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Research Article

AN ANALYSIS OF ANISOTROPIC COSMOLOGICAL MODEL OF UNIVERSE WITH VARIABLE COSMOLOGICAL TERM

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ABSTRACT
Einstein's field equations with variable gravitational and cosmological constant are considered for Bianchi type-I universe by assuming that the cosmological term $\Lambda \propto R^{-n}$(R is scale factor and n is positive constant).We wish here to desire some results in Bianchi I cosmology with variable $G \,\&\, \Lambda$ using a slightly different method from that of fore researchers and, to connect on his results.

INTRODUCTION
The recent most remarkable observational discoveries by many authors showed that the universe expands with acceleration; many models have been proposed to explain this phenomenon. A variable Λ has been studied by many authors, reference to whom could be found in Harko and Mak (1999) who considered particle creation in cosmological models with varying gravitational and cosmological "constants". We vary only the cosmological "constant" with suitable modifications in Einstein field equations. Bianchi models have been studied by several authors in an attempt to achieve better understanding of the observed small account of anisotropy in the Universe. Chodos and Detweller (1980), Lorentz-Petzold (1985), Ibanez and Verdaguer (1986), Gleiser and Diaz(1988), Banerjee and Bhui (1990), Reddy and Venkateswara (2001), Khadekar and Gaikwad (2002), Adhav and others, have studied the multi-dimensional cosmological models in general relativity and in other alternative theories of gravitation.
The law of variation of scale factor which was proposed by Pavon (1991) may result solution to field equation. There are numerous subjective elements which can be explored the behavior of the cosmological scale factor $R(t)$ in solution to Einstein's field equation with Robertson-Walker line element. In recent years, various papers were published where both parameters $G \,\&\, \Lambda$ varied together in a way that leaves Einstein's equations unchanged, this occurs due to numerous reason. The concept of variable gravitational constant G in the framework was first proposed by Dirac(1937), working on this framework of general relativity , Lau(1985) proposed a modification linking the variation of G with Λ.It allows us to apply Einstein's field equations unchanged ,since variation in Λ is accompanied by a variation of G . A number of researchers have used the approach by exploring FRW models and Bianchi models. (Abdel-Rahman 1990; Berman 1990; Sistero 1991; kalligas et al. 1992; Abdussattar & Vishwakarma 1997; Vishwakarma 2000,2005; Pradhan & Otratod 2006; Singh et al. 2007; Singh & Tiwari 2008; Borges & Carneiro 2005) have considered that the cosmological term is proportional to Hubble parameter in FRW model and Bianchi type I model with variable $G \,\&\, \Lambda$. The cosmological models with variable $G \,\&\, \Lambda$ have been recently studied by several authors (Arab 2003; Sistero 1991; Sattar and Vishwakarma 1997; Pradhan et al., 2001, 2002, 2005, 2007; Singh et al., 2006, 2007).
For studying the possible effects of anisotropy in the early universe on present day observations many researchers (Huang 1990; Chimento et al. 1997; Lima 1996; Lima and Carvalho 1994; Pradhan et al. 2004, 2006; Saha 2005, 2006) have investigated Bianchi type-I models from different point of view.
In the present paper, we present a new class of solutions to Einstein's field equations with variable $G \,\&\, \Lambda$ in Bianchi type-I space-time in the presence of a perfect fluid, the behavior of the scale factor, and effect on models due to variable $G \,\&\, \Lambda$ are investigated. The paper has following structure. In

section 2, the metric and field equations are described. In section 3, solution of field equations are given. In section 4, at the end we discuss the models and conclude the results.

Metric and Field Equations

The line element for Bianchi type- I space time is taken in the form-

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 dy^2 + c^2 dz^2 \qquad (1)$$

Where a, b, c are functions of t only.

