International Journal of Applied Engineering and Technology ISSN: 2277-212X (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jet.htm
2014 Vol. 4 (1) January-March, pp.46-53/Mary and Jenitta
Research Article

SECOND OPTIONAL SERVICE QUEUING MODEL WITH FUZZY PARAMETERS

***K. Julia Rose Mary[1] and P. Angel Jenitta[2]**
[1]*Department of Mathematics, Nirmala College for Women (Autonomous), Coimbatore-18, Tamil Nadu, India*
[2]*Nirmala College for Women (Autonomous), Coimbatore-18, Tamil Nadu, India*
**Author for Correspondence*

ABSTRACT
The aim of this paper is to analyze Bi-level threshold policy of $\mathrm{M}^{\mathrm{X}}/(\mathrm{G}_1, \mathrm{G}_2)/1$ with single vacation (SV) queue in a fuzzy environment. A mathematical Non linear programming (NLP) method is used to construct the membership function of the system characteristic in which the batch arrival-rate, expected group size, setup time, vacation time, service time for first essential service(FES), service time for second optional service(SOS) and probability for opting second channel are all fuzzy numbers. The α-cut and zadeh's extension principle are used to transform a fuzzy queue into a family of conventional crisp queue. By means of the membership functions of the system characteristic, a set of parametric NLP is developed to calculate the lower and upper bound of the system characteristics function at α. Moreover a numerical example is also illustrated to check the validity of the proposed approach.

***Keywords**: Bi–Level Threshold Policy, First Essential Service, Second Optional Service, Early Setup, Vacation Policy, α-Cut, Membership Function and Zadeh's Extension Principle*

INTRODUCTION
The first study of batch arrival queuing system with N policy was carried out by Lee and Srinivasan (1989). In their paper they have discussed the mean waiting time of an arbitrary customer and a procedure to find the stationary optimal policy under a linear cost structure. Later, many authors including Lee *et al.*, (1994a, 1994b) have analyzed the N policy of $M^X/G/1$ queuing models with servers having multiple and single vacation. But these models do not involve the server's setup time. In queuing models, server's setup time corresponds to the preparatory work of the server before starting his service. Hur and Park (1999) and Ke (2001) are some of the authors who analyzed the N policy of $M^X/G/1$ queuing models with server's setup time. The batch arrival queuing system with double threshold policy, setup time and vacation are analyzed by Lee *et al.,* (2003) is among the most general queuing system with threshold policies

But in everyday life there are queuing situations where all the arriving customers require the first essential service and some may require the second optional service provided by the same server. Madan (2000) has introduced the concept of second optional service, where the customers may depart from the system either with probability (1-r) or may immediately opt for second optional service with probability r. Choudhury and Paul (2006) have extended the results of Madan. Later Madan and Choudhury (2006) have studied the steady state analysis of the $M^X/(G_1, G_2)/1$ queue with restricted admissibility. Recently Julia Rose Mary *et al.*, (2011) introduced Bi-level threshold policy of $M^X/(G_1, G_2)/1$/SV queue. In this model they obtained the stationary probability generating function of the queue length distribution through supplementary variable techniques and derived the expected length of the cycle (orbit size). The various performance measures were also calculated. In their paper, the inter arrival time, service times, setup time, vacation time are assumed to follow certain probability distributions with fixed parameters.

In real life in many situations the parameter may only be characterized subjectively (i.e) the system parameters are both possibilistic and probabilistic. Thus fuzzy analysis would be potentially much more useful and realistic than the commonly used crisp concepts. Li and Lee (1989) investigated analytical results for two fuzzy queues using a general approach based on Zadeh's extension principle. Nagi and Lee (1992) proposed a procedure using α cut and two variable simulations to analyze fuzzy queues. Using

© Copyright 2014 | Centre for Info Bio Technology (CIBTech)

Research Article

parametric programming Kao *et al.*, (1999) constructed the membership function of system characteristics for fuzzy queues. Chauan *et al.*, (2007) have obtained the membership function of system characteristics of a retrial queuing model with fuzzy arrival, retrial and service rate. Jeeva and Rathnakumari (2012) analyzed a batch arrival single server, Bernoulli feedback queue with fuzzy vacations. Recently Ramesh and Kumara (2013) also introduced a batch arrival queue with multiple servers and fuzzy parameters. With the help of these available literatures we determine the bi-level threshold policy of bulk arrival queue with second optional service and single vacation in fuzzy environment.

Model Description

Consider bi-level threshold policy of $M^X/(G_1, G_2)$/1/SV queue. The orbit size and the normal queue size are assumed to be infinite. The customer arrives according to the compound Poisson process with random arrival size X. The server is turned off and leaves the system for a vacation of random length V, each time when the system becomes empty. After returning from the vacation if the server finds the system size is less than $\boldsymbol{m}$ then he remains idle (build up period) in the system until the queue length reaches at least $\boldsymbol{m}$. On the other hand, if the server finds $\boldsymbol{m}$ or more customers in the system at the end of the vacation then he immediately starts the setup operation which takes the random length D (early setup).After the setup period, if the number of customers in the queue is less than N (N≥ $\boldsymbol{m}$), then the server remains again idle (dormant) until the queue length reaches at least N. Instead, at the end of the set up period if the server finds more than N customers waiting in the system then the server immediately begins to serve the customers. Here, the idle period of the server is made up of vacation period; build up period, setup period and dormant period.

