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Research Article

HIGH-SPEED, NON-LUBRICATED SPUR GEAR PAIR DESIGN OPTIMIZATION USING EVOLUTIONARY ALGORITHM

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ABSTRACT
In this research, genetic algorithm has been used as the optimization approach to increase the power transmission capability of spur gear pair rotating at high speed without lubricating oil. Along with the enhancement of gear efficiency, the weight of the gears also reduced to the minimum by reducing the size of spur gear pair. The developed genetic algorithm module can identify the best optimal parameters for spur gear from the larger solution space. The feasible designs have been obtained by considering the constraints such as bending stress, compressive stress and module. The computational time had been reduced by sensitivity analysis. The optimized result from the developed module had been used for manufacturing and the experimental results were compared with the standard spur gear pair. It was found that the performance has been at par with the standard gear and also the optimized gear pair having lesser weight and size compared to the existing. 12% reduction of gear size and 2 % increase in efficiency were achieved with the optimized spur gear design.

Keywords: *Genetic Algorithm; Optimization; Spur Gear Pair*

INTRODUCTION
Due to digital revolution, manufacturing sectors are focusing towards producing the high quality products with less cost in a sparkling environment. One of the major challenges faced by the industries in achieving the goal is from the lubricating oils. Since these oils will observe the dust particles and results in dirty surroundings. These oils are commonly used to reduce the wear and to carry away the excess heat generated in mating parts. Consequently the cost of the lubricating oils is also added with the product cost. So a necessity arises to eliminate the lubricant both for cost saving and to create a sound environment. Eliminating the lubricating oil will leads to many technical challenges such as plastic deformation, high heat generation, microstructure deformation, wear and tear etc. Gears are the teethed rotating elements used to transmit the power between the machine parts and the lubricating oils are commonly used by the gears to avoid friction between the mating teethes and to avoid the noise at high speed. So in this research, spur gear pair have been taken and optimized for its design parameters to run at high speed without lubricant.

These issues can be overcome by controlling the sliding velocity, contact pressure and by proper design. Bunch of parameters are influencing the gear design and identifying the suitable parameters based on the application will consumes more time. In this research, genetic algorithm (GA) has been used to optimize the spur gear parameters by considering most of the spur gear related constraints. For experimentation purpose, upper and lower limits of the main design parameters such as number of teeth, module, power and tooth thickness were fixed. Also the major constraints such as bending stress, compressive stress and standard values based on design for manufacturability were considered to outline the feasible design solution space. Left over secondary gear parameters such as material, young's modulus, etc have been set with constant values. These solution region have been given to genetic algorithm and the algorithm will identify the best optimal combination of spur gear parameter which satisfy the constraints and suitable for running at high speed without lubricant. An assortment of research and the openings are given in the following section.

Literature Survey

Townsend (1992) correlated the relation between the sliding velocities with line of action. It also proved that the sliding velocity became zero at pitch line and increases when the conjugated teeth contact line, travels away from the pitch line on both directions. Heat is generated by sliding friction of teeth surfaces and is proportional to the distribution of contact pressure and sliding velocity. Variations in temperature along the line of action are calculated by Blok's contact temperature theory. So in this research work, change in temperature also calculated for designing.

The encroachments of computers lead to lot of researches in developing the computer programming for gear design. Madhusudan and Vijayasimha (1987) build a computer program in order to design a required type of gear under a specified set of working conditions. A new computer-aided method for automated gearbox design had been portrayed by Lin *et al.,* (2009). An interactive physical programming had formulated by Huang *et al.,* (2005) in order to optimize a three-stage spur gear reduction unit. An expert system for designing and manufacturing a gearbox unit has illustrated by Abersek *et al.,* (1995). So this research also focused in developing a interactive programming module for spur gear based on the given application. Application of genetic algorithm can be used for optimization problems were introduced by Holland (1975). A non-dominated sorting GA had used by Deb and Jain (2003) in order to solve a multi-objective optimization of a multi-speed gearbox. Tudose (2010) automated the helical gear design for speed reducer application. Thompson *et al.,* (2000) stated a generalized optimal design of two-stage and three-stage spur gear reduction units in a formulation with multiple objectives. The benefits of the particle swarm searches in resolving different engineering designs were revealed by Ray and Shini (2001). Two advanced optimization algorithms known as particle swarm optimization (PSO) and simulated annealing (SA) are brought into play by Savsani *et al.,* (2010) for minimizing the weight of a spur gear train. GA had utilized by Gologlu and Zeyveli (2009) to minimize the volume of two stage helical gear train. A complete automated optimal design of a two-stage helical gear reducer using a two-phase evolutionary algorithm is presented Tudose *et al.,* (2010). Li *et al*., (1996) carried out a study for minimizing the centre distance of a helical gear using American Gear Manufacturers Association (AGMA) procedures. Renner and Ekart (2003) also used genetic algorithm in solving computer aided problems. So GA has been used to optimize the spur gear parameters in this research.