The Energy momentum tensor for a perfect fluid is $T_{ij} = (\rho + p)V_i V_j + pg_{ij}$, (2)

Where ρ energy density of cosmic matter, p is its pressure, v_i is the four velocity s.t. $v_i v^i = 1$; satisfying the equation of state $p = \gamma\rho, o \le \gamma \le 1$ (3)

The Einstein field equations with varying G & Λ in suitable units are

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G(t) T_{ij} + \Lambda(t) g_{ij} \qquad (4)$$

Spatial volume as a average scale factor of the model (1) may be defined as-

$$R = (abc)^{1/3} \qquad (5)$$

The Einstein's field equation (4) for metric and energy momentum tensor (2) read as

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}\dot{c}}{bc} = -8\pi Gp + \Lambda, \qquad (6)$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{c}}{ac} = -8\pi Gp + \Lambda \qquad (7)$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} = -8\pi Gp + \Lambda \qquad (8)$$

$$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{a}\dot{c}}{ac} = 8\pi G\rho + \Lambda \qquad (9)$$

In view of the vanishing divergence of the Einstein's tensor, equation (4) gives

$$\dot{\rho} + (\rho + p)(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}) + \rho\frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0 \qquad (10)$$

We now assume energy conservation equation $T_{ij} = 0$ yields

$$\dot{\rho} + (\rho + p)(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}) = 0 \quad \text{Or,} \quad \dot{\rho} + \rho(1+\gamma)(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}) = 0 \qquad (p = \gamma\rho) \qquad (11)$$

Equation (10) together with (11) is given by

$$\rho\frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0, i.e. \dot{\Lambda} = -8\pi\rho\dot{G} \qquad (12)$$

(Dot denotes differentiation wrt "t")

Equation (12) implies that G increases or decreases as Λ decreases or increases.

Using (5), (11) takes the form $\frac{\dot{\rho}}{\rho} + (1+\gamma)\frac{d(R^3)}{R^3} = 0 \Rightarrow \frac{\dot{\rho}}{\rho} = -(1+\gamma)\frac{d(R^3)}{R^3}$

Integrating we find $\rho = \frac{k_1}{R^{3(1+\gamma)}}$, where k_1 is integration constant (13)

Solutions of Field Equations

Subtracting equation (6) from (7),

$$\frac{\ddot{a}}{a}-\frac{\ddot{b}}{b}+\frac{\dot{a}\dot{c}}{ac}-\frac{\dot{b}\dot{c}}{bc}=0 \text{ i.e. } \ddot{a}bc-a\ddot{b}c+\dot{a}b\dot{c}-a\dot{b}\dot{c}=0$$

Or, $$\frac{d}{dt}(\dot{a}bc)-\frac{d}{dt}(a\dot{b}c)=0 \qquad (14)$$

Similarly subtracting (6) & (7) from (8),

$$\frac{d}{dt}(\dot{a}bc)-\frac{d}{dt}(ab\dot{c})=0 \qquad (15)$$

$$\frac{d}{dt}(a\dot{b}c)-\frac{d}{dt}(ab\dot{c})=0 \qquad (16)$$

Integrating (14), (15) & (16) gives

$$\frac{\dot{a}}{a}-\frac{\dot{b}}{b}=\frac{k_2}{abc}=\frac{k_2}{R^3} \qquad (17)$$

$$\frac{\dot{a}}{a}-\frac{\dot{c}}{c}=\frac{k_3}{R^3} \qquad (18)$$

$$\frac{\dot{b}}{b}-\frac{\dot{c}}{c}=\frac{k_4}{R^3} \qquad (19)$$

Again integrating (17),(18) & (19)

$$\frac{a}{b}=m_1\exp\left[k_2\int\frac{1}{R^3}dt\right] \qquad (20)$$

$$\frac{a}{c}=m_2\exp\left[k_3\int\frac{1}{R^3}dt\right] \qquad (21)$$

$$\frac{b}{c}=m_3\exp\left[k_4\int\frac{1}{R^3}dt\right] \qquad (22)$$

Where $k_2, k_3, k_4, m_1, m_2, m_3$ are integration constants. There are only six equations in seven unknowns ($a, b, c, \rho, p, G, \Lambda$), so we need one extra equation to solve the system completely. We may consider $\Lambda \propto R^{-n}$, since many authors have considered as Λ decay. Chen and Wu (1990) considered $\Lambda \propto R^{-2}$ (R is scale factor) , Hoyle et al. (1997) considered $\Lambda \propto R^{-3}$ and $\Lambda \propto R^{-m}$ was considered by Olson & Jordan (1987), Pavon(1991),Maia & Silva (1994), Silveira & Waga(1994) , Bloomfield Torres & Waga (1996). Thus we take the decaying vacuum energy density $\Lambda \propto R^{-n}$ i.e. $\Lambda = k_5 / R^n$, (23) where k_5 & n are positive constants.

