EVALUATION FUZZY NUMBERS BASED ON RMS

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ABSTRACT
We suggest a new approach to the problem of defuzzification. This approach is a knowledge-based approach in that it uses knowledge in terms of RMS (root mean square) function to help to more intelligently guide the defuzzification process. This approach makes use of a view of the defuzzification process as a kind of clustering problem and uses the basic idea used in the mountain clustering method.

Keywords: Fuzzy Number; Defuzzification; Root Mean Square; Defuzzification Criteria

INTRODUCTION
Defuzzification plays a central role in the implementation of fuzzy system’s modeling techniques, such as those used in fuzzy logic controllers (Lee, 1990; Yager et al., 1994). Defuzzification is essentially a process where guided by the output fuzzy subset of the model. One selects a single crisp value as the system output. In fuzzy logic controller applications the typical methods used for this process are the center of area (COA) method, and the mean of maximal (MOM) method. Both of these methods can be seen to be based on a weighted type aggregation which can be seen as a blending or mixing of different solutions. In (Filev et al., 1991), Filev and Yager show that these are essentially the same approaches, distinguished only by the choice of a parameter. While in many cases these types of blending methods work well, examples have been suggested in the literature which leads to very unsatisfactory solutions when we use these kinds of blending methods (Pfluger et al., 1992; Yager et al., 1993; Yager et al., 1994). The problems that usually arise are rooted in situations where two or more distinct modes of solution exist and the combining (blending) of these distinct modes leads to an unsatisfactory conclusion. For example, if two roads exist for going from point A to point B, and if we try to combine these two roads, we may end up traveling through the woods. A central issue that must be addressed is. When we can and when we cannot combine solutions. In this note we attempt to suggest an approach to the defuzzification process which can avoid some of the pitfalls that have been shown to exist in many defuzzification methods. We note that in (Yager et al., 1993; Yager et al., 1994), we have looked at a closely related issue that of defuzzification under constraints. Today, most applications use either mean of maxima (MOM) defuzzification or centroid (COG) defuzzification method. These two algorithms are well known and used in many implementations. They are working well in many fields, but cannot be regarded as general-purpose algorithms (Pfluger, et al., 1992). Having reviewed the previous concepts, this article proposes here a method to use the concept of (RMS) root mean square, so as to find the order of fuzzy numbers. This method can distinguish the alternatives clearly. The main purpose of this article is that, (RMS) can be used as a crisp approximation of a fuzzy number. Therefore, by the means of this defuzzification, this article aims to present a new method for ranking of fuzzy numbers. In addition to its ranking features, this method removes the ambiguities resulted and overcome the shortcomings from the comparison of previous ranking.

Basic Definitions and Notations
The basic definitions of a fuzzy number are given in (Abbasbandy et al., 2006; Zimmermann, 1991; Saneifard, 2009; Saneifard et al., 2010) as follows:
Definition 1. A fuzzy number $A$ is a mapping $\mu(x): \mathbb{R} \rightarrow [0,1]$ with the following properties:
- $\mu$ is an upper semi-continuous function on $\mathbb{R}$,
- $\mu(x) = 0 \text{ outside of some interval } [a_1, b_2] \subset \mathbb{R}$,
There are real numbers $a_2, b_1$ such as $a_1 \leq a_2 \leq b_1 \leq b_2$ and $\mu(x)$ is a monotonic increasing function on $[a_1, a_2]$, $\mu(x)$ is a monotonic decreasing function on $[b_1, b_2]$, $\mu(x) = 1$ for all $x$ in $[a_2, b_1]$.

Let $\mathbb{R}$ be the set of all real numbers. The researchers assume a fuzzy number $A$ that can be expressed for all $x \in \mathbb{R}$ in the form

$$\mu(x) = \begin{cases} g(x), & \text{when } a \leq x \leq b, \\ 1, & \text{when } b \leq x \leq c, \\ h(x), & \text{when } c \leq x \leq d, \\ 0, & \text{otherwise}. \end{cases}$$

where $a, b, c, d$ are real numbers such as $a < b \leq c < d$ and $g$ is a real valued function that is increasing and right continuous and $h$ is a real valued function that is decreasing and left continuous.