Spur Gear Design

Gears are the toothed wheels used to transmit power between the shafts by mating with each other. Gear in which the power is given is the driver gear and the gear which is rotated by the driver gear is driven wheel. Gears are used to increase or decrease the speed, power, torque, etc. Some of the commonly used gears are Spur gear, Helical gear, Herringbone gear, Bevel gear, Worm gear, Rack, and many more. Spur gears are the far most common type of gear used in machine tools. Spur gears are having teethes running perpendicular to the face of the gear and can be manufactured easily with less cost. So the spur gear is taken in this research to optimize and the objectives given in the following sections along with the constraints considered for the designing.

Objective Function

Spur gear design involves numerous parameters and constraints which makes the design problem as a NP hard problem. NP hard problem are having multiple objective and constraints to be satisfied. Multi-objective function formulated in this research for optimizing the spur gear pair is given in the Equation 1.

$$COF = \{[(F_1 / \max. F_1) + (F_2 / \min. F_2) + (F_3 / \max. F_3) + (F_4 / \min. F_4)] / 4\} \quad (1)$$

Whereas,

F_1 is the maximization function for power transmission and is given in the Equation 2.

$$F_1 = P \qquad \text{where, } P^{(L)} \leq P \leq P^{(U)} \quad (2)$$

'P' is Power transmitting capacity and lies between lower and upper limit.

F_2 is the minimization function by Yokota *et al.,* (1998) for calculating gear weight and is given in the Equation 3.

$$F_2 = \{[\frac{\pi}{4} \times d_1^2 \times b \times \rho] + [\frac{\pi}{4} \times d_2^2 \times b \times \rho]\} \quad (3)$$

'd_1', 'd_2' = Pitch circle diameter of pinion and gear in mm
'b' = Thickness of pinion and gear in mm
'ρ' = Density of the material in kg/mm^3

F_3 is the maximization function for efficiency calculation and is given in the Equation 4.

$$F_3 = 100 - P_L \quad (4)$$

'P_L' = Power loss which is calculated by the Equation 5.

$$P_L = \frac{50f}{\text{Cos}\Phi} \times \frac{(H_s^2 + H_t^2)}{(H_s + H_t)} \quad (5)$$

'H_s' & 'H_t' are Specific sliding velocity at start of approach & end of recess action
'f' = Coefficient of friction
'Φ' = Pressure angle in degrees
'H_s' and 'H_t' are calculated by the Equations 6 & 7 respectively.

$$H_t = \frac{(i+1)}{i} \times \sqrt{\left(\left[\frac{r_0}{r}\right]^2 - \cos^2\Phi\right)} - \sin\Phi \quad (6)$$

$$H_s = (i+1) \times \sqrt{\left(\left[\frac{R_0}{R}\right]^2 - \cos^2\Phi\right)} - \sin\Phi \quad (7)$$

Whereas,
'R' & 'R_o' = Pitch and Outside circle radius of gear in mm.
'r' & 'r_o' = Pitch and Outside circle radius of pinion in mm
R_0 = R + one addendum

One addendum for 20^o full depth involute system = one module = m.

$$r_o = r + m = \frac{d_1}{2} + m$$

$$R_o = R + m = \frac{d_2}{2} + m$$

F_4 is minimization function for calculating center distance and is given in Equation 8.

$$F_4 = \frac{(d_1 + d_2)}{2} = \frac{m}{2}(Z_1 + Z_2) \quad (8)$$

Where, 'Z_1', 'Z_2' = Number of teeth in pinion and gear respectively.