If the queue length reaches N or more either at the end of the setup period or at the end of the dormant period, the server begins a busy period. During busy period the server provides to each unit, two stages of heterogeneous service of which one is optional, that is , the server begins to serve the first phase of essential service(FES) for all the units. After the completion of FES of a unit, the customer may leave the system with probability(1-r) or may opt for a second optional service(SOS) in an additional channel by the same server with probability r (0 ≤ r ≤ 1). The server continues this type of service until the system becomes empty and then turned off the system. Thus a cycle is completed. The system will be turned on again for setup when at least $\boldsymbol{m}$ customers are present in the system. The service times s_1 and s_2 of two channels (FES and SOS) are assumed to be mutually independent of each other. According to this model the expected length of the cycle (orbit size) and mean system size E(N) are obtained as

$$E(T_{cycle}) = \frac{E(D)+E(V)+\sum_{n=0}^{m-1}\frac{\varphi_n}{\lambda}+\sum_{n=m}^{N-1}\frac{\varphi_n}{\lambda}}{1-\lambda E(X)[E(s_1)+rE(s_2)]} \quad (1)$$

$$E(N) = (\lambda E(X))^2(E({s_1}^2) + 2rE(s_1)E(s_2) + rE({s_2}^2)) + \frac{\lambda E(X(X-1))(rE(s_2)+E(s_1))}{2(1-(\lambda E(x)[E(s_1)+rE(s_2)])} + \frac{1}{E(D)+E(V)+\sum_{n=0}^{m-1}\frac{\varphi_n}{\lambda}+\sum_{n=m}^{N-1}\frac{\varphi_n}{\lambda}}\left[\frac{\lambda E(V)(E(D^2)+2E(D)E(V)+E(V^2)}{2} + \frac{\sum_{n=0}^{m-1} n\psi_n}{\lambda} + \sum_{n=0}^{m-1} n\psi_n\, E(D)E(X) + \frac{\sum_{n=m}^{N-1} n\varphi_n}{\lambda}\right] \quad (2)$$

RESULT

We extend the above queuing system in fuzzy environment. Suppose the arrival rate λ, expected group size X, setup time D, vacation time V, service time for FES S_1, service time for SOS S_2 and probability for opting second channel r are approximately known and can be represented as fuzzy sets $\overline{\lambda}, \overline{X}, \overline{D}, \overline{V}, \overline{S_1}, \overline{S_2}$ *and* $\overline{r}$. Using α-cuts for the arrival rate, expected group size, setup time, vacation time, service times and probability for opting second channel, which are represented by different levels of confidence.

Let this interval of confidence be represented by $[x_{1\alpha}, x_{2\alpha}]$. Since the probability distribution for the α cuts can be represented by uniform distributions. We have P $(x_\alpha) = \frac{1}{x_{2\alpha}-x_{1\alpha}} [x_{1\alpha} \le x_\alpha \le x_{2\alpha}]$ Then the

© Copyright 2014 | Centre for Info Bio Technology (CIBTech)

mean and the second order moment of the distribution are obtained as $1/2[x_{2\alpha} + x_{1\alpha}]$ and $\frac{{x_{2\alpha}}^3 - {x_{1\alpha}}^3}{3(x_{2\alpha} - x_{1\alpha})}$. Further its variance is given by $1/12[x_{2\alpha} - x_{1\alpha}]^2$

Let $\eta_{\bar{\lambda}}$(y), $\eta_{\bar{X}}$(x), $\eta_{\bar{D}}$(u), $\eta_{\bar{V}}$(v), $\eta_{\bar{S_1}}$(s), $\eta_{\bar{S_2}}$(t), $\eta_{\bar{r}}$(q) denote the membership function of $\bar{\lambda}, \bar{X}, \bar{D}, \bar{V}, \bar{S_1}, \bar{S_2}, \bar{r}$ respectively. Now we have the following fuzzy sets as,

$\bar{\lambda}$ = { (y , $\eta_{\bar{\lambda},}(y)$) / y ∈ Y} (3)

$\bar{X}$ = { (x , $\eta_{\bar{X}}(x)$ / x ∈ X} (4)

$\bar{D}$ = { (u , $\eta_{\bar{D}}(u)$ / u ∈ U} (5)

$\bar{V}$ = { (v , $\eta_{\bar{V}}(v)$ / v ∈ V} (6)

$\bar{S_1}$ = { (s , $\eta_{\bar{S_1}}(s)$ / s ∈ S } (7)

$\bar{S_2}$ = { (t , $\eta_{\bar{S_2}}(t)$ / t ∈ T } (8)

$\bar{r}$ = { (q , $\eta_{\bar{r}}(q)$ / q ∈ Q} (9)

where Y,X,U,V,S,T and Q are the crisp universal sets of the batch arrival rate, expected group size, setup time, vacation time, service time for FES, service time for SOS and probability for opting second service channel respectively.

