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Research Article

A SYSTEMATIC DERIVATION OF ACTIVE FILTERS FROM PASSIVE FILTERS

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ABSTRACT
It is shown that a third order voltage transfer function requires two conditions to be satisfied, while a second order function of a passive RC circuit cannot yield an all-pass or a BRF function. However, they can be realized with the values less than 1 at null frequency. These circuits are converted into standard active filters using an Op Amp (OA). In place of OA, one can use some other devices, such as CFA, FTFN, CCII. Finally, a current mode circuit is derived.

Keywords: *Bridged-ladder network, Band reject filter, Band-pass filter, All pass filter, CFA, FTFN, CC*

I. INTRODUCTION
Recently, Dutta Roy *et al*., (2024) have considered a third order RC bridged-ladder network as a band reject filter. Then they derived a band pass filter through complementary transformation. We bring out a disadvantage of a 3rd order filter in Section 2 followed by a systematic derivation of active filter structures from their passive counterparts. A current mode circuit is derived from a voltage mode circuit in Section 3. Section 4 gives the conclusion. Although the circuits derived here were proposed independently by various researchers in the past, it is shown that they are closely related.

II. ANALYSIS

2.1 THIRD ORDER CIRCUIT
The voltage transfer function of a 3rd order filter that has the value 1 both at zero and infinite frequencies can be expressed as

$$T(s) = \frac{s^3 + as^2 + bs + c}{s^3 + ds^2 + es + c} \tag{1}$$

Replacing s by $j\omega$, we get

$$T(j\omega) = \frac{(c - a\omega^2) + j\omega(b - \omega^2)}{(c - d\omega^2) - j\omega(e - \omega^2)}. \tag{2}$$

Null is obtained when both the real and imaginary parts reduce to 0, i.e., when

$$(b - \omega^2) = 0, \tag{3}$$

and

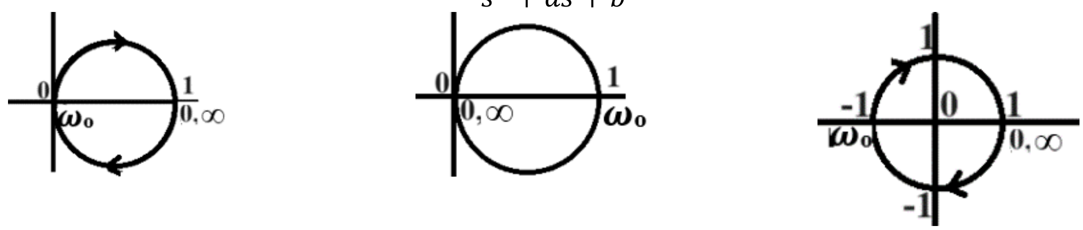
$$c - a\omega^2 = 0. \tag{4}$$

We note that two conditions given by (3) and (4) need to be satisfied simultaneously.

2.2 SECOND ORDER CIRCUITS
Three standard filters BPF, BRF and APF have, respectively, voltage transfer functions

$$T_{BPF}(s) = \frac{as}{s^2 + as + b}, \tag{5}$$

$$T_{BRF}(s) = \frac{s^2 + b}{s^2 + as + b}, \tag{6}$$



(a) BRF (1,1,0) (b) BPF (0,0,1) (c) APF (1,1,-1)

Figure 1: Polar plots of standard filters

$$T_{APF}(s) = \frac{s^2 - as + b}{s^2 + as + b}. \tag{7}$$

The frequency responses (polar plots) of these filters are shown in Fig. 1. We will use the notation (a,b,c) to represent the real values a, b, c at 3 frequencies: 0, ∞, $\omega_o = \sqrt{b}$. Thus, (0,0,1) for BPF, (1,1,0) for BRF and (1,1,-1) for APF. From Figure 1 and eqns. (1)-(3), we see that BPF = 1- BRF. Therefore, BPF and BRF are complementary functions (Rathore *et al.,*1980). Hence, one can be obtained from the other by interchanging the input and ground terminals. Also, APF = -2BRF + 1 = -(1-2BRF). It is the complement of 2BRF with a negative sign. Thus, the three filters are interrelated. If we know one of them, the other two can be derived. The approach we have adopted is to start from a second order passive BRF and then convert it into standard BRF using one OA, and then the other two filters.

