PROBLEM SOLVING OF THE NEW BUTTRESS RETAINING WALL COMPOSED OF RESERVOIRS SHELL AND FORCES IMPOSED ON THEM

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ABSTRACT
There is a wide variety of structures used to retain soil and or water for both temporary works and permanent works. Some of the new types of retaining structures for different purposes are illustrated in this paper. Until now, our concern was with the analysis of flat plates in retaining walls. We now in this paper extend the discussion to curved surface structures termed thin shells. We shall limit our treatment to shells of constant thickness, small in comparison with the other two dimensions. In this paper adaptation of bending theory in cylindrical shell reservoirs with retaining walls by the classical method are presented. In other words, research and problem solving of the new buttress retaining wall composed of reservoirs shell and forces imposed on them have been discussed in this paper.

Keywords: Reservoirs Shell, Cylindrical Shell, Shell Retaining Wall, Buttress Retaining Wall, Forces on Retaining Walls

INTRODUCTION
The analysis of shell structures often embraces two distinct, commonly applied theories. The first of these, the membrane theory, usually applies to a rather large part of the entire shell. A membrane, either flat or curved, is identified as a body of the same shape as a plate or shell, but incapable of conveying moments or shear forces. In other words, a membrane is a two dimensional analog of a flexible string with the exception that it can resist compression. The second, the bending theory or general theory, includes the effects of bending. Thus it permits the treatment of discontinuities in the stress distribution taking place in a limited region in the vicinity of a load or structural discontinuity. It is important to note that membrane forces are independent of bending and are completely defined by the conditions of static equilibrium. As no material properties are used in the derivation of these forces, the membrane theory applies to all shells made of any material (e.g., metal, fabric, reinforced concrete, sandwich shell and soap film). Various relationships developed for bending theory in this paper are, however, restricted to homogeneous, elastic, isotropic shells.

The complete bending theory is mathematically intricate, and the first solutions involving shell bending stresses date back to only 1920 (Majidpourkhoei, 2015). We shall, in this paper, limit consideration to the most significant practical case involving rotationally symmetrical loading. In this paper the function of the triangular bases of the retaining wall against that of the excluded semicircle of the reservoirs is taken into consideration.
In this article, selected some texts about the theory of shells are given in the references (Calladine, 1983; Dikmen, 1982; Goldenveizer, 1961; Koiter, 1969; Kraus, 1967; Leissa, 1973; Lure, 1940; Novozhilov, 1964; Reddy, 2003; Sanders, 1959; Save and Massounet, 1972; Sawczuk, 1980; Timoshenko and Woinowsky, 1959; Ugural, 1996).

MATERIALS AND METHODS
This section detailed about the proposed approach which is based on various parameters and are described below.

New Types of Retaining Walls and Forces Imposed on them
The new types of standing retaining walls used to retain earth or other materials are shown in figure 1. The materials retained exert a push on the retaining wall which tends to overturn or slide. The stability of
the wall against overturning and sliding is maintained by the weight of the Retaining wall and weight of
the earth on the base of the Retaining wall. It is similar to the counterfort wall except that the vertical wall
is tied with the toe of the retaining wall at some spacing. It acts as a compression member to support the
vertical wall and reduces bending moment in it. It also provides support to the toe slab and reduces
bending moment in it. It is provide at spacing equal to approximately one–third of the height of the wall.

Figure 1: The new buttress retaining wall

The Forces acting on retaining wall may be grouped as follows:
a) Self weight of retaining wall.
b) Weight of soil above the foundation base.
c) Earth pressure on retaining wall.
d) Surcharge, i.e. Forces due to loads on the earth surface.
e) Soil reactions on the footing.
f) Frictional Force on the footing due to sliding

Figure 2: Active and passive earth pressures

The main Force that acts on the retaining wall is the lateral pressure induced by the retained earth. It
exists in three different states, i.e. at rest in the active state and in the passive state. The lateral pressure
exerted by soil on an immovable able and in the rigid wall is known as the pressure at rest. Usually such a
state of earth pressure does not exist and the wall tilts and moves slightly under the action of earth
pressure. Even a small movement of the top of the wall away from the earth fills by about 0.1 to 0.5 per
cent of the height of the wall reduces earth pressure appreciably from that occurring at rest state. This
is known as active earth pressure. It is induced by the sliding tendency of the wedge a b c of earth about
plane a b (figure 2). The sliding plane a b makes an angle of 45+ϕ/2 with the horizontal where ϕ is the
angle of repose or angle of internal friction of soil. If the wall is pushed towards the backfill, the earth
pressure increases appreciably above that occurring in the rest state. This is known as passive earth
pressure. It is induced by the sliding tendency of the wedge a c d about the plane a d (figure 2). The
passive earth pressure occurs on the opposite side of the retained earth when the wall yields slightly under the pressure of the backfill. Under normal conditions, the wall deflects slightly resulting in active earth pressure on the side of earth retained and passive earth pressure on the opposite side of the earth retained. The earth pressure is calculated by the Rankin theory based on the assumption that the soil is homogenous, incompressible and cohesion less. The active and passive earth pressure is assumed to increase linearly with depth as a function of the weight of soil. Generally, the following conditions of earth fill occur (figure 3).