Let z = f(y,x,u,v,s,t,q) denote the system characteristics of interest. Since $\bar{\lambda}, \bar{X}, \bar{D}, \bar{V}, \bar{S_1}, \bar{S_2}, \bar{r}$ are fuzzy numbers, f($\bar{\lambda}, \bar{X}, \bar{D}, \bar{V}, \bar{S_1}, \bar{S_2}, \bar{r}$) is also a fuzzy number. By Zadeh's extension principle the membership function of the system characteristics f($\bar{\lambda}, \bar{X}, \bar{D}, \bar{V}, \bar{S_1}, \bar{S_2}, \bar{r}$) is defined as

$\eta_{f(\bar{\lambda}, \bar{X}, \bar{D}, \bar{V}, \bar{S_1}, \bar{S_2}, \bar{r})}(z) = \sup_{\substack{y\in Y, x\in X, u\in U, v\in V, \\ s\in S, t\in T, q\in Q}} \min\{\eta_{\bar{\lambda}}(y), \eta_{\bar{X}}(x), \eta_{\bar{D}}(u), \eta_{\bar{V}}(v), \eta_{\bar{S_1}}(s), \eta_{\bar{S_2}}(t), \eta_{\bar{r}}(q)$ / z = f(y,x,u,v,s,t,q)} (10)

Also assume if the system characteristics of interest are expected cycle length E(T_{cycle}), and mean system size E(N). Then we have to find the membership function of E(T_{cycle}), and E(N). Thus we consider E(T_{cycle}) as

$$f(y,x,u,v,s,t,q) = \frac{E(u) + E(v) + \sum_{n=0}^{m-1} \frac{\varphi n}{y} + \sum_{n=m}^{N-1} \frac{\varphi n}{y}}{1 - yE(x)[E(s) + qE(t)]}$$

and the membership function of expected cycle length E(T_{cycle}) is given by

$\eta_{\overline{E(T_{cycle})}}(z) = \sup_{\substack{y\in Y, x\in X, u\in U, v\in V, \\ s\in S, t\in T, q\in Q}} \min\{\eta_{\bar{\lambda}}(y), \eta_{\bar{X}}(x), \eta_{\bar{D}}(u), \eta_{\bar{V}}(v), \eta_{\bar{S_1}}(s), \eta_{\bar{S_2}}(t), \eta_{\bar{r}}(q)$ /

$z = \frac{E(u) + E(v) + \sum_{n=0}^{m-1} \frac{\varphi n}{y} + \sum_{n=m}^{N-1} \frac{\varphi n}{y}}{1 - yE(x)[E(s) + qE(t)]}$} (11)

Likewise the membership function of the E(N) is also obtained. The membership function in the above equation is not in the usual forms thus making it very difficult to imagine its shapes. For this we approach the problem by using the mathematical programming techniques. Parametric NLPs are developed to find α cut of f($\bar{\lambda}, \bar{X}, \bar{D}, \bar{V}, \bar{S_1}, \bar{S_2}, \bar{r}$) based on the extension principle.

To construct the membership function of $\eta_{\overline{E(T_{cycle})}}$(z) = α, we have to derive the α cuts of E(T_{cycle}). The α cuts of $\bar{\lambda}, \bar{X}, \bar{D}, \bar{V}, \bar{S_1}, \bar{S_2}, \bar{r}$ are represented as follows

λ(α) = $[y_\alpha^L \ y_\alpha^U]$ = [min{ y∈ Y/ $\eta_{\bar{\lambda},}(y) \geq \alpha$ }, max{ y∈ Y/ $\eta_{\bar{\lambda},}(y) \geq \alpha$ }] (12a)

X(α) = $[x_\alpha^L \ x_\alpha^U]$ = [min{ x∈ X/ $\eta_{\bar{X}}(x)$ ≥ α }, max { x∈ X/ $\eta_{\bar{X}}(x)$ ≥ α}] (12b)

D(α) = $[u_\alpha^L \ u_\alpha^U]$ = [min{ u∈ U/ $\eta_{\bar{D}}(u) \geq \alpha$ }, max{u∈ U/ $\eta_{\bar{D}}(u) \geq \alpha$ }] (12c)

V(α) = $[v_\alpha^L \ v_\alpha^U]$ = [min{ v∈ V/ $\eta_{\bar{V}}(v) \geq \alpha$ }, max{ v∈ V/ $\eta_{\bar{V}}(v) \geq \alpha$ }] (12d)

S_1(α) = $[s_\alpha^L \ s_\alpha^U]$ = [min { s∈ S / $\eta_{\bar{S_1}}(s)$ ≥ α }, max{ s∈ S / $\eta_{\bar{S_1}}(s)$ ≥ α }] (12e)

S_2(α) = $[t_\alpha^L \ t_\alpha^U]$ = [min { t∈ T / $\eta_{\bar{S_2}}(t)$ ≥ α }, max { t∈ T / $\eta_{\bar{S_2}}(t)$ ≥ α }] (12f)

r(α) = $[q_\alpha^L \ q_\alpha^U]$ = [min{ q∈ Q/ $\eta_{\bar{r},}(q) \geq \alpha$ }, max { q∈ Q/ $\eta_{\bar{r},}(q) \geq \alpha$ }] (12g)

Further, the bounds of these intervals can be described as functions of α and can be obtained as