The second order voltage transfer function of a passive RC circuit can be expressed as

$$T_o(s) = \frac{s^2 + cs + b}{s^2 + as + b}, \qquad 0 < c < a. \tag{8}$$

In view of the inequalities in Eqn. (4), it cannot reduce to an APF or BRF. However, it can be converted into standard form of the filter using an active device as shown below. From Eqn. (4),

$$T_o(j\omega) = \frac{(b - \omega^2) + jc\omega}{(b - \omega^2) + jc\omega}. \tag{9}$$

At the centre frequency

$${\omega_o}^2 = b, \tag{10}$$

$$T_o(j\omega_o) = \frac{c}{a} = \text{real value} = \ x \text{ (say)} < 1. \tag{11}$$

It is a BRF with a minimum value of $x < 1$ at ω_o. Thus, it is not a standard BRF filter (which has 0 at the null frequency). To convert it into a standard one, we multiply the function by (1+*m*) and subtract *m*, so that the values at 0 and ∞ frequencies remain the same 1. However, from Eqn. (7),

$$T_o(j\omega_o) = x(1 + m) - m. \tag{12}$$

The condition for standard BRF is

$$x(1 + m) - m = 0 \quad \rightarrow \ m = \frac{1}{\left(\frac{1}{x}\right) - 1}. \tag{13}$$

Similarly, the condition for a standard APF is

$$x(1 + m) - m = -1 \quad \rightarrow \ m = \frac{1 + x}{1 - x}. \tag{14}$$

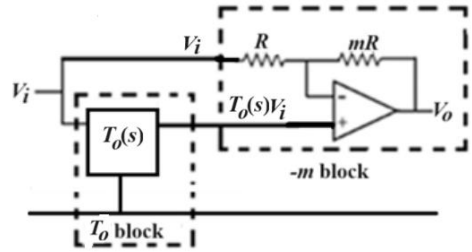


Figure 2: Circuits for BRF ($m = m_1$), for APF ($m = m_2$)

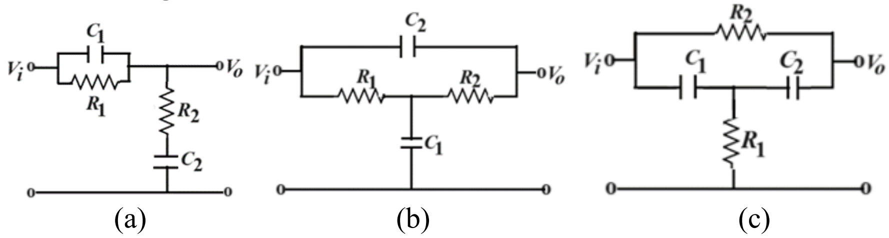


Figure 3: Simple RC circuits for $T_o(s)$.

The implementation of these filters using OA is shown in Fig. 2. Here (-*m* block) is receiving two inputs, $T_o(s)V_i$ and V_i. The voltage transfer function can easily be obtained by superposition, and using the standard formulae for non-inverting and inverting amplifier.

Note that the values at frequencies 0 and ∞ are still 1. When

$$m=\frac{c}{a-c}=m_1, \tag{15}$$

it becomes a BRF and when

$$m=\frac{a+c}{a-c}=1+2m_1=m_2 \tag{16}$$

it becomes an APF.

Example: Three simple passive RC circuits shown in Fig. 2 (Rathore *et al.* 1975a), (Rathore 1975), (Rathore *et al.* 1975b), (Rathore 1976) have the transfer function given by eqn. (4), where

$$a=\frac{(C_1R_1+C_2R_2+C_1R_2)}{C_1R_1C_2R_2}, \quad b=\frac{1}{C_1R_1C_2R_2}, \quad c=\frac{(C_1R_1+C_2R_2)}{C_1R_1C_2R_2}.$$

For convenience, we choose $R_1 = R_2 = 1$ and $C_1 = C_2 = 1$. Then

$$T_o(s)=\frac{s^2+2s+1}{s^2+3s+1}. \tag{17}$$

Therefore,

$$c=2, a=3, \quad x=2/3. \tag{18}$$

We choose as per Eqns. (12) and (13) $m_1 = 2$ and $m_2 = 5$ to realize the following standard BRF and APF, respectively

$$T(s)=\frac{s^2+1}{s^2+3s+1}, \tag{19}$$

$$T(s)=\frac{s^2-3s+1}{s^2+3s+1}. \tag{20}$$

If we interchange the input and ground terminals of T_o-block, the transfer function becomes (Rathore, 1976), (Rathore *et al.*, 1980)

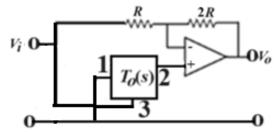


Figure 4: Implementation of BPF (*m* = 2 or 5).

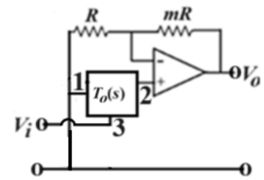


Fig. 5: Implementation of BPF (*m* = 2 or 5).

$$T_{oc}(s) = 1 - T_o(s) = \frac{(a-c)s}{s^2 + as + b}. \quad (21)$$

$T_{oc}(s)$ is 0 at both 0 and ∞ frequencies. This is a BPF with the peak value $(1-x) < 1$ at $\omega_o = 1$. It can be converted into a standard BPF (which has the value 1 at the centre frequency) by multiplying with (1+*m*). Because of the 0 value at 0 and ∞ frequencies, we need not subtract anything, such as *m* above.