Substituting equation (13) & (23) into (12), $\dot{G}=-\dot{\Lambda}/8\pi\rho=\frac{nk_5}{8\pi k_1}R^{-\{(n+1)-3(1+\gamma)\}}$

Integrating, $$G=\frac{nk_5}{8\pi k_1(-n+3+3\gamma)}R^{(-n+3+3\gamma)} \qquad (24)$$

The Hubble parameter H , deceleration parameter q are given by $H = \frac{\dot{R}}{R}$ (25)

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{R\ddot{R}}{\dot{R}^2} \quad (26)$$

Equations 6-9 and (11) can be written in terms of $H, \sigma, \& q$ as

$$H^2(2q-1) - \sigma^2 = 8\pi G p - \Lambda \quad (27) \ ,$$

$$3H^2 - \sigma^2 = 8\pi G\rho + \Lambda \quad (28)$$

$$\dot{\rho} + 3(p+\rho)\frac{\dot{R}}{R} = 0 \quad (29)$$

From (27) and (28) $4H^2 - 2qH^2 + 8\pi G(p-\rho) - 2\Lambda = 0$, or

$2\left(\frac{\dot{R}}{R}\right)^2 - \frac{k_5}{R^n} - qH^2 + 4\pi G(p-\rho) = 0$, using $p = \gamma\rho \ \& \ q = -1 - \frac{\dot{H}}{H^2}$

$$2\left(\frac{\dot{R}}{R}\right)^2 - \frac{k_5}{R^n} - \left(-1 - \frac{\dot{H}}{H^2}\right)H^2 + 4\pi G(\gamma - 1)\rho = 0$$

$$2\left(\frac{\dot{R}}{R}\right)^2 - \frac{k_5}{R^n} + \left(H^2 + \dot{H}\right)H^2 - 4\pi G(1-\gamma)\rho = 0$$

$$2\left(\frac{\dot{R}}{R}\right) - \frac{k_5}{R^n} + \frac{\ddot{R}}{R} - 4\pi G(1-\gamma)\rho = 0 \qquad \left(H^2 + \dot{H} = \frac{\ddot{R}}{R}\right)$$

$$2\left(\frac{\dot{R}}{R}\right)^2 - \frac{k_5}{R^n} + \frac{\ddot{R}}{R} - 4\pi G(1-\gamma)\frac{k_1}{R^{3(1+\lambda)}} = 0$$

i.e. $\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2 - \frac{k_5}{R^n} - \frac{n(1-\gamma)k_5}{2R^n(3\gamma + 3 - n)} = 0$ (30)

Now we will discuss for γ =0, by which we will analyze nature of model, $\Lambda, \rho, G, \Omega, \rho_c, \rho_v$.

Case I:

For $\gamma = 0$, equation (30) gives

$$\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2 - \frac{k_5(n-6)}{2R^n(n-3)} = 0 \quad (31)$$

$$\Rightarrow \frac{R\ddot{R} + 2\dot{R}^2}{R^2} = \frac{k_5(n-6)}{2(n-3)R^n}$$

$$\Rightarrow \frac{d(R^2\dot{R})}{R} = \frac{k_5(n-6)R^{2-n}}{2(n-3)}$$

$$\Rightarrow d(R^2\dot{R}) = \frac{k_5(n-6)R^{3-n}}{2(n-3)}$$

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Integrating, we obtain

$$R^2\dot{R} = \frac{k_5(n-6)R^{4-n}}{2(n-3)(4-n)} \Rightarrow \dot{R} = \frac{k_5(n-6)R^{2-n}}{2(n-3)(4-n)} \Rightarrow \frac{dR}{dt} = \frac{k_5(n-6)R^{2-n}}{2(n-3)(4-n)}$$

Again Integrating,

$$, \quad R = \left[\frac{k_5(n-1)(n-6)}{2(n-3)(4-n)}t + c_1\right]^{1/n-1} \tag{32}$$

$$\Rightarrow \frac{\dot{R}}{R} = \frac{k_5(n-6)}{2(n-3)(4-n)}\left[\frac{k_5(n-1)(n-6)}{2(n-3)(4-n)}t + c_1\right]^{-1} \tag{33}$$

$$\Rightarrow \frac{\dot{R}}{R} = A[Bt + c_1]^{-1}$$

Where $A = \frac{k_5(n-6)}{2(n-3)(4-n)}$, $B = \frac{k_5(n-1)(n-6)}{2(n-3)(4-n)}$ and c_1 is constant of integration.

