TEMPORAL INSTABILITY OF LONGITUDINAL ELECTROKINETIC WAVE IN SEMICONDUCTOR PLASMA DOPED WITH A METAL NANOPARTICLE

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ABSTRACT
Temporal instability of longitudinal electrokinetic wave in a nanoparticle impinged semiconductor plasma medium is investigated in this present report. Using hydrodynamic approach, we examined dispersion and gain characteristics of wave and found that two new models are induced within the medium due to the inclusion of a nanoparticle. The dependency of the characteristics of four modes propagating in the medium on carrier drift and wave number are explored.

Keywords: Semiconductor Plasma, Hydrodynamical Model of Plasma, Electrokinetic Wave, Propagation Characteristics, Nanoparticles

INTRODUCTION
Of all the dynamical phenomena that can occur in plasma, perhaps the most important is a relative oscillation of plasma’s electrons and positive ions, i.e. the Plasma Oscillations. The propagation of these plasma oscillations gives rise to electrostatic wave in the medium. On the other edge, semiconductors are having electrons and holes as free carriers those behave as solid state plasma medium. Because of its dilute nature in terms of parameters of interest, semiconductors are found to be more apposite materials amongst all solid state plasmas to explore the propagation characteristics of electrokinetic waves. Researchers have investigated the propagation of longitudinal electrokinetic waves in different plasma medium under different physical situations and found various modifications in characteristics of propagating modes and explored the possibilities of device manufacturing (Ghosh and Thakur, 2004; 2005; 2007; Ghosh et al., 2006).

On the other side, nanoparticles are important scientific tools that have been and are being explored in various technological areas. They are the link between bulk material and atomic or molecular structures. Nanoparticles are unique because of the properties they inherit. They have high surface to bulk ratio, which makes thin films of nanoparticles of significant interest for gas sensor applications. They used to show unexpected optical and dielectric properties as they are small enough to confine orbital electrons and produce quantum effects. Other properties unique among nanoparticles are surface plasmon resonance in metal particles, quantum confinement in semiconductors and supermagnetism in magnetic materials.

Several researchers have reported wave-nanoparticle interaction in many reports (Wei et al., 2004; Weber and Ford, 2004; Lindberg et al., 2007). In this sequence of work an important contribution is made by Jain and Parashar (2011), they studied the propagation of electrostatic and electromagnetic waves in medium consisting of a single nanoparticle, collection of nanoparticles and equally spaced (periodically arranged) nanoparticles.

Although many works are dedicated in field of wave- nanoparticle interaction, but to the best of our knowledge no serious attempt has been made to investigate wave characteristics in semiconductor plasma medium consisting of a nanoparticle except the one recently reported by the present authors (Ghosh and Dubey, 2014; hereafter referred as paper I). In this report, while studying the convective (spatial) instabilities of longitudinal electrokinetic wave in semiconductor medium impinged with a single metallic nanoparticle, authors found that the presence of nanoparticle drastically modify the dispersion and amplification profiles of pre-existing modes and no novel mode has been introduced. Authors believed that the presence of novel mode can be a possibility if one confines one’s study to the temporal (absolute)
nature of instability of the particular wave. Motivated by this background discussion, we made a theoretical study of temporal instability of longitudinal electrokinetic wave in semiconductor plasma medium and found the existence of two new modes induced by the existence of a single metal nanoparticle in the medium.

**Theoretical Formulation**

A metal nanoparticle of electron density $n_{0e}$ and radius $r$ is assumed to be impinged within the n-type semiconductor plasma medium. An external dc electric field $\hat{E}_0 = -E_0 \hat{z}$ is applied due to which a drift is produced and free electrons of the medium moves along z-axis with an average drift velocity $\hat{\mathbf{v}}_{0z}$. To study the propagation characteristics, we assumed that free electrons of the medium exhibits longitudinal oscillations governed by plasma frequency $\omega_{pe} = \left( n_{0e} q^2 / m_e e_0 c_L \right)^{1/2}$. A perturbation of the kind $\exp[i(\omega t - kz)]$ is assumed to propagate through the medium along the applied dc electric field. These field geometry and the physical conditions are identical to those considered in paper I. Hence, we may safely use the general dispersion relation [equation (5)] obtained in paper I and given as below

$$
\epsilon(\omega, k) = \left[ 1 - \frac{\omega_{pe}^2}{(\omega - k \partial_{0z})(\omega - k \partial_{0z} - i \mathbf{v}_e)} - \frac{\partial_{0z}^2}{k^2} \right] + \frac{\omega_{pe}^2}{\omega^2 - \frac{\omega_{pe}^2}{3}} = 0 \quad (1)
$$