The combined objective function value has to be normalized as it involves different sub functions and the aim is to maximize the COF by satisfying the design constraints.

Constraints

The foremost constraint is that the obtained optimal design values should be within the allowable stress values for its safety operation by Grote and Antonsson (2009). Also the optimal design parameters should have the standard values for ease of manufacturing and standardization. Some of the major constraints considered in this research are as follows.

Bending Stress Constraint

During mating of gear teethes, the top surface of the gear teeth will subject to bending stress and the condition for the bending stress is given in the Equation 9.

$\sigma_b \leq [\sigma_b]_{al}$ (9)

Whereas, '$[\sigma_b]_{al}$' = Allowable bending stress in N/mm^2.
'σ_b' = Induced bending stress in N/mm^2 and is given in Equation 10.

$$\sigma_b = \frac{(i+1)}{(a\,m\,b\,y)} \times [M_t] \quad (10)$$

Where, 'i' = Gear ratio / Speed ratio
'a' = Center distance between gear and pinion
'y' = Form factor
'$[M_t]$' = Design twisting moment in Nmm, and is given in Equation 11.

$$[Mt] = M_t \times k \times k_d \quad (11)$$

'M_t' = Normal twisting moment transmitted by the pinion in Nmm
'K' & 'k_d' are the Load Concentration factor and Dynamic load factor

Compressive Stress Constraint

During mating of the gears, compressive stresses are created at the bottom surface of teethes of the mating gears. The compressive stress function is given in the Equation 12.

$\sigma_c \leq [\sigma_c]_{al}$ (12)

Where, '$[\sigma_c]_{al}$' = Allowable crushing stress in N/mm^2.
'σ_c' = Induced crushing stress in N/mm^2 and is given in Equation 13.

$$\sigma_c = 0.74\left(\frac{i+1}{a}\right) \times \sqrt{\left[\left(\frac{i+1}{ib}\right) \times E \times [M_t]\right]} \quad (13)$$

Where, 'E' = Young's Modulus of the gear material in N/mm^2

Number of Teeth in Pinion and Gear

The number of teeth must be an even integer number. Since the odd numbered gears are difficult to manufacture in milling machine. Also the number of teeth should be between the lower and upper limits to make the solution a feasible design. The limits set after conducting the sensitivity analysis are as follows.

Z_i ε N, N = {14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38… 56, 58, 60}.

Module Constraint

Module plays a major role in the gear design and in manufacturing. The condition for module is given in the Equation 14.

$m \geq m_{min}$ (14)

'm_{min}' is the minimum module and is given in the Equation 15.

$$m_{min} = 1.26 \times \sqrt[3]{\frac{[M_t]}{(y\,\sigma_b \Psi_m Z_1)}} \quad (15)$$

'Ψ_m' is the ratio between the gear pair thickness and module.
The obtained module value should be standardized to the 'R' series values which is stored in the database.

Spur Gear Parameters Calculation

The basic parameters of the spur gear pair are calculated using the following equations.

Gear Ratio (i)

The gear ratio is the ratio of speed of the pinion and the gear wheel. The formula for calculating gear ratio is given in the Equation 16.

$$i = \frac{Z_2}{Z_1} \text{ (or) } \frac{d_2}{d_1} = \frac{N_1}{N_2} \quad (16)$$

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Center Distance between Pinion and Gear (a)

The size of the gear wheels decides the centre distance and the formula for calculating the centre distance is given in the Equation 17 and the condition to be satisfied is given in the Equation 18.

$$a = \frac{(d_1 + d_2)}{2} = \frac{m}{2}\,[Z_1 + Z_2] \tag{17}$$

$$a \geq a_{min} \tag{18}$$

The minimum center distance can be calculated from the Equation 19.

$$a_{min} = (i+1)\sqrt[3]{\left[\left(\frac{0.74}{[\sigma_c]}\right)^2 \times \left(\frac{E\,[M_t]}{i\Psi}\right)\right]} \tag{19}$$

'Ψ' is the ratio between the gear pair thickness and center distance.

