RELATIONSHIP BETWEEN THE SATURATED HYDRAULIC
CONDUCTIVITY AND THE PARTICLE SIZE DISTRIBUTION

Mohammad Aminifard¹ and *Maaroof Siosemarde²

¹Department of Civil Engineering, Mahabad Branch, Islamic Azad University, Mahabad, Iran
²Department of Water Engineering, College of Agriculture and Natural Resources, Mahabad Branch, Islamic Azad University, Mahabad, Iran

*Author for Correspondence

ABSTRACT
Hydraulic conductivity is one important soil parameter in hydrological studies. The hydraulic conductivity is measured on soil samples in the lab and sometimes tests carried out in the field. In this study, 50 set of soil samples with sand texture selected. The results show approximately success in predicting hydraulic conductivity from particle diameters data. The following relationship obtained from regression on data ($R^2 = 0.40$): $K_s = 8.4477 + 52.911(d_{10})$. Where $d_{10}$ is the soil particle diameter (mm) that 10% of all soil particles are finer (smaller) by weight and $K_s$, saturated hydraulic conductivity is expressed in m/day. Comparison between linear and quadratic single parameter equations showed $K_s$, saturated hydraulic conductivity predicted from linear equation based on $d_{10}$, estimated better than quadratic single parameter equation based on $d_{10}$ but saturated hydraulic conductivity predicted from quadratic equations based on $d_{50}$ and $d_{60}$, estimated better than linear single parameter equations based on $d_{50}$ and $d_{60}$. Also the results showed that among seven empirical formulae (Hazen, Kozeny-Carman, Breyer, Slitcher, Terzaghi, USBR and Alyamani&Sen), the Slitcher formula predicted $K_s$, saturated hydraulic conductivity, better than other formulae with 0.629 R; 6.78 RMSE; 5.73 MAE; 26.71% RE & 1.46 DT and the Breyer formula estimated $K_s$, with largest prediction error with 0.634 R; 58.11 RMSE; 48.80 MAE; 227.54% RE & 3.21 DT.

Keywords: Empirical Equation, Hydraulic Conductivity, Particle Diameter, Particle-size Distribution and Soil

INTRODUCTION
Saturated hydraulic conductivity is one of the most important of soil physical characteristics that are very important in ground water studies. Various studies indicate that the hydraulic conductivity is dependent on the particle size distribution. Curve of particle size distribution (PSD) is one of the basic physical properties of soils that usually expressed the cumulative probability distribution of the diameter of the soil particles (Hwang et al., 2002). Since, direct measurement of hydraulic conductivity is time consuming and costly, indirect methods such as predicting from readily available soil properties e.g. particle-size distribution have been developed (Alyamani and Sen, 1993; Chakraborty et al., 2006; Mualem, 1976; Shao and Robert, 1998; Van and Leji, 1989). In recent years, many studies have been to estimate soil hydraulic functions based on soil particle size distribution (Hwang and Powers, 2003b; Hwang and Hong, 2006).

Although in hydromechanics, it would be more useful to characterize the diameters of pores rather than those of the grains, the pore size distribution is very difficult to determine, so that approximation of hydraulic properties are mostly based on the easy-to-measure grain size distribution as a substitute. There have been attempts to estimate saturated hydraulic conductivity based on particle-size distribution (PSD) (Boadu, 2000; Mualem, 1976; Tyler and Wheatcraft, 1989; Van et al., 1992; Van, 1980; Van Genuchten and Leji, 1989).

Many different relationships have been developed from readily available soil properties e.g. particle-size distribution. Research findings showed that USBR and Slitcher equation calculated the amounts of hydraulic conductivity less than the other equations (Cheng and Chen, 2007). The research results showed

© Copyright 2014 | Centre for Info Bio Technology (CIBTech)
Indian Journal of Fundamental and Applied Life Sciences ISSN: 2231–6345 (Online)
An Open Access, Online International Journal Available at www.cibtech.org/sp.ed/jls/2014/04/jls.htm
2014 Vol. 4 (S4), pp. 73-80/Aminifard and Siosemarde

Research Article

that the Breyer equation is very suitable for soils with low uniformity coefficient (Pinder and Celia, 2006).


Odong (2007) have been evaluated several empirical equations to calculate hydraulic conductivity using grain size distribution of unconsolidated aquifer materials. Odong (2007) noted that all the seven empirical formulae reliably estimated hydraulic conductivities of the various soil samples well within the known ranges. Kozeny-Carmen formula proved to be the best estimator of most samples analyzed, and may be, even for a wide range of other soil types (Odong, 2007).

Han et al., (2008) developed a new model to estimate saturated hydraulic conductivity from soil structural properties derived from water retention curve (Han et al., 2008).

Chakraborty et al., (2006) reported that considerable success in predicting hydraulic conductivity from PSD data of soils (Chakraborty et al., 2006).