$y_\alpha^L = \min \eta_{\bar{\lambda},}^{-1}(\alpha)$ $y_\alpha^U = \max \eta_{\bar{\lambda}}^{-1}(\alpha)$
$x_\alpha^L = \min \eta_{\bar{X}}^{-1}(\alpha)$ $x_\alpha^U = \max \eta_{\bar{X}}^{-1}(\alpha)$
$u_\alpha^L = \min \eta_{\bar{D}}^{-1}(\alpha)$ $u_\alpha^U = \max \eta_{\bar{D}}^{-1}(\alpha)$
$v_\alpha^L = \min \eta_{\bar{V}}^{-1}(\alpha)$ $v_\alpha^U = \max \eta_{\bar{V}}^{-1}(\alpha)$
$s_\alpha^L = \min \eta_{\bar{S_1}}^{-1}(\alpha)$ $s_\alpha^U = \max \eta_{\bar{S_1}}^{-1}(\alpha)$
$t_\alpha^L = \min \eta_{\bar{S_2}}^{-1}(\alpha)$ $t_\alpha^U = \max \eta_{\bar{S_2}}^{-1}(\alpha)$
$q_\alpha^L = \min \eta_{\bar{r}}^{-1}(\alpha)$ $q_\alpha^U = \max \eta_{\bar{r}}^{-1}(\alpha)$

Thus by making use of the α-cuts for E(T_{cycle}) we construct the membership functions of (11) which is parameterized by α. Now to derive the membership function of E(T_{cycle}) it is suffice to find the left and right shape function of $\eta_{\overline{E(T_{cycle})}}$(z). This can be achieved by following the Zadhe's extension principle for $\eta_{\overline{E(T_{cycle})}}$(z) which is the minimum of $\eta_{\bar{\lambda},}(y)$, $\eta_{\bar{X}}(x)$, $\eta_{\bar{D}}(u)$, $\eta_{\bar{V}}(v)$, $\eta_{\bar{S_1}}(s)$, $\eta_{\bar{S_2}}(t)$, $\eta_{\bar{r}}(q)$. Then at least one the following cases to be hold which satisfies $\eta_{\overline{E(T_{cycle})}}$(z) = α. hence,

Case(i)
$\eta_{\bar{\lambda},}(y) = \alpha, \eta_{\bar{X}}(x) \geq \alpha, \eta_{\bar{D}}(u) \geq \alpha, \eta_{\bar{V}}(v) \geq \alpha, \eta_{\bar{S_1}}(s) \geq \alpha, \eta_{\bar{S_2}}(t) \geq \alpha, \eta_{\bar{r}}(q) \geq \alpha$
Case(ii)
$\eta_{\bar{\lambda},}(y) \geq \alpha, \eta_{\bar{X}}(x) = \alpha, \eta_{\bar{D}}(u) \geq \alpha, \eta_{\bar{V}}(v) \geq \alpha, \eta_{\bar{S_1}}(s) \geq \alpha, \eta_{\bar{S_2}}(t) \geq \alpha, \eta_{\bar{r}}(q) \geq \alpha$
Case (iii)
$\eta_{\bar{\lambda},}(y) \geq \alpha, \eta_{\bar{X}}(x) \geq \alpha, \eta_{\bar{D}}(u) = \alpha, \eta_{\bar{V}}(v) \geq \alpha, \eta_{\bar{S_1}}(s) \geq \alpha, \eta_{\bar{S_2}}(t) \geq \alpha, \eta_{\bar{r}}(q) \geq \alpha$
Case(iv)
$\eta_{\bar{\lambda},}(y) \geq \alpha, \eta_{\bar{X}}(x) \geq \alpha, \eta_{\bar{D}}(u) \geq \alpha, \eta_{\bar{V}}(v) = \alpha, \eta_{\bar{S_1}}(s) \geq \alpha, \eta_{\bar{S_2}}(t) \geq \alpha, \eta_{\bar{r}}(q) \geq \alpha$
Case(v)
$\eta_{\bar{\lambda},}(y) \geq \alpha, \eta_{\bar{X}}(x) \geq \alpha, \eta_{\bar{D}}(u) \geq \alpha, \eta_{\bar{V}}(v) \geq \alpha, \eta_{\bar{S_1}}(s) = \alpha, \eta_{\bar{S_2}}(t) \geq \alpha, \eta_{\bar{r}}(q) \geq \alpha$
Case (vi)
$\eta_{\bar{\lambda},}(y) \geq \alpha, \eta_{\bar{X}}(x) \geq \alpha, \eta_{\bar{D}}(u) \geq \alpha, \eta_{\bar{V}}(v) \geq \alpha, \eta_{\bar{S_1}}(s) \geq \alpha, \eta_{\bar{S_2}}(t) = \alpha, \eta_{\bar{r}}(q) \geq \alpha$
Case (vii)
$\eta_{\bar{\lambda},}(y) \geq \alpha, \eta_{\bar{X}}(x) \geq \alpha, \eta_{\bar{D}}(u) \geq \alpha, \eta_{\bar{V}}(v) \geq \alpha, \eta_{\bar{S_1}}(s) \geq \alpha, \eta_{\bar{S_2}}(t) \geq \alpha, \eta_{\bar{r}}(q) = \alpha$