Spatial Volume $V = R^3 = [Bt + c_1]^{3/n-1}$ (34)

By inserting equation (32) into (17) & (18)

$$a = bm_1 \exp\left[k_2\{Bt + c_1\}^{n-1/n-4}\left(\frac{n-1}{n-4}\right)\right] \tag{35}$$

$$b = cm_3 \exp\left[k_4\{Bt + c_1\}^{n-4/n-1}\left(\frac{n-1}{n-4}\right)\right] \tag{36}$$

$$a = cm_2 \exp\left[k_3\{Bt + c_1\}^{n-4/n-1}\left(\frac{n-1}{n-4}\right)\right] \tag{37}$$

From (5), (35), (36) and (37), we obtain

$$(Bt + c_1)^{3/n-1} = cm_2\, e^{k_3\left(\frac{n-1}{n-4}\right)\{Bt+c_1\}^{\frac{n-4}{n-1}}} .cm_3 e^{k_4\left(\frac{n-1}{n-4}\right)\{Bt+c_1\}^{\frac{n-4}{n-1}}} .c \tag{38}$$

$$c^3 = \frac{1}{m_2 m_3}\{Bt + c_1\}^{\frac{3}{n-1}} .e^{-\left(\frac{n-1}{n-4}\right)\left[(k_3+k_4)\{Bt+c_1\}^{\frac{n-4}{n-1}}\right]} \tag{39}$$

$$c = \frac{1}{(m_2 m_3)^{1/3}}\{Bt + c_1\}^{1/n-1} .e^{-\frac{1}{3}\left(\frac{n-1}{n-4}\right)\left[(k_3+k_4)\{Bt+c_1\}^{\frac{n-4}{n-1}}\right]} \tag{40}$$

Putting this value of c into equation (37), we get

$$a = \frac{(m_2)^{2/3}}{(m_1)^{1/3}}\{Bt + c_1\}^{1/n-1}\, e^{-\left(\frac{n-1}{n-4}\right)\left[k_4\{Bt+c_1\}^{n-4/n-1}\right]} \tag{41}$$

$$b = \frac{(m_3)^{2/3}}{(m_2)^{1/3}}(Bt + c_1)^{1/n-1} .e^{\frac{1}{3}\left(\frac{n-1}{n-4}\right)(2k_4-k_3)(Bt+c_1)^{n-4/n-1}} \tag{42}$$

Using (40), (41), (42), the metric (1) has the form

$$ds^2 = -dt^2 + \left\{ \frac{(m_2)^{2/3}}{(m_1)^{1/3}} \{Bt + c_1\}^{1/n-1} e^{-\left(\frac{n-1}{n-4}\right)\left[k_4 \{Bt+c_1\}^{n-4/n-1}\right]} \right\}^2 dx^2 + \left\{ \frac{(m_3)^{2/3}}{(m_2)^{1/3}} \left(Bt + c_1\right)^{1/n-1} .e^{\frac{1}{3}\left(\frac{n-1}{n-4}\right)(2k_4 - k_3)(Bt+c_1)^{n-4/n-1}} \right\}^2 dy^2$$

$$+ \left\{ \frac{1}{(m_2 m_3)^{1/3}} \{Bt + c_1\}^{1/n-1} .e^{-\frac{1}{3}\left(\frac{n-1}{n-4}\right)\left[(k_3+k_4)\{Bt+c_1\}^{\frac{n-4}{n-1}}\right]} \right\}^2 dz^2$$

Where m_1, m_2, m_3 are constants. (43)

From equation (13) and (32), we obtain

$$\rho = k_1 R^{-3(1+\gamma)} = k_1 \{Bt + c_1\}^{-\frac{3(1+\gamma)}{n-1}}$$

$$\Rightarrow \rho = k_1 \left(Bt + c_1\right)^{-\frac{3}{n-1}} \qquad \text{(Since } \gamma=0\text{)} \qquad (44)$$

From eqn. (23) and eqn. (32)

$$\Lambda = k_5 R^{-n} = k_5 \left(Bt + c_1\right)^{-\left(\frac{n}{n-1}\right)} \qquad (45)$$

