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Research Article

A NEW DIVISION -METHOD TO SOLVE FOR ASSIGNMENT PROBLEMS

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ABSTRACT
Time minimizing assignment problem dealing with the allocation of n jobs and n persons is considered in this paper. We consider not only difference its also may apply division method of assignment problems. In Hungarian method only consists of subtraction of row and column minimum but in this paper row and column division method.The division method is a systematic procedure , easy to apply for solving assignment problems .

***Keywords*: ** Assignment problem, Hungarian method, New division Method, Optimization.

INTRODUCTION
The Hungarian method is combinatorial optimization algorithm which solves the Assignment problem in Polynomial time and which efficient workers can be find and provide efficient and quality work. In classical assignment problem was made by Harold W.Khun(1955), who gave the name Hungarian method because of the algorithm was largely based on the earlier works of two Hungarian Mathematicians: D Konig and Egarvary. Now a days modified that classical assignment problem method into several methods and changed that procedure and algorithm but the solutions are same. This is called new method of solving assignment problems method.

MATHEMATICAL FORMULATION OF ASSIGNMENT PROBLEM
Each assignment problem has a matrix associated with it. Generally the row contains the objects or people we wish to assign and the column comprise the jobs or tasks we want to them assigned to. Consider a problem of assignments of n resource to m activities so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job.

The cost matrix $C_{ij}$is given under:
A_1 A_2 A_3 .... A_n Available Required

	C_{11} C_{12} C_{13} ....C_{1n}	R_1
R_2	C_{21} C_{22} C_2 .... C_{2n}	
	C_{11} C_{11} C_{11} ....C_{11}	R_3
	.	
	.	
	.	
R_m	C_{n1} C_{n2} C_{n3} .... C_{mn}	

Let C_{ij} be the cost of assigning the ith resource to the jth task. We define the cost matrix to be the n × n matrix. An assignment is a set of n entry positions in the cost matrix, no two of which lie in the same row or column. The sum of the n entries of an assignment is its cost. An assignment with the smallest possible cost is called an optimal assignment.
The assignment problem can be explained as follows:
Let X_{ij} be the variable defined by

$$X_{ij}=\begin{cases} 1, \text{ if } j^{th} \text{ job is assigned to the } i^{th} \text{ machine} \\ 0, \text{ otherwise (if the } j^{th} \text{ job is not assigned)} \end{cases}$$

Where i=1,2,......n; j=1,2,.....n.
Let us assume that there are n jobs and n machines available. Since each machine is available to exactly one job , we have $\sum_{j=1}^{n} X_{ij}$ =1,i=1,2,….n. Similarly each job is assigned to exactly one machine, we have $\sum_{i=1}^{n} X_{ij}$=1, j=1,2,….n, for all variable $X_{ij}\geq 0$.
The problem is minimized $z=\sum_{i=1}^{n}\sum_{j=1}^{n} X_{ij}C_{ij}$ Subject to the given constraints
$\sum_{j=1}^{n} X_{ij}$=1, i=1,2,….n.
$\sum_{i=1}^{n} X_{ij}$=1, j=1,2,…..n, for all variable $X_{ij}\geq 0$.

Method of solving
Step1: Divide the smallest number from each row of the given matrix.
Step2: Divide the smallest number from each column of the given matrix.
Step3: Draw the straight lines covering maximum 1 of the resultant matrix.
Step4: If the minimum number of straight lines equal to the number of assignments, then the optimal solution is obtained. Otherwise to go to step 6.
Step5: Select independent 1 from each row and calculate the optimum.
Step6: Find the minimum number from the uncovered by the straight lines.
a) We have to divide the minimum number to the remaining numbers not covered by these straight lines.
b) We have to multiply the minimum number at the intersection of two straight lines of the resulting matrix.
c) A single straight line numbers are not changed of the resulting matrix.
Return to step 5, we have select the independent 1 of the resulting matrix and finally we will get the optimal assignments.

Numerical Comparison of New Division - Method

1.Solve the following assignment problem using new method
A department head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimate of the time each men would take to perform each tasks is given in the matrix below:

Task	Men			
	E	F	G	H
A	1	4	6	3
B	9	7	10	9
C	4	5	11	7
D	8	7	8	5

How should the tasks be allocated, one to a man, so as to minimize the total man-hours?
Solution:
Given that

1	4	6	3
9	7	10	9
4	5	11	7
8	7	8	5

Step1:Divide the smallest number from each row of the given matrix.

1	4	6	3
1.2857	1	1.4285	1.2857
1	1.25	2.75	1.75
1.6	1.4	1.6	1

Step2:Divide the smallest number from each column of the given matrix.