This can be accomplished by using parametric NLP techniques. The NLP techniques to find the lower and upper bounds of α cut of $\eta_{\overline{E(T_{cycle})}}$(z) for case (i) is

$$[E(T_{cycle})]_\alpha^{L1} = \min \left\{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \right\} \text{ (13 a)}$$

$$[E(T_{cycle})]_\alpha^{U1} = \max \left\{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \right\} \text{ (13 b)}$$

For case (ii) as

$$[E(T_{cycle})]_\alpha^{L2} = \min \left\{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \right\} \text{ (13c)}$$

$$[E(T_{cycle})]_\alpha^{U2} = \max \left\{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \right\} \text{ (13d)}$$

For case (iii) as

$$[E(T_{cycle})]_\alpha^{L3} = \min \left\{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \right\} \text{ (13e)}$$

$$[E(T_{cycle})]_\alpha^{U3} = \max \left\{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \right\} \text{ (13f)}$$

For case (iv) as

© Copyright 2014 | Centre for Info Bio Technology (CIBTech)

$[\mathrm{E}(T_{cycle})]_\alpha^{L4} = \min \{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \}$ (13g)

$[\mathrm{E}(T_{cycle})]_\alpha^{U4} = \max \{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \}$ (13h)

For case (v) as

$[\mathrm{E}(T_{cycle})]_\alpha^{L5} = \min \{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \}$ (13i)

$[\mathrm{E}(T_{cycle})]_\alpha^{U5} = \max \{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \}$ (13j)

For case (vi) as

$[\mathrm{E}(T_{cycle})]_\alpha^{L6} = \min \{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \}$ (13k)

$[\mathrm{E}(T_{cycle})]_\alpha^{U6} = \max \{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \}$ (13l)

For case (vii) as

$[\mathrm{E}(T_{cycle})]_\alpha^{L7} = \min \{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \}$ (13m)

$[\mathrm{E}(T_{cycle})]_\alpha^{U7} = \max \{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \}$ (13n)

As λ(α),X(α),D(α),V(α),$S_1(\alpha)$, S_2 (α),r(α) are given in equations (12 a-g) y ∈ λ(α), x ∈ X(α), u ∈ D(α) , v ∈ V(α), s ∈ $S_1(\alpha)$, t ∈ S_2 (α), q ∈ r(α) can be replaced by y ∈ $[y_\alpha^L\ y_\alpha^U]$, x ∈ $[x_\alpha^L\ x_\alpha^U]$, u ∈ $[u_\alpha^L\ u_\alpha^U]$, v ∈ $[v_\alpha^L\ v_\alpha^U]$, s ∈ $[s_\alpha^L\ s_\alpha^U]$, t ∈ $[t_\alpha^L\ t_\alpha^U]$, q ∈ $[q_\alpha^L\ q_\alpha^U]$ and which are given by the α cuts and in turn they form a nested structure with respect to α which are expressed in (13a-n). Hence for given 0< α_2< α_1< 1, we have $[y_{\alpha 1}^L\ y_{\alpha 1}^U] \subseteq [y_{\alpha 2}^L\ y_{\alpha 2}^U]$, $[x_{\alpha 1}^L\ x_{\alpha 1}^U] \subseteq [x_{\alpha 2}^L\ x_{\alpha 2}^U]$, $[u_{\alpha 1}^L u_{\alpha 1}^U] \subseteq [u_{\alpha 2}^L\ u_{\alpha 2}^U]$, $[v_{\alpha 1}^L v_{\alpha 1}^U] \subseteq [v_{\alpha 2}^L\ v_{\alpha 2}^U]$, $[s_{\alpha 1}^L\ s_{\alpha 1}^U] \subseteq [s_{\alpha 2}^L\ s_{\alpha 2}^U]$, $[t_{\alpha 1}^L\ t_{\alpha 1}^U] \subseteq [t_{\alpha 2}^L\ t_{\alpha 2}^U]$, $[q_{\alpha 1}^L\ q_{\alpha 1}^U] \subseteq [q_{\alpha 2}^L\ q_{\alpha 2}^U]$. Thus equations (13a), (13c), (13e), (13g), (13i), (13k) and (13m) have the unique smallest element and equations (13b), (13d), (13f), (13h),(13j), (13l) and (13n) have the unique largest element. Now, to find the membership function of $\eta_{\mathrm{E}(T_{cycle})}$(z) which is equivalent to find the lower bound of $[\mathrm{E}(T_{cycle})]_\alpha^L$ and upper bound of $[\mathrm{E}(T_{cycle})]_\alpha^U$ are written as,

$[\mathrm{E}(T_{cyc\,le})]_\alpha^L = \min\{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \}$ (14a) where, $y_\alpha^L \le y \le y_\alpha^U$, $x_\alpha^L \le x \le x_\alpha^U$, $u_\alpha^L \le u \le$ *uαU, vαL≤v≤vαU, sα L≤s≤sαU,tαL≤t≤ tαU, qαL*≤q≤*qαU.*

$[\mathrm{E}(T_{cycle})]_\alpha^U = \max\{ \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi n}{y}+\sum_{n=m}^{N-1}\frac{\varphi n}{y}}{1-yE(x)[E(s)+qE(t)]} \}$ (14b) where, $y_\alpha^L \le y \le y_\alpha^U$, $x_\alpha^L \le x \le x_\alpha^U$, $u_\alpha^L \le u \le$ *uαU, vαL≤v≤vαU, sα L≤s≤sαU, tαL≤t≤ tαU, qαL*≤q≤*qαU.*