The symbols used here are well defined in paper I. To study the temporal instability, we may rewrite the equation (1) as polynomial in terms of wave angular frequency $\omega$ as-

$$
A_5 \omega^5 + A_4 \omega^4 + A_3 \omega^3 + A_2 \omega^2 + A_1 \omega + A_0 = 0 \quad (2)
$$

where,

- $A_5 = 1$
- $A_4 = (-2k \partial_{0z} - i \mathbf{v}_e)$
- $A_3 = (k^2 \partial_{0z}^2 + ik \partial_{0z} \mathbf{v}_e - i \mathbf{v}_e^2 k^2 + \frac{2}{3} \omega_{pe}^2 - \omega_{pe}^2)$
- $A_2 = \frac{2}{3} \omega_{pe}^2 (-2k \partial_{0z} - i \mathbf{v}_e)]$
- $A_1 = \frac{2}{3} \omega_{pe}^2 (k^2 \partial_{0z}^2 + ik \partial_{0z} \mathbf{v}_e - \partial_{0z}^2 k^2 + \frac{\omega_{pe}^2}{2})$

As dispersion relation (2) is a fourth order polynomial in $\omega$ hence infers existence of four longitudinal modes with angular frequencies $\omega_1, \omega_2, \omega_3$ and $\omega_4$ for a given wave number $k$.

In absence of nanoparticle the polynomial given in equation (2) reduces to

$$
B_0 \omega^2 + B_2 \omega + B_1 = 0 \quad (3)
$$

- $B_3 = 1$
- $B_2 = (-2k \partial_{0z} - i \mathbf{v}_e)$
- $B_1 = (k^2 \partial_{0z}^2 + ik \partial_{0z} \mathbf{v}_e - i \mathbf{v}_e^2 k^2 - \omega_{pe}^2)$

Equation (3) indicates the existence of only two longitudinal electrokinetic modes. On comparing equations (2) and (3), one may infer that the presence of nanoparticle introduces two new modes while modifying the characteristics of the two pre-existing modes also.

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RESULTS AND DISCUSSION

To study the nature of these four modes, we have solved polynomial [equation (2)] for candidate material n-Ge at 300K using following set of parameters

\[ m_e = 1.588m_0, \quad \varepsilon_L = 15.8, \quad n_{oe} = 2 \times 10^{24}m^{-3}. \]

Since we have assumed that all perturbations vary as \( \exp[i(\alpha - kz)] \) the temporal instability shall occur only when \( \omega_{\text{Im}} < 0 \) for a real value of \( k \).

Figures 1-4 depict the dispersion \( (\omega_k \text{ Vs } k) \) and amplification \( (\omega_{\text{Im}} \text{ Vs } k) \) profiles of four possible modes propagating through the medium.

**Figure 1a:** Variation of phase constant of I-mode with wave number \( k \) at \( E_0 = 5 \times 10^{10}V/m \)

**Figure 1b:** Variation of growth rate of I-mode with wave number \( k \) at \( E_0 = 5 \times 10^{10}V/m \)

Figure 1a infers that I-mode is copropagating and non-dispersive in absence of nanoparticle and its frequency increases with \( k \). In presence of nanoparticle the mode becomes contrapropagating and
dispersive in nature. Initially its frequency decreases abruptly upto \( k \approx 9.06 \times 10^2 \text{m}^{-1} \) at this point frequency achieves a magnitude equals to \( 1.4 \times 10^5 \text{s}^{-1} \) and then becomes independent of \( k \) for \( k > 9.06 \times 10^2 \text{m}^{-1} \). Figure 1b indicates that in absence of nanoparticle I-mode is decaying, having decay constant independent of wave number \( k \). While in presence of nanoparticle the mode is decaying and decay constant is independent of \( k \) upto \( k \approx 8.05 \times 10^2 \text{m}^{-1} \), afterwards, it decreases suddenly to zero value and saturates at this wave number.