Support function is defined as follows:

$$\text{supp}(A) = \{x | \mu(x) > 0\},$$

where $\{x | \mu(x) > 0\}$ is closure of set $\{x | \mu(x) > 0\}$.

**Proposed RMS Defuzzification Algorithms**

Many defuzzifiers have been proposed so far. But no one method gives a right effective defuzzified output because each method gives different results. This paper introduces new defuzzification algorithms based on RMS value, since RMS always leads to an effective value. This word effective is taken from electrical engineering where RMS is always referred to as an effective value.

**3.1 RMS Algorithm 1 (Aarthi et al., 2006)**

The root mean square (RMS) of a variate $x$ of the mean squared value of $x$ is the squared value of $x$, given by the following algebraic expression for an input ($x$) varying function:

$$RMS_1 = \sqrt{\frac{\int_A^B (f(\mu(x)))^2 dx}{B-A}}.$$  

(2)

$A$ and $B$ are the lower and upper limits of the function and, $f$ represents the aggregated membership function and $\mu(x)$ is the degree of membership. The $RMS_1$ value, which is per unit, depends on the interval chosen. If the values of $A$ and $B$ are changed the $RMS$ value of the function across the interval from $A$ to $B$ will also change. In electrical engineering, this definition is invariably used for a periodic function.

**3.2 RMS Algorithm 2 (Aarthi et al., 2006)**

The $RMS$ of a variate $x$ of the mean squared value of $x$ is the squared value of $x$ given by the following algebraic expression:

$$RMS_2 = \sqrt{\frac{\int_A^B (f(\mu(x)))^2 dx}{\int_A^B f(\mu(x)) dx}}.$$  

(3)

$f$ represents the aggregated membership function and $\mu(x)$ is the degree of membership. This function calculates the $RMS$ value based on the area under the membership function $f(x)$.

**A New Method for Ranking Fuzzy Numbers**

In this section, we present a new approach for ranking fuzzy numbers based on the distance method. The method not only considers the $RMS$ of a fuzzy number, but also considers the minimum crisp value of fuzzy numbers.

For ranking fuzzy numbers, this study firstly defines a minimum crisp value $\mu_{\min}(x)$ to be the benchmark and its characteristic function $\mu_{\min}(x)$ is as follows:

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\[ \mu_{\tau_{\min}}(x) = \begin{cases} 1 & \text{when } x = \tau_{\min}, \\ 0 & \text{when } x \neq \tau_{\min}. \end{cases} \]  

(4)

When ranking \( n \) fuzzy numbers \( A_1, A_2, \cdots, A_n \) the minimum crisp value \( \mu_{\tau_{\min}} \) is defined as:

\[ \tau_{\min} = \min\{x|x \in \text{Domain}(A_1, A_2, \cdots, A_n)\}. \]  

(5)

The advantages of the definition of minimum crisp value are two-fold: firstly, the minimum crisp values will be obtained by themselves, and another is it is easy to compute.

Assume that there are \( n \) fuzzy numbers \( A_1, A_2, \cdots, A_n \). The proposed method for ranking fuzzy numbers \( A_1, A_2, \cdots, A_n \) is now presented as follows:

Step 1. Use formulas (2) and (3) to calculate the \( \text{RMS} \) of each fuzzy numbers \( A_j \), where \( 1 \leq j \leq n \).

Step 2. Calculate the maximum crisp value \( \tau_{\min} \) of all fuzzy numbers \( A_j \), where \( A_j \), where \( 1 \leq j \leq n \).

Step 3. Use the point \( \text{RMS} \) to calculate the ranking value \( d_{\text{RMS}}(A_j, \tau_{\min}) \) of the fuzzy numbers \( A_j \), where \( A_j \), where \( 1 \leq j \leq n \), as follows:

\[ d_{\text{RMS}}(A_j, \tau_{\min}) = \left| \text{RMS}(A_j) - \tau_{\min} \right|. \]  

(6)

From formula (6), we can see that \( d_{\text{RMS}}(A_j, \tau_{\min}) \) can be considered as the Euclidean distance between the point \( (\text{RMS}(A_j), 0) \) and the point \( (\tau_{\min}, 0) \). We can see that the bigger the value of \( d_{\text{RMS}}(A_j, \tau_{\min}) \), the better the ranking of \( A_j \), where \( 1 \leq j \leq n \).