1	4	4.2002	3
1.2857	1	1	1.2857
1	1.25	1.9250	1.75
1.6	1.4	1.12	1

Step3: Draw the straight lines covering maximum 1 of the resultant matrix.

1	4	4.2002	3
1.2857	1	1	1.2857
1	1.25	1.9250	1.75
1.6	1.4	1.12	1

Step4: The minimum number of straight lines is not equal to the number of assignments, so we cannot apply step 5 at this stage, after complete the step 6 then we return to the step3 to step5.
Step6: Find the minimum number from the uncovered by the straight lines.

a) We have to divide the minimum number from the remaining numbers not covered by these straight lines.

b) We have to multiply the minimum number at the intersection of two straight lines of the resulting matrix.

c) A single straight line numbers are not changed of the resulting matrix.

1	3.2	3.3601	2.4
1.6071	1	1	1.2857
1	1	1.54	1.4
2	1.4	1.12	1

Return to step3 to step5
Step3: Draw the straight lines covering maximum 1 of the resultant matrix.

1	3.2	3.3601	2.4
1.6071	1	1	1.2857
1	1	1.54	1.4
2	1.4	1.12	1

Step4: The minimum number of straight lines are equal to the number of assignments, then the optimal solution is obtained.
Step5: Select independent 1 from each row and column.

[1]	3.2	3.3601	2.4
1.6071	1	[1]	1.2857
1	[1]	1.54	1.4
2	1.4	1.12	[1]

Step5: The optimal assignment is
A→E, B→G, C→F, D→H
The optimal solution is 1+10+5+5=21.

HUNGARIAN METHOD

Step1: Subtracting row minimum from each row of the given matrix
Step 2: Subtracting Column minimum from each column of the given matrix
Step 3: We have draw the minimum number of straight lines vertically or horizontally to cover all the zeros of the resulting matrix. If the minimum number of straight lines equal to the number of assignments, then go to step5.Otherwise go to step4.
Step 4: We have to find out the minimum of the number not covered by these straight lines of the resulting matrix. (a)We have subtracting the minimum number from the remaining numbers not covered by these straight lines.
(b). We have add the minimum number to the numbers at the intersection of two straight lines of the resulting matrix.
(c) A single straight line numbers are not changed of the resulting matrix.
Return to step3.
Step 5: We have select the independent zeros of the resulting matrix and finally we will get the optimal assignments.

1. Solve the following assignment problem using Hungarian method

A department head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimate of the time each men would take to perform each tasks is given in the matrix below:

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C	4	5	11	7
D	8	7	8	5

How should the tasks be allocated, one to a man, so as to minimize the total man-hours?
Solution:
Given that

1	4	6	3
9	7	10	9
4	5	11	7
8	7	8	5

Step1: Subtract row minimum

0	3	5	2
2	0	3	2
0	1	7	3
3	2	3	0

Step2: Subtract column minimum

0	3	2	2
2	0	0	2
0	1	4	3
3	2	0	0

Step3: Draw a minimum number of straight lines it covers maximum and all zeros

0	3	2	2
2	0	0	2
0	1	4	3
3	2	0	0

Number of straight lines are not equal to number of assignments, then we go to step4
Step4:We have to find out the minimum of the number not covered by these straight lines of the resulting matrix.
(a)We have subtracting the minimum number from the remaining numbers not covered by these straight lines.
(b). We have add the minimum number to the numbers at the intersection of two straight lines of the resulting matrix.
(c) A single straight line numbers are not changed of the resulting matrix.

0	2	1	1
3	0	0	2
0	0	3	2
4	2	0	0

Step3: Draw a minimum number of straight lines it covers maximum and all zeros

0	2	1	1
3	0	0	2
0	0	3	2
4	2	0	0

Number of rows equal to number of assignments.

Step4: Select independent zeros for each row and column

[0]	2	1	1
3	0	[0]	2
0	[0]	3	2
4	2	0	[0]

Step5: The optimal assignment is
A→E, B→G, C→F, D→H
The optimal solution is 1+10+5+5=21.

Comparison of Hungarian Method and another Method

Example	New Division-Nethod	HA-Method
01	21	21

CONCLUSION

Assignment problems are solving by the steps forsubtraction of row minimum and then column minimum but now in this new division method of solving t step is divide row minimum and then the column minimum then the solution of the problem is same for the two types of new division method and Hungarian method. Therefore we can solve the assignment problem is divide row and column minimum gives the optimal solution and the results are equal. This paper is an new method of solving assignment problems.

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