From (24)

$$G = \frac{nk_5}{8\pi k_1(-n+3+3\gamma)} R^{(-n+3+3\gamma)} = \frac{nk_5}{8\pi k_1(-n+3+3\gamma)} \left(Bt + c_1\right)^{\frac{3\gamma+3-n}{n-1}}$$

$$\Rightarrow G = \frac{nk_5}{8\pi k_1 (3-n)} \left(Bt + c_1\right)^{3-n/n-1} \qquad (46)$$

The vacuum energy density ρ_v and critical density ρ_c are given by

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{3A^2 k_1 (3-n)}{nk_5} \left(Bt + c_1\right)^{-\frac{(n+1)}{n-1}} \qquad (47)$$

(Using 33 and 46)

And $\rho_v = \dfrac{\Lambda}{8\pi G} = \dfrac{k_1(3-n)}{n} \left(Bt + c_1\right)^{-\frac{3}{n-1}}$ (48)

(Using 45 and 46)

Density parameter $\Omega = \dfrac{\rho}{\rho_c} = \dfrac{8\pi G \rho}{3H^2} = \dfrac{8\pi n k_5 k_1}{3A^2} \left(Bt + c_1\right)^{\frac{n-2}{n-1}}$ (49)

(Using 33, 44, 46)

Expansion Scalar $\theta = 3H = 3A\left(Bt + c_1\right)^{-1}$ (50) (Using 33)

Shear $\sigma = \dfrac{k_1}{\sqrt{3}R^3} = \dfrac{k_1}{\sqrt{3}} \left(Bt + c_1\right)^{-\frac{3}{n-1}}$ (51) (Using 33)

For the model (43), we find that 0<n<4, the spatial volume V is zero at $t = -\dfrac{c_1}{B}$ and expansion scalar is infinite at $t = -\dfrac{c_1}{B}$.It shows that the universe starts evolving with zero volume with an infinite rate of expansion. The scale factor R is also zero at $t = -\frac{c_1}{B}$, it means that during initial age the space –time exhibits a point type singularity. At $t = -\frac{c_1}{B}$, $\rho \to \infty, \sigma \to \infty$ .As t increases ,the scale factor and spatial volume increases but the expansion scalar decreases .i.e. rate of expansion slows down. Grav. Constant $G \to 0$ at $t = -\frac{c_1}{B}$ and when $t \to \infty$ then $G \to \infty$ .When $t \to \infty$ then $R \to \infty, V \to \infty, \Lambda \to 0,$ and ρ, ρ_c, ρ_v all tends to zero.Therefore model gives an empty universe for $t \to \infty$. This result is in agreement with observations obtained by many astronormers(Knop et al. 2003; Riess et al. 1998, 2004; Perlmutter et al. 1998, etc.).

Conclusion

In this paper, we have presented a class of solutions to Einstein's field equations with variable $G \& \Lambda$ in Bianchi type-I space time in the presence of a perfect fluid. By using a law of variation of scale factor with a variable cosmological term the solution of field equations has been obtained. A cosmological term $\Lambda \propto R^{-n}$ (where R is scale factor). In some cases, it is found that G is an increasing function of time. The possibility of an increasing G has been suggested by several authors [16,21-25].Beesham (1994), Lima and Carvalho (1994), Kallingas et al. (1995) and Lima (1996) have also derived the Bianchi type-I cosmological models with variable $G \& \Lambda$ assuming a particular form for Λ.The scale factor also vanish at $t = -\frac{c_1}{B}$ and hence the space -time exhibits a point singularity during the initial epoch. Thus, the model would essentially give an empty universe for large time t. The cosmological term Λ determines the behavior of model in the universe. A positive value of Λ corresponds to a negative effective mass density. Therefore, we expect that in the universe with a positive value of Λ, the expansion will tend to accelerate; whereas in the universe with negative value of Λ, the expansion will slow down, stop and reverse. Recent cosmological observations (Garnavich et al. 1998; Perlmutter et al. 1997, 1998, 1999; Riess et al. 1998, 1999, 2004; Schmidt et al. 1998) suggest the existence of a positive cosmological constant Λ.

In this paper we have extended previous work by Kalligas; Wesson and Everitt (1995) and other many researchers on isotropic cosmologies with variable $G \& \Lambda$ to anisotropic Bianchi type-I cosmologies.

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