Let \( A_j \) is a fuzzy number characterized by (1) and \( d_{\text{RMS}}(A_j, \tau_{\min}) \) is the Euclidean distance between the point \( (\text{RMM}(A_j), 0) \) and the point \( (\tau_{\min}, 0) \) of its.

Since this article wants to approximate a fuzzy number by a scalar value, thus the researchers have to use an operator \( d_{\text{RMS}}: F \rightarrow \mathbb{R} \) (A space of all fuzzy numbers will be denoted by \( F \)) which transforms fuzzy numbers into a family of real line. Operator \( \text{dist} \) is a crisp approximation operator. Since ever above defuzzification can be used as a crisp approximation of a fuzzy number, therefore the resultant value is used to rank the fuzzy numbers. Thus, \( d_{\text{RMS}} \) is used to rank fuzzy numbers.

Let \( A_1, A_2 \in F \) be two arbitrary fuzzy numbers. Define the ranking of \( A_1 \) and \( A_2 \) by \( d_{\text{RMS}} \) on \( F \) as follows:

\begin{align*}
1) & \quad d_{\text{RMS}}(A_1, \tau_{\min}) < d_{\text{RMS}}(A_2, \tau_{\min}) \text{ if only if } A_1 < A_2, \\
2) & \quad d_{\text{RMS}}(A_1, \tau_{\min}) > d_{\text{RMS}}(A_2, \tau_{\min}) \text{ if only if } A_1 > A_2, \\
3) & \quad d_{\text{RMS}}(A_1, \tau_{\min}) = d_{\text{RMS}}(A_2, \tau_{\min}) \text{ if only if } A_1 \approx A_2.
\end{align*}

Then, this article formulates the order \( \geq \) and \( \leq \) as \( A_1 \geq A_2 \) if and only if \( A_1 > A_2 \) or \( A_1 \sim A_2 \), \( A_1 \leq A_2 \) if and only if \( A_1 < A_2 \) or \( A_1 \sim A_2 \). The new ranking index can sort many different fuzzy numbers simultaneously. In addition, the calculation is simple, and the index also satisfies the common properties of ranking fuzzy numbers:

(a) Transitivity of the order relation, i.e. if \( A_1 \leq A_2 \) and \( A_2 \leq A_3 \), then we should have \( A_1 \leq A_3 \).

(b) Compatibility of addition, that is if there is \( A_1 \leq A_2 \) on \( \{A_1, A_2\} \), then there is \( A_1 + A_3 \leq A_2 + A_3 \) on \( \{A_1 + A_3, A_2 + A_3\} \).

Remark 3.1. If \( A \preceq B \), then \( -A \succeq -B \).

Hence, this article can infer ranking order of the images of the fuzzy numbers.

Examples

In this section, we want compare proposed method with others methods.

Example 4.1.

Consider the following sets: \( A = (2,1,3), B(3,3,1), \) and \( C(2.5,0.5,0.5) \). By using this new approach, \( \text{RMS}(A)=2.33, \text{RMS}(B) = 2.66 \) and \( \text{RMS}(C) = 2.5 \). Hence, the ranking order is \( A < C < B \) too. It
seems that, the result obtained by Distance Minimization method is unreasonable. To compare with some of the other methods in (Chu and Tsao, 2002), the readers can refer to Table (4.1). Furthermore, to aforesaid example $RMS(-A) = -2.33 \text{\ and} \ RMS(-B) = -2.66 \text{\ and} \ RMS(-C) = -2.50$, consequently the ranking order of the images of three fuzzy number is $-B < -C < -A$. Clearly, this proposed method has consistency in ranking fuzzy numbers and their images, which could not be guaranteed by CV-index method. It easy to see that neither of them is consistent with human intuition.