(i.e) At least any one of y, x, u, v, s, t, q must hit the boundaries of their α cut that satisf $\eta_{\mathrm{E}(T_{cycle})}$(z) = α

The crisp interval $[\mathrm{E}(T_{cycle})_\alpha^L, \mathrm{E}(T_{cycle})_\alpha^U]$obtained from (14a) and (14b) represent α cuts of $\mathrm{E}(T_{cycle})$. By applying the results of Zimmerman (2001) and convexity property, we obtain $[\mathrm{E}(T_{cycle})]_{\alpha 1}^L \ge [\mathrm{E}(T_{cycle})]_{\alpha 2}^L$ and $[\mathrm{E}(T_{cycle})]_{\alpha 1}^U \le [\mathrm{E}(T_{cycle})]_{\alpha 2}^U$, where 0< α_2< α_1< 1.

In other words $\mathrm{E}(T_{cycle})_\alpha^L$ increases and $\mathrm{E}(T_{cycle})_\alpha^U$ decreases as α increases, consequently the membership function $\eta_{\mathrm{E}(T_{cycle})}$(z) can be found from (14) If both $[\mathrm{E}(T_{cycle})]_\alpha^L$ and $[\mathrm{E}(T_{cycle})]_\alpha^U$ are invertible with respect to α then the left shape function L(z) =$[\mathrm{E}(T_{cycle})_\alpha^L]^{-1}$ and right shape function R(z) =$[\mathrm{E}(T_{cycl\,e})_\alpha^U]^{-1}$ can be derived, such that

© Copyright 2014 | Centre for Info Bio Technology (CIBTech)

$$\eta_{\overline{\mathrm{E}(T_{cycle})}}(z) = \begin{cases} L(z)\ [\mathrm{E}(T_{cycle})]^L_{\alpha=0} \le z \le [\mathrm{E}(T_{cycle})]^L_{\alpha=1} \\ 1\ [\mathrm{E}(T_{cycle})]^L_{\alpha=1} \le z \le [\mathrm{E}(T_{cycle})]^U_{\alpha=1} \\ R(z)\ [\mathrm{E}(T_{cycle})]^U_{\alpha=1} \le z \le [\mathrm{E}(T_{cycle})]^U_{\alpha=0} \end{cases}$$

In many cases, the value of $\{[\mathrm{E}(T_{cycle})]^L_\alpha\ [\mathrm{E}(T_{cycle})]^U_\alpha\ /\ \alpha \in [0\ 1]\}$ cannot be solved analytically, consequently a closed form membership function of $\mathrm{E}(T_{cycle})$ cannot be obtained. The numerical solutions for $[\mathrm{E}(T_{cycle})]^L_\alpha$ *and* $[\mathrm{E}(T_{cycle})]^U_\alpha$ at different levels of α can be collected that approximate the shape of L (z) and R(z) (i.e.,) the set of intervals$\{[\mathrm{E}(T_{cycle})]^L_\alpha\ [\mathrm{E}(T_{cycle})]^U_\alpha\ / \in [0\ 1]\}$ will estimate the shapes.

Numerical Example

Consider a fuzzy bi level threshold policy of $M^X/(G_1, G_2)/1/\mathrm{SV}$ queuing system. The corresponding parameters such as the arrival rate λ, expected group size X, setup time D, vacation time V, service time for FES S_1, service time for SOS S_2 and probability for opting second channel r are fuzzy numbers.

Let $\bar{\lambda}$ = [0.4,0.401,0.402,0.403], $\overline{X}$ = [1.55,1.65,1.75,1.85] $\overline{D}$ =[1,1.5,2,2.5], $\overline{v}$ =[0.5, 0.6,0.7,0.8], $\overline{S_1}$ = [0.1, 0.15, 0.2, 0.25], $\overline{S_2}$ = [0.5,0.6,0.7,0.8], $\overline{r}$ =[0.2,0.3,0.4,0.5]

Then the expected length of the cycle, and the mean system size are given by,

$$\mathrm{E}(T_{cycle}) = \frac{E(u)+E(v)+\sum_{n=0}^{m-1}\frac{\varphi_n}{y}+\sum_{n=m}^{N-1}\frac{\varphi_n}{y}}{1-yE(x)[E(s)+qE(t)]}$$

$$\mathrm{E(N)} = (yE(x))^2(\mathrm{E}(s^2) + 2qE(s)E(t) + q\mathrm{E}(t^2)) + \frac{yE(x(x-1))(qE(t)+E(s))}{2(1-(yE(x)[E(s)+qE(t)])} + \frac{1}{E(D)+E(V)+\sum_{n=0}^{m-1}\frac{\varphi_n}{\lambda}+\sum_{n=m}^{N-1}\frac{\varphi_n}{\lambda}}\left[\frac{\lambda E(V)(E(D^2)+2E(D)E(V)+E(V^2)}{2} + \frac{\sum_{n=0}^{m-1} n\psi_n}{\lambda} + \sum_{n=0}^{m-1} n\psi_n\, E(D)E(X) + \frac{\sum_{n=m}^{N-1} n\varphi_n}{\lambda}\right]$$