Gear Surface Temperature

In this research, the optimization is done for the gears which are rotating without lubricating oil, so more heat energy will be developed on the gear surface. The surface temperature of the gear wheels should be kept within the allowable limits while optimizing the design, because the gear life and the lubrication depend mainly on the amount of heat generated. The maximum contact temperature is obtained by Equation 20.

$$\theta_{B\,max} = \theta_M + \theta_{fl\,max} \tag{20}$$

Whereas, 'θ_M' is tooth temperature,

'$\theta_{fl\ max}$' is maximum flash temperature along the line-of-action, which is calculated by Blok's relation given in the Equation 21.

Primary Gear Parameters

For experimentation purpose, the primary gear parameters are set with the limits and the limit can be allowed to change by the user depend upon the application. The primary gear parameters used to form the solution space are given in the Table 1.

Table 1: Primary Spur Gear Parameters

Power	30 to 40 Kw
Module	01 to 20 mm
Tooth Thickness	10 to 100 mm
Number of Teeth	14 to 60

Secondary Gear Parameters

In order to reduce the computational time and search space, secondary parameters are fixed with the standard values. Some of the secondary parameters are Coefficient of friction, Thermal conductivity, Density, Specific heat, Material of gear and pinion, Gear ratio, Young's modulus, Pressure Angle, Tool dedendum coefficient, Backlash coefficient, Minimum top-land coefficient, Minimum root clearance, Allowable bending stress, Allowable compressive stress, etc.

Genetic Approach

The genetic algorithm is the most well known and best of all evolution-based search algorithms. The basic concepts of GA was developed by Holland [7], described the biological processes of evolutionary systems. The main objective of GA is that the better offspring will survive and the worst offspring will die in the population by Bentley (1999). After many generations, most of the offspring will be better, as that offspring are reproduced form the best parents. The individual offspring called genetic chromosome represents a solution for the problem and each element called as genes represents the parameters to be optimized. The various stages of the genetic algorithm are explained in the following sections.

User Input Stage

The input module is used to interact with the user for getting the required primary and secondary data. These data need to be encoded to genetic chromosomal format and given as input to the genetic algorithm module.

Encoding

Encoding is the process of converting the input data into chromosomal format for further processing. In this work, decimal encoding has been implemented; the four parameter values are given directly as input. The sample encoded parents are shown in the Figure 1. From the Figure 1, the first four digits of the genetic parent represent the power followed by module in the next four digits. The ninth and tenth digits represent the number of teeth and next four digits represent tooth thickness. This encoded data will be given as input to the genetic algorithm module.

Input : P = 30.41 KW; *m = 20.22 mm*; $\mathbf{Z_1}$ **= 18**; t = 4.73 mm

Chromosome : Parent 1 /1 3041*2022***18**0473

Figure 1: Randomly Generated Parent

4.3 Initial Parent Generation

Initially parents have to be generated at random with the population size of 100 and parent size of 14 strings. Figure 2 shows a sample set of five parents. The generated parents have been allowed for crossover operation to inherit the best properties from the parents.

Parent 1 /1 30412022180473,
Parent 2 /1 30063612180483,
Parent 3 /1 30074423220774,
Parent 4 /1 31273053240628,
Parent 5 /1 31294636160507,

Figure 2: Five Set of Randomly Generated Parents

4.4 Crossover

Crossover is the process of interchanging a certain set of strings at random between two parent to generate children having properties of both the parents. Crossover parents and crossover site are selected randomly. Figure 3 explains crossover with a sample set of parents. The next GA stage is the mutation.

Parent No.	Initial Parent	Cross. Parent	Cross. Site	Crossover Offspring
Parent 1 /1	30412022180473	2	2	3041**3612180483**
Parent 2 /1	**30063612180483**	1		**3006**2022180473
Parent 3 /1	*30074423220774*	4	3	*30074423*240628
Parent 4 /1	31273053240628	3		31273053*220774*

Figure 3: Crossover

4.5 Mutation

Stagnation of fitness function value could be resolved by applying the mutation operator. Figure 4 explains the mutation with a sample parents.