a) Level backfill  
b) Sloping backfill  
c) Level backfill with surcharge  
d) Level backfill with submerged soil

Figure 3: Earth pressure
Problem Solving of the New Wall Composed of Reservoirs Shell

Pipes, reservoirs, boilers, and various other vessels under internal pressure exemplify the ax symmetrically loaded cylindrical shell. Expression 2 is an ordinary differential equation with constant coefficients. It also represents the equation of a beam of flexural rigidity $D$, resting on an elastic foundation and subject to loading $P_r$.

\[ \frac{du}{dx} = \nu \frac{w}{a} \]  
\[ \frac{d^2 w}{dx^2} + 4 \beta^4 w = \frac{P_r}{D} \]  
\[ \beta^4 = \frac{Et}{4a^2 D} = \frac{3(1 - \nu^2)}{a^2 \gamma^2} \]  

Where $D$ is the flexural rigidity of the shell, Here:

\[ \beta^4 = \frac{Et}{4a^2 D} = \frac{3(1 - \nu^2)}{a^2 \gamma^2} \]  

We here consider a cylindrical retaining wall (figure 4). The wall bottom is assumed to be built in and the top is open. The physical conditions indicate that the upper edge is free to deform and that no force exists there ($N_x = 0$).

Figure 4: Parameters of shell retaining wall

At the lower edge, locating the coordinates in the figure, we have:

\[ W = 0, \quad \frac{dw}{dx} = 0 \]  

The differential equation is, from Eq. 2:

\[ P_r = - c_a \gamma (H - x) \]  

Where:

\[ \gamma = \text{specific weight of earth}, \]  
\[ C_a = \text{active-earth-pressure coefficient}, \]  

We have:

\[ \frac{d^4 w}{dx^4} + 4 \beta^4 w = - \frac{c_a \gamma (H - x)}{D} \]  

Where the outward pressure acting at any point on the wall, $P_r$, is replaced by:
- cₐ, γ (H-x)

The particular solution is found to be:

\[ F(x) = \frac{P}{4 E t D} = \frac{P a^2}{E t} = - cₐ \gamma (H - x) a^2 \]  

(6)

Let \( F(x) \) represent the particular solution \( w \). It is noted that the results of membrane theory can always be considered as the particular solutions of the equations of bending theory. The general solution is thus:

\[ W = e^{-\beta x} (c₁ \cos \beta x + c₂ \sin \beta x) + e^{\beta x} (c₃ \cos \beta x + c₄ \sin \beta x) - \frac{cₐ \gamma (H - x) a^2}{E t} \]  

(7)

Where \( c₁, c₂, c₃, c₄ \) are arbitrary constants of integration, determined on the basis of the appropriate boundary conditions. If \( t \) is small relative to \( a \) and \( h \), as is usually the case, a longitudinal slice of unit width of the cylinder may be considered infinitely long. Because \( w \) must be finite for all \( x \), \( c₃ = c₄ = 0 \) in the above expression. We now satisfy the remaining conditions.

\[ W = c₁ \frac{cₐ \gamma a^2 H}{E t} = 0 \]

\[ \frac{dW}{dx} = \beta (C₂ - C₁) + \frac{cₐ \gamma a^2}{E t} = 0 \]

From which:

\[ C₁ = \frac{cₐ \gamma a^2 H}{E t} \quad C₂ = \frac{cₐ \gamma a^2}{E t} (H - \frac{1}{\beta}) \]

The radial deflection of the wall is then:

\[ W = - \frac{cₐ \gamma a^2 H}{E t} \left\{ 1 - \frac{x}{H} \right\} e^{\beta x} \left[ \cos \beta x + (1 - \frac{1}{\beta H}) \sin \beta x \right] \]  

(9)

Equation 1, after integration, provides the axial displacement:

\[ \frac{du}{dx} = \nu \frac{w}{a} \quad \rightarrow \quad u = \int_{0}^{x} \nu \frac{w}{a} \, dx + u₀ \]  

(10)

Where the constant, from \( u(0) = 0, \ u₀ = 0 \).

