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Research Article

THE CONNECTIVE ECCENTRIC INDEX FOR AN INFINITE FAMILY OF DENDRIMERS

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ABSTRACT
Let $G=(V,E)$ be a simple connected graph, where $V(G)$ and $E(G)$ be the vertex and edge sets of G. A topological index is a numeric quantity related to G which is invariant under graph automorphisms. The *Connective Eccentric index* $C^{\xi}(G)$ is defined as $\sum_{v\in V(G)} d_v/\varepsilon(v)$. where d_v and $\varepsilon(v)$ denote the degree of vertex v in G and the largest distance between v and any other vertex u of G. In this paper we compute the connective eccentric index for an infinite family of Nanostar Dendrimer.

Keywords: *Nanostar Dendrimer, Connective Eccentric Index, Eccentric Connectivity Index*

INTRODUCTION
Let $G=(V,E)$ be a simple connected molecular graph, the vertex and edge sets of a graph G are denoted by $V(G)$ and $E(G)$, respectively.

Throughout this paper, graph means simple connected graph (Wiener, 1947; Gutman *et al.*, 1972; Consonni, 2000).

If $x, y\in V(G)$ then the distance $d(x,y)$ between x and y is defined as the length of a minimum path connecting x and y.

The *eccentric connectivity index* of the molecular graph G, was proposed by *Sharma*, *Goswami* and *Madan* (Sharma *et al.*, 1997). It is defined as

$$\xi(G)=\sum_{v\in V(G)} d_v \times \varepsilon(v)$$

where d_v denotes the degree of the vertex v in G and, $\varepsilon(v)$ denote the largest distance between v and any other vertex u of G or $\varepsilon(v)=Max\{d(v,u)|\ \forall v\in V(G)\}$. See (Gutman *et al.*, 1986; Johnson *et al.*, 1990; Ashrafi *et al.*, 2009; Alikhani *et al.*, 2011; Ghorbani *et al.*, 2010; Kumar *et al.*, 2004; Fischermann *et al.*, 2002; Sardana *et al.*, 2001; Ashrafi *et al.*, 2009; Dureja *et al.*, 2007; YarAhmadi, 2010; Farahani 2013) for more details.

In 2000, the *Connective Eccentric index* $C^{\xi}(G)$ of the molecular graph G was proposed by Gupta *et al.*, (2000, 2002) and is defined as**:**

$$C^{\xi}(G)=\sum_{v\in V(G)} \frac{d_v}{\varepsilon(v)}$$

The aim of this paper is to compute the Connective Eccentric index for an infinite family of Nanostar Dendrimer.

The Nanostar Dendrimer is a part of a new group of macromolecules that seem photon funnels just like artificial antennas and also is a great resistant of photo bleaching. Recently some people investigated the mathematical properties of these nanostructures in (Newkome *et al.*, 2002; Heydari *et al.*, 2007; Eliasi *et al.*, 2008; Ashrafi *et al.*, 2008; Ashrafi *et al.*, 2009; Ashrafi *et al.*, 2009; Karbasioun *et al.*, 2009; Mirzagar, 2009; Alikhani *et al.*, 2010; Alikhani *et al.*, 2010; Alikhani *et al.*, 2011; Golriz *et al.*, 2011; Husin *et al.*, 2013; Farahani, 2013; Farahani, 2013; Farahani, 2015).

MATERIALS AND METHODS

In this section, we study the Connective eccentric index $C^{\xi}(G)$ and compute its index of an infinite family of Dendrimer.