![Figure 2a: Variation of phase constant of II-mode with wave number \( k \) at \( E_0 = 5 \times 10^10 \text{V/m} \)](image)

\[ \omega_{ph}(k) \]

Figure 2a depicts the dispersion characteristics of II-mode. In absence of nanoparticle II-mode is copropagating and non-dispersive but wave frequency increases due to increase in wave number \( k \).
While in presence of nanoparticle initially the mode is contrapropagating with wave frequency increasing with \( k \) upto \( k \approx 8 \times 10^2 \, m^{-1} \). Now increase in \( k \) from this value depicts a sharp decrement in wave frequency till wave frequency reaches zero value at \( k \approx 8.8 \times 10^2 \, m^{-1} \). Above this value of \( k \) direction of propagation of II-mode inverts and it becomes copropagating with wave frequency increasing upto \( \omega_{2\text{Re}} \approx 4.5 \times 10^6 \, s^{-1} \). After attaining this maximum value, frequency again starts shrinking and becomes zero at \( k \approx 1.9 \times 10^3 \, m^{-1} \). Figure 2b reveals that in absence of nanoparticle II-mode is decaying and decay constant is independent of wave number; whereas in presence of nanoparticle, initially II-mode is growing and after \( k \approx 8.73 \times 10^2 \, m^{-1} \) it becomes decaying in nature.

\[ \omega_{\text{3Re}} \text{ with nanoparticle (10}^{-4}, s^{-1}) \]

\[ k (10^4 \, m^{-1}) \]

\[ \omega_{3\text{Re}} \text{ with nanoparticle (10}^{-4}, s^{-1}) \]

\[ k (10^4 \, m^{-1}) \]

**Figure 3a:** Variation of phase constant of III-mode with wave number \( k \) at \( E_0 = 5 \times 10^{10} \, Vm^{-1} \)

\[ \omega_{\text{3im}} \text{ with nanoparticle (10}^{-9}, s^{-1}) \]

\[ k (10^5 \, m^{-1}) \]

\[ \omega_{3\text{Im}} \text{ with nanoparticle (10}^{-9}, s^{-1}) \]

\[ k (10^5 \, m^{-1}) \]

**Figure 3b:** Variation of growth rate of III-mode with wave number \( k \) at \( E_0 = 5 \times 10^{10} \, Vm^{-1} \)
Figures 3 and 4 show the dispersion and amplification characteristics of new modes (III and IV). Figure 3a shows that III-mode is copropagating and non-dispersive. It has increasing wave frequency with increase in wave number $k$. Figure 3b infers III-mode is decaying whose decay constant retains zero value up to $k \approx 8.73 \times 10^2 \text{ m}^{-1}$. After this value of $k$ decay constant increases sharply and sticks at $\omega_{3\text{m}} \approx 1.53 \times 10^{11} \text{ s}^{-1}$. Figure 4a and 4b depict that IV-mode is copropagating and decaying in nature. Its frequency increases with wave number and decay constant is independent of wave number $k$.
Figures 5 to 8 depict the characteristics of all the four modes propagating in the medium with varying dc electric field $E_0$. Figures 5a and 5b show the variation of $\omega_1\text{Re}$ and $\omega_1\text{Im}$ with applied electric field $E_0$ for I-mode of propagation. Figure 5a shows that in absence of nanoparticle in the medium the mode is contrapropagating and propagating with decreasing phase velocity. At $E_0 \approx 4.46 \times 10^9 \text{Vm}^{-1}$ the mode inverts its direction of propagation and becomes copropagating. Now it’s phase velocity increases with $E_0$. While in presence of nanoparticle, the mode is contrapropagating. Its phase velocity decreases with $E_0$ and becomes zero at $E_0 \approx 4.46 \times 10^9 \text{Vm}^{-1}$. Figure 5b shows the gain characteristics of I-mode which reveals that the mode is decaying in nature in absence of nanoparticle and its decay constant is
independent of applied electric field $E_0$. Due to presence of nanoparticle, I-mode changes its nature and becomes growing with varying electric field. Initially its gain decreases with electric field upto $E_0 \approx 2.35 \times 10^{11} Vm^{-1}$; beyond this value gain becomes nearly independent of electric field.