\[ \{ \epsilon₀ = \frac{w}{a}, \ N₀ = \frac{Et}{1 - \nu²} (\epsilon₀ + \nu \epsilonₐ), \ \nu = 0 \} \quad \rightarrow \quad N₀ = - \frac{Etw}{a} \]

\[ N₀ = cₐ \gamma a H \left\{ 1 - \frac{x}{H} \right\} e^{\beta x} \left[ \cos \beta x + (1 - \frac{1}{\beta H}) \sin \beta x \right] \]

(11)

\[ N₀ = cₐ \gamma a \left[ H - H \nu \cos \beta x + \left( \frac{1}{\beta} \right) - H e^{\beta x} \sin \beta x \right] \]

\[ Mₓ = - \frac{d²w}{dx²} \quad D = \frac{Et³}{12(1 - \nu²)} \quad \beta = \left[ \frac{3(1 - \nu²)}{a²t²} \right]^{1/4} \]

\[ Mₓ = \frac{-cₐ \gamma a t}{\sqrt{12(1 - \nu²)}} \left[ H e^{\beta x} \sin \beta x + \left( \frac{1}{\beta} \right) - H e^{\beta x} \cos \beta x \right] \]  

(13)
The maximum bending moment occurs at the bottom of the wall, at x=0:

\[ M_{x, \text{Magz.}} = (1 - \frac{1}{\beta H'}) \left( \frac{c_0 y a H t}{\sqrt{12(1-\nu^2)}} \right) \]  

(14)

**RESULTS AND DISCUSSION**

**Numerical Results of the Shell Retaining Wall**

Amount of bending moment \( (M_x) \) and circumferential forces \( (N_\theta) \) created in the fig. 6 and 7 are presented.

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**Figure 5: Geometrical parameters of new retaining wall**
The Forces acting on retaining walls are customarily taken per unit of width for both gravity and cantilever walls. The new buttress retaining walls may be considered as a unit between joints, or as a unit centered on two buttresses: the gravity wall retains earth entirely by its own dead weight of unreinforced mass of masonry or concrete. This dead weight, gives stability against the thrust of the earth pressure. These walls must be proportioned such that no tension develops in the wall, because the permissible tensile stress of plain concrete is very small. The reaction must lie in the middle third of the base, to overturning of the wall. In a new cylindrical shell retaining wall, much less concrete is used. The confined soil or backfill on the heel provides the dead weight. The reinforced concrete members are designed to with stand the stresses coming on them. The stem acts as a cantilever under the lateral earth pressure, the heel is loaded by the dead weight of the soil above it. The toe is loaded by the soil reaction from below. So reinforcing steel is provided on the tension face of each of these three cantilever elements.

**Figure 6: Amount of circumferential forces (N_0)**

![Graph showing amount of circumferential forces (N_0)](image1)

**Figure 7: Amount of bending moment (M_x)**

![Graph showing amount of bending moment (M_x)](image2)

**Conclusion**

The Forces acting on retaining walls are customarily taken per unit of width for both gravity and cantilever walls. The new buttress retaining walls may be considered as a unit between joints, or as a unit centered on two buttresses: the gravity wall retains earth entirely by its own dead weight of unreinforced mass of masonry or concrete. This dead weight, gives stability against the thrust of the earth pressure. These walls must be proportioned such that no tension develops in the wall, because the permissible tensile stress of plain concrete is very small. The reaction must lie in the middle third of the base, to overturning of the wall. In a new cylindrical shell retaining wall, much less concrete is used. The confined soil or backfill on the heel provides the dead weight. The reinforced concrete members are designed to with stand the stresses coming on them. The stem acts as a cantilever under the lateral earth pressure, the heel is loaded by the dead weight of the soil above it. The toe is loaded by the soil reaction from below. So reinforcing steel is provided on the tension face of each of these three cantilever elements.
Membrane theory cannot, in all instances, provide solutions compatible with the actual conditions of deformation. This theory also fails to predict the state of stress at the boundaries and in certain other areas of the shell. These shortcomings are avoided by application of bending theory, considering membrane forces, and moments to act on the shell structure. Membrane theory is valid at sections away from fixed end of a long, thin cylinder. For a comparatively thick-walled and short cylinder, however, the differences in the results given by the theories are pronounced in the lower half of the shell. In any case, membrane theory applies reasonably well to the upper portion of the retaining wall. In any case, in this paper, the function of the triangular bases of the retaining wall against that of the excluded semicircle of the reservoirs is taken into consideration. In other words, the triangular bases most function as substitutes for the excluded semicircle (the excluded part) of the reservoirs.

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REFERENCES