### Table 4.1: Comparative results of Example (4.1)

<table>
<thead>
<tr>
<th>Fuzzy number</th>
<th>New approach</th>
<th>Sign Distance with $p=2$</th>
<th>Distance Minimization</th>
<th>Chu and Tsao (Revisited)</th>
<th>$CV$ index</th>
<th>Magnitude method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>2.33</td>
<td>3.9157</td>
<td>2.5</td>
<td>0.74</td>
<td>0.32</td>
<td>2.16</td>
</tr>
<tr>
<td>$B$</td>
<td>2.66</td>
<td>3.9157</td>
<td>2.5</td>
<td>0.74</td>
<td>0.36</td>
<td>2.83</td>
</tr>
<tr>
<td>$C$</td>
<td>2.50</td>
<td>3.5590</td>
<td>2.5</td>
<td>0.75</td>
<td>0.08</td>
<td>2.50</td>
</tr>
<tr>
<td>Results</td>
<td>$A &lt; C &lt; B$</td>
<td>$C &lt; A &lt; B$</td>
<td>$C &lt; A &lt; B$</td>
<td>$A &lt; B &lt; C$</td>
<td>$B &lt; A &lt; C$</td>
<td>$A &lt; C &lt; B$</td>
</tr>
</tbody>
</table>

Example 4.2

Consider the three fuzzy numbers, $A = (6,1,1)$, $B(6,0,1,1)$, and $C(6,0,1)$. According to Eq. (6), the ranking index values are obtained i.e. $RMS(A) = 6$, $RMS(B) = 6.15$ and $RMS(C) = 6.16$. Accordingly, the ranking order of fuzzy numbers is $C > B > A$. However, by Chu and Tsao’s approach (Chu and Tsao, 2002), the ranking order is $B > C > A$. Meanwhile, using $CV$ index proposed (Cheng, 1999), the ranking order is $B > C > A$. It easy to see that the ranking results obtained by the existing approaches (Cheng, 1999; Chu and Tsao, 2002) are unreasonable and are not consistent with human intuition. On the other hand in (Abbasbandy and Asady, 2006), the ranking result is $C > B > A$, which is the same as the one obtained by our approach. However, our approach is simpler in the computation procedure. Based on the analysis results from (Abbasbandy and Asady, 2006), the ranking results using our approach and other approaches are listed in Table (4.2).

### Table 4.2: Comparative results of Example (4.2)

<table>
<thead>
<tr>
<th>Fuzzy number</th>
<th>New approach</th>
<th>Sign Distance with $p=1$</th>
<th>Sign Distance with $p=2$</th>
<th>Chu-Tsao</th>
<th>Cheng Distance</th>
<th>$CV$ index</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>6.00</td>
<td>6.12</td>
<td>8.52</td>
<td>3.000</td>
<td>6.021</td>
<td>0.028</td>
</tr>
<tr>
<td>$B$</td>
<td>6.15</td>
<td>12.45</td>
<td>8.82</td>
<td>3.126</td>
<td>6.349</td>
<td>0.009</td>
</tr>
<tr>
<td>$C$</td>
<td>6.16</td>
<td>12.50</td>
<td>8.85</td>
<td>3.085</td>
<td>6.351</td>
<td>0.008</td>
</tr>
<tr>
<td>Results</td>
<td>$C &gt; B &gt; A$</td>
<td>$C &gt; B &gt; A$</td>
<td>$C &gt; B &gt; A$</td>
<td>$B &gt; C &gt; A$</td>
<td>$C &gt; B &gt; A$</td>
<td>$B &gt; C &gt; A$</td>
</tr>
</tbody>
</table>

All the above examples show that this method is more consistent with institution than the previous ranking methods.

**Conclusion**

In this paper, the researchers proposed a defuzzification using RMS of the fuzzy number and by using this defuzzification, we proposed a method for ranking of fuzzy numbers. Roughly, there not much difference in our method and theirs. The method can effectively rank various fuzzy numbers and their images.

**REFERENCES**


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