Hazen (1882) proposed the relationship between saturated hydraulic conductivity and soil particle diameter, \( K_s = c(d_{10})^2 \), Where \( K_s \) is expressed in cm/sec, \( c \) is a constant that varies from 1.0 to 1.5, and \( d_{10} \) size gives the diameter for which 10% (by mass) of the particles in a soil sample are finer (Cronican and Gribb, 2004); Hazen, 1892). Also, Hazen equation is presented in the following form.

\[
K = \frac{g}{v} \times 6 \times 10^{-4} \left[ 1 + 10(n - 0.26) \right] d_{10}^2
\]  

(1)

Kozeny-Carmen formula is one of the most applicable equations used for determine of hydraulic conductivity as a function of the soil properties

\[
K = \frac{g}{v} \times 8.3 \times 10^{-3} \left[ \frac{n^3}{(1-n)^2} \right] d_{10}^2
\]  

(2)

Breyer presented the following formula (Odong, 2007):

\[
K = \frac{g}{v} \times 6 \times 10^{-4} \log \frac{500}{U} d_{10}^2
\]  

(3)

Where \( U \) is the grain uniformity coefficient.

Slitcher presented the following formula (Odong, 2007):

\[
K = \frac{g}{v} \times 1 \times 10^{-2} n^{3.287} d_{10}^2
\]  

(4)

Terzaghi presented the following formula:

\[
K = \frac{g}{v} \times C_i \left( \frac{n-0.13}{\sqrt[3]{1-n}} \right)^2 d_{10}^2
\]  

(5)

Where the \( C_i = \) sorting coefficient and \( 6.1 \times 10^{-3} < C_i < 10.7 \times 10^{-3} \). In this study, an average value of \( C_i \) is used (Terzaghi and Peck, 1964).

The following formula presented by U.S. Bureau of Reclamation (USBR) (Odong, 2007):

\[
K = \frac{g}{v} \times 4.8 \times 10^{-4} d_{20}^{0.3} \times d_{20}^2
\]  

(6)

Alyamani and Sen (1993) proposed the relationship between saturated hydraulic conductivity and soil particle diameters (Alyamani and Sen, 1993):
Research Article

\[ K = 1300 \left[ I_o + 0.025(d_{50} - d_{10}) \right]^2 \]  

(7)

Where \( K \) is expressed in cm/sec, \( I_o \) is the x-intercept of the straight line formed by joining \( d_{50} \) and \( d_{10} \) of the grain-size distribution curve (mm). \( d_{50} \) is the mean grain-size for which 50% of the particles are finer by weight (mm).

The aim of the study was to determine relationship between saturated hydraulic conductivity and particle-size distribution. Also several empirical equations have been evaluated to calculate hydraulic conductivity using grain size distribution.

MATERIALS AND METHODS

In this study the 50 sets of soil samples were collected to estimate hydraulic conductivity based on particle-size distribution (PSD). Standard methods were applied to investigate particle size distribution (grain size curve), and finally determine of parameters of \( d_{10} \), \( d_{50} \) and \( d_{60} \). Where \( d_{10} \), \( d_{50} \) and \( d_{60} \) are the soil particle diameter (mm) where \( d_{10} \), \( d_{50} \) and \( d_{60} \) size gives the diameter for which 10%, 50% and 60% (by mass) of the particles in a soil sample are finer.

Soil texture was classified according to the International Society of Soil Science (ISSS) classification system. The soil texture was sand. The values of parameters of \( d_{10} \), \( d_{50} \) and \( d_{60} \) and saturated hydraulic conductivity are summarized in Table 1.

Table 1: Values of statistics of \( d_{10} \), \( d_{50} \), \( d_{60} \) and saturated hydraulic conductivity

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( d_{10} )</td>
</tr>
<tr>
<td>Mean</td>
<td>0.246</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.13</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.38</td>
</tr>
</tbody>
</table>

\( a. d_{10}, d_{50} \) and \( d_{60} \) are the soil particle diameter (mm) that 10%, 50% and 60% of all soil particles are finer (smaller) by weight and \( K_s \), saturated hydraulic conductivity is expressed in m/day.

The mean values of parameters of \( d_{10} \), \( d_{50} \) and \( d_{60} \) were 0.246, 0.641 and 0.840 [mm], respectively, also the mean values of saturated hydraulic conductivity was 21.45 (m/day). In this study saturated hydraulic conductivity was measured by the falling head method. The samples were first wetted by capillarity for 24 hours. The water is then allowed to flow through the soil with maintaining a constant pressure head and saturated hydraulic conductivity was measured when outflow rate becomes constant.

