and

Let \( x = a_0 + a_1 x_1 + \cdots + a_{n-1} x_{n-1} + a_n x_n, a_i \in F \ and \ n \in N, \)

\[ y = b_0 + b_1 x_1 + \cdots + b_{n-1} x_{n-1} + b_n x_n, b_i \in F \ and \ n \in N, \]

\[ z = c_0 + c_1 x_1 + \cdots + c_{n-1} x_{n-1} + c_n x_n, c_i \in F \ and \ n \in N, \]

\[ -x = (-a_0) + (-a_1) x_1 + \cdots + (-a_{n-1}) x_{n-1} + (-a_n) x_n, a_i \in F \ and \ n \in N \]

\[ O = 0 + 0 x_1 + \cdots + 0 x_{n-1} + 0 x_n \]
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1 = 1 + 0x_1 + \cdots + 0x_{n-1} + 0x_n

\[ cx = (ca_0) + (ca_1)x_1 + \cdots + (ca_{n-1})x_{n-1} + (ca_n)x_n, \ c \in F \]

\[ x = y \text{ iff } a_i = b_i, \ \forall \ i \]

Now we define first binary operation \( \oplus \) on \( A \) as

\[ x \oplus y = (a_0 + a_1x_1 + \cdots + a_{n-1}x_{n-1} + a_nx_n) \oplus (b_0 + b_1x_1 + \cdots + b_{n-1}x_{n-1} + b_nx_n) \]

\[ = (a_0 + b_0) + (a_1 + b_1)x_1 + \cdots + (a_{n-1} + b_{n-1})x_{n-1} + (a_n + b_n)x_n \]

\[ \text{........... (1)} \]

\[ \Rightarrow \ x \oplus y = y \oplus x, \forall \ x, y, \in A \]

\[ x \oplus (y \oplus z) = (x \oplus y) \oplus z, \forall \ x, y, z \in A \]

\[ 0 \oplus x = x \oplus 0, \forall \ x \in A \]

\[ x \oplus (-x) = (-x) \oplus x = 0, \forall \ x \in A \]

\((A, \oplus)\) is an abelian group. \text{........... (2)}

Now we define second binary operation \( \otimes \) on \( A \) as

\[ c \otimes x = cx, \forall \ c \in F \ \& \ x \in A \text{ ........... (3)} \]

\[ \Rightarrow c \otimes (x \oplus y) = (c \otimes x) \oplus (c \otimes y), \forall \ c \in F \ \& \ x, y \in A \]

\[ (c_1 \otimes c_2) \otimes x = (c_1 \otimes x) \oplus (c_2 \otimes x), \forall \ c_1, c_2 \in F \ \& \ x \in A \]

\[ c_1 \otimes (c_2 \otimes x) = (c_1 \otimes c_2) \otimes x, \forall \ c_1, c_2 \in F \ \& \ x \in A \]

\[ 1 \otimes x = x, \forall \ x \in A \]

\[ \Rightarrow (A, \oplus, \otimes) \text{ is a vector space. ........... (4)} \]

Now we define inner product on \( A \) as

\[ (x, y) = (a_0 + a_1x_1 + \cdots + a_{n-1}x_{n-1} + a_nx_n) (b_0 + b_1x_1 + \cdots + b_{n-1}x_{n-1} + b_nx_n) \in F \]

\[ \Rightarrow (x, y) = (y, x), \forall \ x, y \in A \]

\[ (x, x) \geq 0 \text{ and } (x, x) = 0 \iff x = 0 \text{ or } \sum a_i = 0 \]

\[ (c_1x \oplus c_2y, z) = c_1(x, z) + c_2(y, z), \forall c_1, c_2 \in F \ \& \ x, y, z \in A \]

\[ \text{........... (5)} \]

From (1) to (5) we come to the Conclusion that

\( A = \{ a_0 + a_1x_1 + \cdots + a_{n-1}x_{n-1} + a_nx_n \ \mid \ a_i \in F \ \& \ n \in N \} \), is a modified inner product space.
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Conclusion

From the above discussion, I come to the following conclusions

\((A, \oplus)\) is an abelian group. \((A, \oplus, \otimes)\) is a vector space.

\[ A = \{ a_0 + a_1 x_1 + \cdots + a_{n-1} x_{n-1} + a_n x_n / a_i \in F \text{ and } n \in N \}, \]

is a modified inner product space.

REFERENCES