and y,x,u,v,s,t,q are the fuzzy variable corresponding to $\bar{\lambda}, \overline{X}, \overline{D}, \overline{V}, \overline{S_1}, \overline{S_2}, \overline{r}$ respectively. Thus, $[y^L_\alpha\ y^U_\alpha] = [0.4 + 0.0001\alpha\ 0.403 - 0.0001\alpha]$, $[x^L_\alpha\ x^U_\alpha] = [1.55 + 0.1\alpha\ 1.85 - 0.1\alpha]$, $[u^L_\alpha\ u^U_\alpha]$= [1+0.5α, 2.5-0.5α], $[v^L_\alpha\ v^U_\alpha]$= [0.5+0.1α, 0.8-0.1α], $[s^L_\alpha\ s^U_\alpha] = [0.1 + 0.05\alpha\ 0.25 - 0.05\alpha]$ $[t^L_\alpha\ t^U_\alpha] = [0.5 + 0.1\alpha\ 0.8 - 0.1\alpha]$ $[q^L_\alpha\ q^U_\alpha]$= [0.2+0.1α, 0.5-0.1α]. By substituting the above values, the effect of parameters on the expected cycle length $E(T_{cycle})$, and mean system size E(N) are tabulated and their graphical representations are also shown below

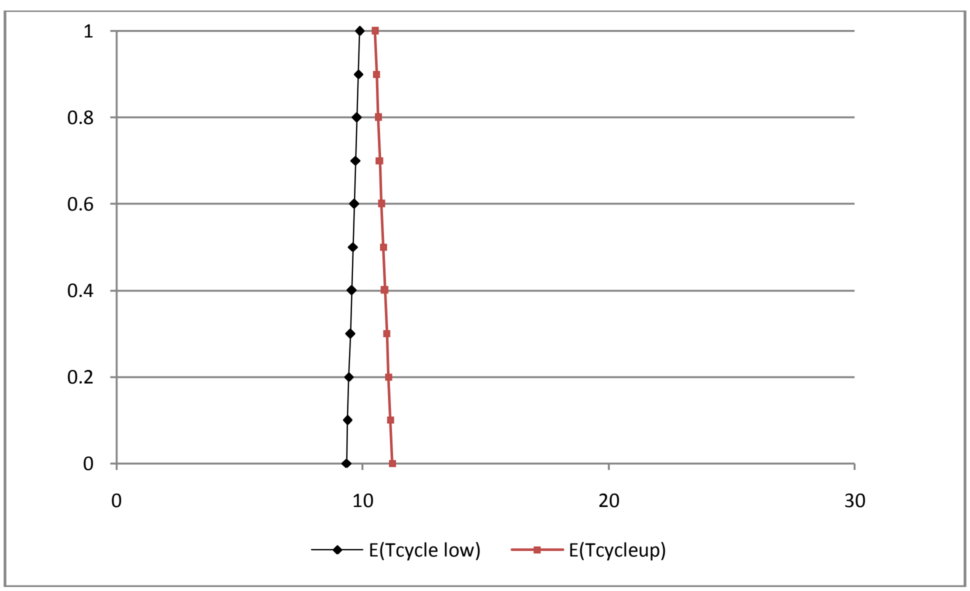

Figure 1: The membership function for fuzzy $E(T_{cycle})$

© Copyright 2014 | Centre for Info Bio Technology (CIBTech)

Table 1: The α cuts for the performance measure of $E(T_{cycle})$

α	E(T_{Cycle}) low	E(T_{Cycle})upp
0	9.33636134	11.19899
.1	9.38776351	11.1241
.2	9.43977988	11.05026
.3	9.49242148	10.97746
.4	9.54569961	10.90568
.5	9.59962582	10.83488
.6	9.65421197	10.76506
.7	9.70947018	10.69619
.8	9.7654129	10.62825
.9	9.82205287	10.56122
1	9.87940315	10.4951

Table 2: The α cuts for the performance r measure of E(N)

	E(N) low	E(N) upp
0	1.486717	1.674968
.1	1.491894	1.665163
.2	1.497128	1.655577
.3	1.502423	1.646204
.4	1.507778	1.637039
.5	1.513197	1.628079
.6	1.518681	1.61932
.7	1.524231	1.610755
.8	1.529851	1.602382
.9	1.535541	1.594197
1	1.541305	1.586194