Theorem 1. Let $D_3[n]$ be the n^{th} growth of Nanostar Dendrimer ($\forall n\geq 1$). Then the Connective Eccentric index $C^{\xi}(G)$ of $D_3[n]$ is equal to

$$C^{\xi}(D_3[n])= 3\times\left(\sum_{i=1,\dots,n}\left(2^{i-1}\right)\left(\frac{3}{5i+5n+5}+\frac{6}{5i+5n+6}+\frac{12}{5i+5n+7}+\frac{12}{5i+5n+8}+\frac{6}{5i+5n+9}\right)+\frac{2^{n-1}-\left(2n+1/_{n+2}\right)}{5(n+1)}\right)$$

Proof of Theorem 1. $\forall n\in\mathbb{N}\cup\{0\}$, consider the n^{th} growth of Nanostar Dendrimer $D_3[n]$. $D_3[n]$ has $21(2^{n+1})-20$ vertices/atoms and $24(2^{n+1}-1)$ bonds/edges (see Figure 1). For further study and more detail of this Benzenoid family, see the paper series (Husin *et al.*, 2013; Farahani, 2013; Farahani, 2013; Farahani, 2015).

The Nanostar Dendrimer $D_3[n]$ has a core depicted in Figure 1 and we define an element as Figure 2 by "*Leaf*".

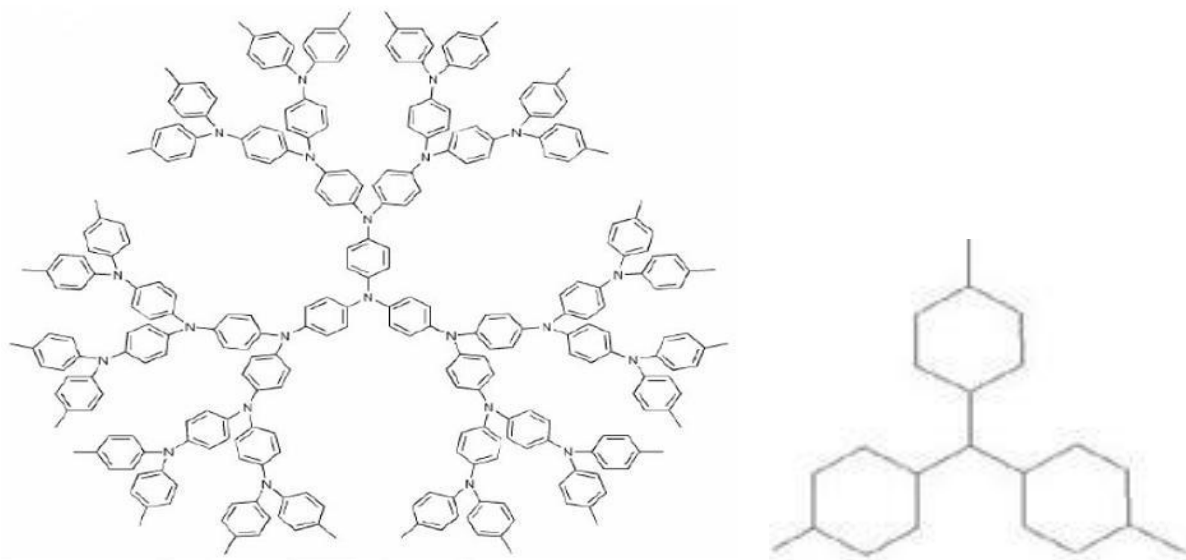

Figure 1: The first kind of Dendrimer of generation 1-3 has grown 3 stages and $D_3[0]$ is the primal structure of Nanostar Dendrimer $D_3[n]$.