Parent No.	Crossover Offspring	Mut. Parent	Mut. Site	Crossover Offspring
Parent 1 /1	30412022180473		0	30412022180473
Parent 2 /1	30063612180483		0	30063612180483
Parent 3 /1	30074423220774		0	30074423220774
Parent 4 /1	31273053**24**0628	4	3	31273053**18**0628

Figure 5: Mutation Operation

4.6 Fitness Function Calculation

Fitness function is also called as objective function, which is used to select the best parent from the generated population and is given in Equation 1.

4.7 Termination Conditions

Theoretically, expected value for the COF is one, which represents best optimal design and will be difficult for multi-objective and multi-constrained problems. Thus a set of conditions are required to identify the optimal design and called as termination conditions. A set of termination conditions given below is used to identify the optimal solution in this research work.

1. Minimum Criterion Condition: The obtained fitness value should be greater than the threshold value. In this work, the threshold value was set to 0.92.
2. Generation Condition: Maximum number of generations should be reached, in this work maximum number of generation is set to 100 generations.
3. Stagnation Condition: Successive five iterations no longer produce better results.

The above three conditions will be checked at each stage of the population generation and the generation terminates on satisfying any one condition. Then the parent with the best fitness value should be considered as the best parent. This obtained genetic chromosome has been decoded to user understandable format.

4.8 Decoding Module

Decoding is the reverse of encoding process. Decoding process converts genetic chromosomal output into user understandable format. Once the genetic chromosome is decoded to design data, then the complete gear specification has to be calculated in the output module.

4.9 Output Module

The output module utilized the decoded data and generated the values to calculate the gear specification.

1. GEAR DESIGN OPTIMIZATION

The major challenge faced by the researchers are the multi-objective optimization of gear parameters by considering the constraints such as bending stress, compressive stress, standard values for ease of manufacturing, tooth surface temperature and noise. The objectives considered for optimization of spur gear pair are minimization of gear size, weight, thickness, module, centre distance and maximization of power transmitted capacity, efficiency. In order to obtain the feasible solution in less computational time, these gear parameters had fixed with standard prioritized values and within defined ranges. Thereby the search region having only feasible data sets. In every genetic generation, the candidate solution which violates allowable bending and compressive stresses have to be eliminated from the population, even though the contact temperature and noise were within the acceptable level and vice-versa. The best candidate solution obtained after satisfying the stopping conditions are stored. Similarly 52 best candidate solutions have to be formed and the best design geometry among the solution, which is having higher combined objective function value will be chosen as the best optimized gear parameter. In this research, a gear pair had been made of 40 Ni 2 Cr 1 Mo 28 (Cr-Mo) series steel and surface hardened up to 56 HRC with the density and young's modulus values as 8.836×10^{-6} kg/mm^3 and 2.15×10^5 N/mm^2 respectively. Tooth flanks were ground to achieve AGMA grade A4. This gear pair had been tested with the standard gear pair.

RESULTS

An existing standard spur gear set and the gear set manufactured based on optimization result had compared for its performance in an automotive test rig that runs at high speed of 38 m/s without lubrication. Both the gear sets were run for 100 hours in the same condition and the results obtained are as follows.

- The wear rate of driver gear is more than the driven gear.
- The air temperature remains more or less same for both the sets as 170 °C.
- The surface temperature is average of 400 °C for standard gear pair and average of 460°C for the optimized gear pair with the maximum temperature of 540°C.
- 12% reduction of gear size compared to standard gear size.
- 2 % increase in efficiency was achieved with the optimized gear design.

Conclusion

Thus a good correlation exists between the standard and the optimized gear pairs. By maintaining the surface temperature and noise in the optimized design can significantly contribute to prevent tooth surface damage under no lubricant operating condition. Also the optimized design is less in size and weight with same life time and produces same effect of the existing standard gears. Thus the costs have been saved and the weight of the machine tools also gets reduced if the same procedure extended for all other machine elements. As the standard values have been used which make the manufacturing process simpler.

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