The results were analyzed with SPSS 21.0 and EXCEL software with statistics such as, Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Relative Error (RE) and Deviation Time (DT) that calculated using equation (8), (9) and (10) respectively, where \( n \) represents the number of instances presented to the model and \( O_i \) and \( P_i \) represents measured and predicted, and \( O_{ave} \) and \( P_{ave} \) represents mean values of measured and predicted respectively.

\[ RMSE = \left( \frac{1}{n} \sum_{i=1}^{n} (P_i - O_i)^2 \right)^{0.5} \]  

(8)

\[ MAE = \frac{1}{n} \sum_{i=1}^{n} |P_i - O_i| \]  

(9)

\[ RE = \frac{(MAE / O_{ave}) \times 100}{\} } \right) \]
Research Article

\[ \log_{10}^{DT} = \left[ n^{-1} \sum_{i=1}^{n} [\log_{10}(P_i/O_i)]^2 \right]^{0.5} \] (10)

Empirical equations including Hazen, Kozeny-Carmen, Breyer, Slitcher, Terzaghi, USBR and Alyamani and Sen Equation have been evaluated.

RESULT AND DISCUSSION

The following equations for \( K_s \), saturated hydraulic conductivity (m/day), were obtained from multiple regressions on data.

\[ K_s = 8.4477 \times 10^5 + 52.911(d_{10}) \] (11)

\[ K_s = 13.792 \times 10^5 + 11.935(d_{50}) \] (12)

\[ K_s = 13.641 \times 10^5 + 9.2877(d_{60}) \] (13)

\[ K_s = 14.683 \times 10^5 + 105.17(d_{10})^2 \] (14)

\[ K_s = 17.836 \times 10^5 + 8.2169(d_{50})^2 \] (15)

\[ K_s = 17.803 \times 10^5 + 4.8243(d_{60})^2 \] (16)

\[ K_s = 17.987 \times 10^5 + 8.7434(d_{50} - d_{10}) \] (17)

\[ K_s = 16.524 \times 10^5 + 8.2761(d_{60} - d_{10}) \] (16)

\[ K_s = 36.019 \times 10^5 - 4.2149(d_{60}/d_{10}) \] (19)

\[ K_s = 37.225 \times 10^5 - 5.9843(d_{50}/d_{10}) \] (20)

Where \( d_{10}, d_{50} \) and \( d_{60} \), are the soil particle diameter (mm) that 10%, 50% and 60% of all soil particles are finer (smaller) by weight and \( K_s \), saturated hydraulic conductivity is expressed in m/day. Table 2 was indicated the various statistics of equations mentioned above.

Table 2: Values of statistics of multiple regressions equations estimate to hydraulic conductivity

<table>
<thead>
<tr>
<th>Equation</th>
<th>Statistics</th>
<th>RMSE</th>
<th>MAE</th>
<th>RE</th>
<th>DT</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td></td>
<td>4.11</td>
<td>3.28</td>
<td>15.31</td>
<td>1.22</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>6.77</td>
<td>5.46</td>
<td>25.46</td>
<td>1.37</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>7.41</td>
<td>6.07</td>
<td>28.28</td>
<td>1.42</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>19.79</td>
<td>19.07</td>
<td>88.94</td>
<td>1.98</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>5.23</td>
<td>4.22</td>
<td>19.69</td>
<td>1.27</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>5.67</td>
<td>4.51</td>
<td>21.02</td>
<td>1.30</td>
</tr>
<tr>
<td>17</td>
<td></td>
<td>5.13</td>
<td>4.16</td>
<td>19.41</td>
<td>1.27</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>5.76</td>
<td>4.55</td>
<td>21.20</td>
<td>1.30</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td>14.60</td>
<td>13.54</td>
<td>63.12</td>
<td>1.79</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>15.34</td>
<td>14.31</td>
<td>66.72</td>
<td>1.82</td>
</tr>
</tbody>
</table>

\( a. \) RMSE is the Root Mean Square Error; MAE, Mean Absolute Error; RE, is the Relative Error and DT is Deviation Time.
The results showed as per the table the equation (11) was the best model for predicting $K_s$, saturated hydraulic conductivity (m/day), with 4.11 RMSE, 3.28 MAE, 15.31 RE and 1.22 DT. Comparison of observed vs. predicted values of saturated hydraulic conductivity obtained from the equation (11) as a 1:1 scale has been depicted in figure (1) that indicates good match.