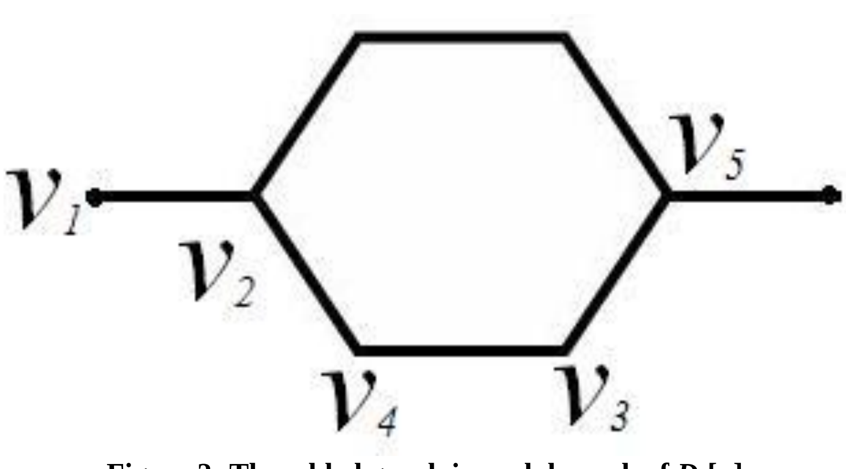

Figure 2: The added graph in each branch of $D_3[n]$.

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It is obvious that a leaf consist of a cycle C_6 and add $3(2^n)$ leafs to $D_3[n-1]$ in the n^{th} growth of Nanostar Dendrimer. Therefore, there exist the number of $\xi_n = 3\sum_{i=0}^{n}(2^i) = 3\left(\frac{2^{n+1}-1}{2-1}\right)$ of leafs (C_6) in Nanostar Dendrimer $D_3[n]$.

Consider the n-1th growth of Nanostar Dendrimer *in* $D_3[n\text{-}1]$ and we would like to construct $D_3[n]$. In every branch of $D_3[n]$, the leaf graph added. From Figure 2, one can see that the maximum eccentricity of a leaf of $D_3[n]$ is 6, and also, the eccentricity of previous vertices of core $D_3[0]$ are equal to 10.

Thus, for eccentric of vertices in added leaf of Nanostar Dendrimer $D_3[n\text{-}1]$ to $D_3[n]$, we can following results:

We have $3(2^{n-1})$ vertices of kind labeled v_1 with eccentricity $5n+5(n+1)$ and have $3(2^n)$ vertices of kind labeled v_2 with eccentricity $5n+1+5(n+1)$.

Also there are $3(2^{n+1})$ vertices of v_3 and v_4, with eccentricity $10n+7$ and $5n+8$, respectively. And $3(2^n)$ vertices v_5, its eccentricity is $10n+9$.

Also, from the general representation of Nanostar Dendrimer $D_3[n]$ in Figure 1, it is easy to see that all Hydrogen atoms in added leafs have degree 1 and all other vertices/atoms have degree three.

Thus, we have following computations for the Connective Eccentric index C^{ξ} of the n^{th} growth of Nanostar Dendrimer $D_3[n]$, $\forall n \in \mathbb{N}$ as:

$$C^{\xi}(D_3[n]) = \sum_{v \in V(D_3[n])} \frac{d_v}{\varepsilon(v)}$$

$$= \sum_{\substack{vertex\ labeled\ v_1 in D_3[i] \\ i=1,\dots,n}} \frac{d_{v_1}}{\varepsilon(v_1)} + \sum_{\substack{vertex\ labeled\ v_2 in D_3[i] \\ i=1,\dots,n}} \frac{d_{v_2}}{\varepsilon(v_2)} + \sum_{\substack{vertex\ labeled\ v_3 in D_3[i] \\ i=1,\dots,n}} \frac{d_{v_3}}{\varepsilon(v_3)}$$

$$+ \sum_{\substack{vertex\ labeled\ v_4 in D_3[i] \\ i=1,\dots,n}} \frac{d_{v_4}}{\varepsilon(v_4)} + \sum_{\substack{vertex\ labeled\ v_5 in D_3[i] \\ i=1,\dots,n}} \frac{d_{v_5}}{\varepsilon(v_5)} + \sum_{\substack{Hydrogen\ atoms\ of \\ D_3[n]}} \frac{d_H}{\varepsilon(H)}$$