Figure 6a: Variation of phase constant of II-mode with applied electric field $E_0$ at $k = 10^3 m^{-1}$

![Figure 6a](image)

Figure 6b: Variation of growth rate of II-mode with applied electric field $E_0$ at $k = 10^3 m^{-1}$

![Figure 6b](image)

Figures 6a and 6b shows propagation and gain characteristic of II-mode. Figure 6a infers that in absence of nanoparticle II-mode is copropagating and its phase velocity increases with electric field. Whereas, in presence of nanoparticle the mode is initially contrapropagating with increasing phase velocity upto $E_0 \approx 4.22 \times 10^{10} Vm^{-1}$. With further increase in electric field $E_0$, phase velocity starts decreasing. At $E_0 \approx 4.45 \times 10^{10} Vm^{-1}$ phase velocity becomes zero and mode changes its direction of propagation by $180^\circ$ and becomes copropagating in nature. Again phase velocity increases upto $E_0 \approx 4.86 \times 10^{10} Vm^{-1}$
and afterwards it starts decreasing and becomes zero at $E_0 \approx 7.76 \times 10^{10} V m^{-1}$. Figure 6b depicts that in absence of nanoparticle II-mode is decaying and its decay constant is independent of $E_0$. In presence of nanoparticle the mode is growing with increasing growth rate upto $E_0 \approx 4.26 \times 10^{10} V m^{-1}$, then growth rate starts decreasing and becomes zero at $E_0 \approx 4.37 \times 10^{10} V m^{-1}$ and then afterwards mode becomes decaying in nature. Its decay constant increases abruptly upto $E_0 \approx 4.92 \times 10^{10} V m^{-1}$, then again it starts decreasing and becomes nearly independently of $E_0$ at $E_0 \approx 9.52 \times 10^{10} V m^{-1}$.

Figure 7a: Variation of phase constant of III-mode with applied electric field $E_0$ at $k = 10^3 m^{-1}$

Figure 7b: Variation of growth rate of III-mode with applied electric field $E_0$ at $k = 10^3 m^{-1}$
The two new modes that are produced due to the presence of nanoparticle in the medium are depicted in Figures 7 and 8. Figure 7a infers that III-mode is contrapropagating initially with increasing phase velocity. But at \( E_0 \approx 4.23 \times 10^{10} \text{Vm}^{-1} \) phase velocity starts decreasing and the mode changes its direction of propagation and becomes copropagating at \( E_0 \approx 4.4 \times 10^{10} \text{Vm}^{-1} \). Its phase velocity increases up to \( E_0 \approx 4.82 \times 10^{10} \text{Vm}^{-1} \), afterwards it starts decreasing and becomes zero at \( E_0 \approx 7.60 \times 10^{10} \text{Vm}^{-1} \). Figure 7b reflects that III-mode is decaying in nature and its decay constant is nearly independent of \( E_0 \) up to \( E_0 \approx 2.5 \times 10^{10} \text{Vm}^{-1} \). Beyond this, the decay constant increases abruptly.

Figure 8a: Variation of phase constant of IV-mode with applied electric field \( E_0 \) at \( k = 10^3 \text{m}^{-1} \)

Figure 8b: Variation of growth rate of IV-mode with applied electric field \( E_0 \) at \( k = 10^3 \text{m}^{-1} \)
Figures 8a and 8b represent that IV-mode is decaying in nature with decay constant independent of $E_0$ and this mode is copropagating in nature. The phase velocity increases with increase in dc electric field $E_0$.

**Conclusion**

In this work we have studied the temporal (absolute) instability of longitudinal electrokinetic wave and found that two new models are induced due to the presence of a metal nanoparticle in semiconductor plasma medium. In this study we investigated the characteristics with respect to electric field $E_0$ and wave number $k$. It is found that in case of varying carrier drift $E_0$ I-mode becomes growing in presence of nanoparticle, whereas it was decaying without nanoparticle. In absence of nanoparticle II-mode was decaying with unchanging decay constant, while in presence of nanoparticle initially II-mode is growing then after $E_0 \approx 4.37\times 10^{10} \text{Vm}^{-1}$ it becomes decaying in nature. The two newly induced modes show decaying behaviour with varying $E_0$. While exploring the characteristic of all four modes with varying wave number $k$, it is found that all modes except the II-mode shows decaying nature. II-mode is initially growing and adopts decaying nature after $k \approx 8.82\times 10^2 \text{m}^{-1}$. Authors hope that the obtained informations may become useful in extending the fundamental knowledge about the characteristics of longitudinal electrokinetic wave in presence of a metallic nanoparticle.

**REFERENCES**