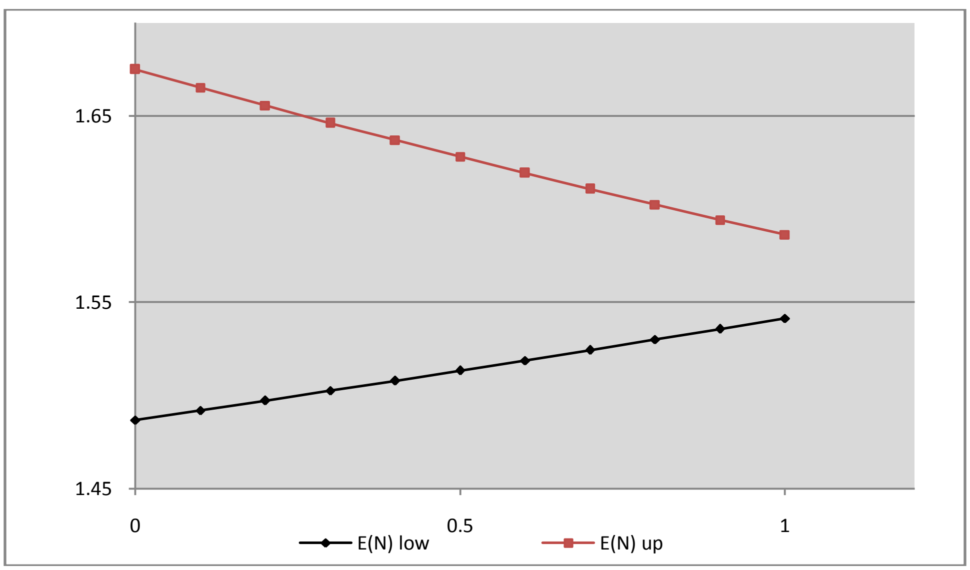

Figure 2: The membership function for fuzzy E(N)

Here we perform α cuts of fuzzy E(T_{Cycle}) and E(N) at eleven distinct α levels 0,0.1,0.2,……1.0. Crisp intervals for fuzzy E(T_{Cycle}) with single vacation at different possible α levels are presented in table 1,

Research Article

similarly the other performance measure such as E(N) are presented in table 2. Figure 1 depicts the rough shape of E(T_{Cycle}) constructed from α values. The rough shape turns out rather fine and look like a continuous function. The other performance measure E(N) is represented by Figure 2.The α cut depicts that these two performance measures will lie only in the specified associated range. We find that the above information is very useful for designing the fuzzy queuing system.

Conclusion

The fuzzy queuing model has more applicability in the real environments than the crisp systems. This paper applies the concept of α cut and Zadeh's extension principle to fuzzy Bi level threshold policy of $M^X/(G_1,G_2)$/1/SV queuing system and thereby deriving the membership function of the expected length of the cycle, for this model . We find that it is more meaningful to express E(T_{Cycle}) and E(N) as a membership function rather than by crisp values (i.e) as fuzzy performance measures. The benefit and significance of such a fuzzy performance measure include maintaining the fuzziness of input information completely and the results can be used to represent the fuzzy system more accurately.

REFERENCES

Chaun KJ, Hung Hsim I and Hong LC (2007). On retrial queuing model with fuzzy parameters. *Physica A: Statistical Mechanics and its Applications* **374**(1) 272-280.

Choudhury G and Paul M (2006). A batch arrival queue with a second optional service channel under N policy. *Stochastic Analysis and Application* **24** 1-21.

Hur S and Park J (1999). The effect of different arrival rates on N policy of M/G/1 with server setup. *Applied Mathematical Modeling* **23** 289-299.

Jeeva M and Rathnakumari E (2012). Bulk arrival single server Bernoulli Feedback queue with fuzzy vacations and fuzzy parameters. *ARPN Journal of Science and Technology* **2**(5) 492-498.

Julia Rose Mary K, Afthab Begum MI, and Jamila PM (2011). Bi-level threshold policy of $M^X/(G_1,G_2)$/1 queue with early setup and single vacation. *International Journal of Operational Research* **10**(4) 469-492.

Kao C, Li C and Chen S (1999). Parametric programming to the analysis of fuzzy queues. *Fuzzy Sets and System* **107** 93-100.

Ke JC (2001). The control policy of $M^X/G/1$ Queing system with startup and two vacation types. *Mathematical Methods of Operations Research* **54**(31) 471-490**.**

Lee HS and Srinivasan MM (1989). Control Policies for the $M^X/G/1$ queuing system. *Management Sciences* **35**(6) 708-721.

Lee HW, Lee SS and Park JO (1994 a). Operating Characteristics of $M^X/G/1$ queue with N Policy. *Queuing Systems* **15** 378-399.

Lee HW, Lee SS and Park JO (1994 b). Analysis of the $M^X/G/1$ queue with N Policy and multiple vacations. *Journal of the Applied Probability* **31** 476-496.

Lee HW, Park NL and Jeon J (2003). Queue length analysis of a batch arrival queues under bi-level threshold control with early setup. *International Journal of System Science* **34**(3) 195-204.

Li RJ and Lee ES (1989). Analysis of fuzzy queues. *Computers and Mathematics with Application* **17**(7) 1143-1147.

Madan KC (2000). An M/G/1 queue with second optional service. *Queuing Systems* **34** 37-46.

Madan KC and Choudhury G (2006). Steady state analysis of an $M^X/(G_1,G_2)$/1 queue with restricted admissibility and random setup time. *Information and Management Science* **17**(2) 33-56.

Nagi DS and Lee ES (1992). Analysis and simulation of fuzzy queues. *Fuzzy Sets and Systems* **46** 321-30.

Ramesh R and Kumara G G (2013). A batch arrival queue with multiple servers and fuzzy parameters parametric programming approach. *International Journal of Science and Research* **2**(9) 135-140.

Zimmermann HF (2001). *Fuzzy Set Theory and its Application*, 4th edition (Kluwer Academic, Boston).

© Copyright 2014 | Centre for Info Bio Technology (CIBTech)