$$= \frac{3}{5n+5} + \sum_{i=2,\dots,n} 3\left(2^{i-1}\right)\frac{3}{5i+5n+5} + \sum_{i=1,\dots,n} 3\left(2^{i}\right)\frac{3}{5i+5n+6} + \sum_{i=1,\dots,n} 3\left(2^{i+1}\right)\frac{2}{5i+5n+7}$$

$$+ \sum_{i=1,\dots,n} 3\left(2^{i+1}\right)\frac{2}{5i+5n+8} + \sum_{i=1,\dots,n} 3\left(2^{i}\right)\frac{3}{5i+5n+9} + \frac{3\left(2^n\right)}{10(n+1)}$$

$$= \sum_{i=1,\dots,n} \frac{3^2\left(2^{i-1}\right)}{5i+5n+5} + \sum_{i=1,\dots,n} \frac{3^2\left(2^{i}\right)}{5i+5n+6} + \sum_{i=1,\dots,n} \frac{3\left(2^{i+2}\right)}{5i+5n+7} + \sum_{i=1,\dots,n} \frac{3\left(2^{i+2}\right)}{5i+5n+8}$$

$$+ \sum_{i=1,\dots,n} \frac{3^2\left(2^{i}\right)}{5i+5n+9} + \frac{3\left(2^n\right)}{10(n+1)} - \frac{3(2n+1)}{5\left(n^2+3n+2\right)}$$

$$= 3\times\left(\sum_{i=1,\dots,n}\left(2^{i-1}\right)\left(\frac{3}{5i+5n+5}+\frac{6}{5i+5n+6}+\frac{12}{5i+5n+7}+\frac{12}{5i+5n+8}+\frac{6}{5i+5n+9}\right)+\frac{2^{n-1}-\left(2n+1\big/ n+2\right)}{5(n+1)}\right)$$

And this completed the proof of Theorem 1. □

Reader can see some values of the Connective Eccentric index C^{ξ} of $D_3[n]$ for integer n=1, 2, 3,..., 1000 in following table.

Table 1: Some values of the Connective Eccentric index $C^{\xi}(D_3[n])$ for integer n=1, 2, 3,..., 1000

n	$\|V(D_3[n])\|$	$\|E(D_3[n])\|$	Connective Eccentric index $C^{\xi}(D_3[n])$
1	64	72	6.79001547987616
2	148	168	13.9928138178663
3	316	360	24.490067851166
4	652	744	41.550751144813
5	1324	1512	117.660936869584
6	2668	3048	121.30647812691
7	5356	6120	211.314999770378
8	10732	12264	373.056233480523
9	21484	24552	666.56944159841
10	42988	49128	1203.57519017428
20	44040172	50331624	622223.87780502
30	45097156588	51539607528	426199600.797565
40	46179488366572	52776558133224	327912915489.941
50	4.72877960873902E16	5.40431955284459E16	268928103252621
60	4.84227031934876E19	5.53402322211287E19	2.2966089766362E17
70	4.95848480701313E22	5.66683977944357E22	2.01688378022574E20
80	5.07748844238144E25	5.80284393415022E25	1.8078866983509E23
90	5.1993481649986E28	5.94211218856983E28	1.64612017251685E26
100	5.32413252095856E31	6.0847228810955E31	1.51746610005702E29
200	6.74913978588776E61	7.71330261244315E61	9.61943682075734E58
300	8.55555110060484E91	9.77777268640553E91	8.13551180348495E88
400	1.0845449487965E122	1.23947994148172E122	7.73769523182308E118
500	1.37482405531638E152	1.57122749179015E152	7.84876851937877E148
600	1.74279653893002E182	1.99176747306288E182	8.29253049846576E178
700	2.20925707865032E212	2.52486523274322E212	9.01131242489797E208
800	2.80056606180954E242	3.20064692778233E242	9.99613017109719E238
900	3.55013924923167E272	4.05730199912191E272	1.12643772146726E269
1000	4.50033615018232E302	5.14324131449408E302	1.28520346166618E299

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