![Figure 1 Comparison of measured saturated hydraulic conductivity, $K_s$ (m/day) and $K$, estimated by equation (11)](image)

The results showed that among single parameter linear equations (equation 11, 12 and 13) in this study, the equation that predicted $K_s$, saturated hydraulic conductivity (m/day), from $d_{10}$ estimated better (less prediction error) than $d_{50}$ and $d_{60}$ with 4.11 RMSE; 3.28 MAE; 15.31 RE, and 1.22 DT and the equation that predicted $K_s$, from $d_{50}$ and $d_{60}$ estimated with larger prediction error and the higher trend is evident between $K_s$ and $d_{10}$. The results of single parameter regression analysis showed that when $d_{10}$, $d_{50}$ and $d_{60}$ increase, $K_s$, saturated hydraulic conductivity (m/day), increases.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE</td>
</tr>
<tr>
<td>Hazen</td>
<td>54.45</td>
</tr>
<tr>
<td>Kozeny-Carmen</td>
<td>43.92</td>
</tr>
<tr>
<td>Breyer</td>
<td>58.11</td>
</tr>
<tr>
<td>Slitcher</td>
<td><strong>6.78</strong></td>
</tr>
<tr>
<td>Terzaghi</td>
<td>19.63</td>
</tr>
<tr>
<td>USBR</td>
<td>27.04</td>
</tr>
<tr>
<td>Alyamani and Sen</td>
<td>22.76</td>
</tr>
</tbody>
</table>

*a. RMSE is the Root Mean Square Error; MAE, Mean Absolute Error; RE, is the Relative Error and DT is Deviation Time.*

© Copyright 2014 | Centre for Info Bio Technology (CIBTech)
**Research Article**

The results showed that among single parameter quadratic equations (equation 14, 15 and 16), the equation that predicted $K_s$, saturated hydraulic conductivity (m/day), from $d_{50}$ estimated better (less prediction error) than $d_{10}$ and $d_{60}$ with 5.23 RMSE; 4.22 MAE; 19.69 RE, and 1.27 DT.

Also the results showed that among equation 17, 18, 19 and 20, the equation 17 that predicted $K_s$ from $d_{50}$-$d_{10}$ (without $d_{60}$) estimated better than other tow parameter equations with 5.13 RMSE; 4.16 MAE; 19.41 RE, and 1.27 DT and $K_s$ predicted based on $d_{50}$/$d_{10}$ estimated with largest prediction error. Overall variations between predicted and observed $K_s$ are reported in the literature (Alyamani and Sen, 1993; Chakraborty et al., 2006; Jabro, 1992; Mualem, 1976; Rawls and Brakensiek, 1989; Tyler and Wheatcraft, 1989; Uma et al., 1989; Van et al., 1992; Van, 1980; Van and Leji, 1989). The values of various statistics of empirical equations were indicated in Table 3.

The results showed as per the table the slitcher equation was the best model for predicting $K_s$, saturated hydraulic conductivity, with 6.78 RMSE, 5.73 MAE, 26.71% RE and 1.46 DT. Also the results showed that among studied equations, the Breyer equation predicted $K_s$, saturated hydraulic conductivity with high prediction error with 58.11 RMSE, 48.80 MAE, 227.54% RE and 3.21 DT. Meanwhile, the high value of DT statistic indicates that the accuracy and efficiency of the model in estimating the saturated hydraulic conductivity is low.

Comparison of observed vs. predicted values of saturated hydraulic conductivity of slitcher equation as a 1:1 scale has been depicted in figure (2) that indicates good match.

![Comparison of measured saturated hydraulic conductivity, $K_s$ (m/day) and $K_s$ estimated by slitcher equation](image)

Odong (2007) have been evaluated several empirical equations to calculate hydraulic conductivity using grain size distribution and Results showed that Kozeny-Carman formula proved to be the best estimator of most samples analyzed, and may be, even for a wide range of other soil types. However, some of the formulae underestimated or overestimated hydraulic conductivity; even of the same soils (Jabro, 1992). The difference between measured values of saturated hydraulic conductivity with estimated values has been reported by various researchers (Jabro, 1992; Van et al., 1992; Van, 1980).

It is concluded that the values of hydraulic conductivity calculated by the Slitcher equation is lower than the other equations, which is consistent with the conclusions by Vukovic and Soro (1992), Cheng and Chen (2007) and Odong (2007) (Hwang and Powers, 2003b; Jabro, 1992; Shao and Robert, 1998). Alyamani and Sen, Terzaghi and USBR equations calculated saturated hydraulic conductivity lower than Breyer, Hazen and Kozeny-Carmen equations.
In this study several multiple regression and empirical equations to calculate hydraulic conductivity based on grain size distribution have been evaluated. The results showed approximately success in predicting hydraulic conductivity from particle diameters data. The following relationship obtained from regression on data ($R^2 = 0.40$):

$$K_s = 8.4477 + 52.911(d_{10})$$

Where $d_{10}$ is the soil particle diameter (mm) that 10% of all soil particles are finer (smaller) by weight and $K_s$, saturated hydraulic conductivity is expressed in m/day. The results showed that the slitter equation was the best formula for predicting saturated hydraulic conductivity among studied equations. The results showed that the Breyer equation predicted saturated hydraulic conductivity with high prediction error.

REFERENCES


**Research Article**