The condition for BPF is

$$(1-x)(1+m) = 1 \quad \to m = \frac{1}{(1/x)-1} = 2. \quad (22)$$

The complete circuit of the BPF is shown in Figure 4.
If the input and ground terminals of the circuit shown in Figure 2 are interchanged, we get two complementary functions (Rathore *et al.,* 1980), (Rathore *et al.,* 1980), as BPFs

$$T_C(s) = 1 - T(s) = \frac{(3 \text{ or } 6)s}{s^2 + 3s + 1}. \quad (23)$$

All the passive circuits (including those presented in Dutta Roy *et al*., (2024) cannot be cascaded unless a buffer is employed, except when the load has an infinite input impedance (Rathore, 1980), Rathore *et al*., 2010), (Rathore *et al*., 2005). Then they become active circuits. We have given a flexible active configuration of Figure 2 which can realize BRF, BPF and APF by changing a single resistance and a few connections, and is cascadable.

2.3 FIRST ORDER CIRCUITS

Consider the first order high pass voltage transfer function

$$T_o(s) = \frac{s}{s+1}. \quad (24)$$

Here we have 0 at zero frequency and 1 at ∞ frequency. To get an APF, we need to adjust the value at 0 frequency to -1. Multiply by (1+*m*) and subtract *m* so that

$$T(s) = \left(\frac{s}{s+1}\right)(1+m) - m = \left(\frac{s-m}{s+1}\right). \quad (25)$$

Thus, choosing *m* = 1, we get

$$T(s) = \left(\frac{s-1}{s+1}\right). \quad (26)$$

Consider the first order passive RC high pass circuit shown in Figure 5.
If we interchange input and ground of high pass filter of Figure 6,

$$T_{oc}(s) = 1 - T_o(s) = \frac{1}{s+1}. \quad (27)$$

It has 0 at ∞ and 1 at 0 frequency. We have to bring the value at ∞ to 1. Again, multiplying by (1+*m*) and subtracting *m*, we get

$$T(s) = \left(\frac{1}{s+1}\right)(1+m) - m = -\frac{sm-1}{s+1}. \quad (28)$$

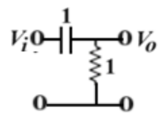
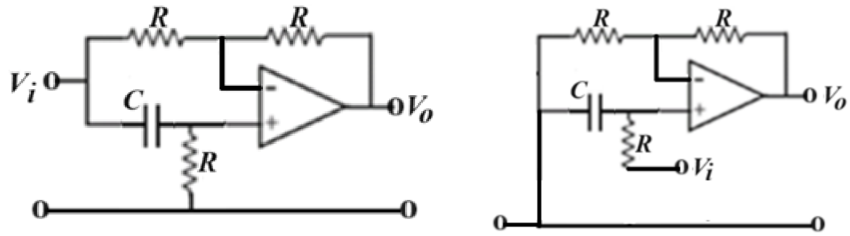


Figure 6: First order high pass filter



(a) (b)

Figure 7: First order APF with (a) positive gain (b) negative gain.

Choosing m = 1 will make it an APF shown in Figure 6(b).
If we interchange input and ground of high pass filter of Figure 6,

$$T_{oC}(s) = 1 - T_o(s) = \frac{1}{s+1} \tag{29}$$

This is a low pass function.

If the input and ground terminals of the circuit shown in Figure 2 are interchanged, we get two complementary functions (Rathore, 1980), (Rathore *et al.* 1980a), as BPFs.

III. CIRCUITS WITH OTHER DEVICES

The OA in Figure 2 is replaced by other active devices as shown in Figure 8. In Figure 8(a), FTFN satisfy eqns. (27) and (28). Therefore, OA can directly be replaced by FTFN. Output is taken at z terminal which follows the voltage of w terminal, but offers zero output impedance. In Figure 8(b), CCII satisfies Equation (27), but does not Equation (28). The current at x terminal is not 0 but equal to I_z, the current through the feedback resistance is halved. Hence, to give the same output voltage, the feedback resistance is doubled. In Figure 8(c), CFA satisfies eqn. (27) and $I_y = 0$, but $I_x \neq 0$. To force $I_x = 0$, I_z is made 0 by keeping the z terminal open. Circuits of Figures 8(a) and (b) have appeared in Rathore *et al.* (2008), Rathore *et al.* (1975b), Rathore *et al.* (2005c).

IV. CURRENT MODE CIRCUIT

In sections II, we have used the following terminal characteristics of the OA given in Table 1.

$$V_x = V_y \tag{30}$$

$$I_x = I_y = 0. \tag{31}$$

The voltage transfer function of the circuit shown in Figure 2 can be expressed as

$$T(s) = T_S(s)(1+m) - m \quad \rightarrow (-m)\{1 - T(s)\} + T(s). \tag{32}$$

From this relation, we see that (-m) and $T(s)$ blocks can be interchanged. Thus, we get an alternative circuit shown in Figure 8. This voltage mode circuit has a virtual ground. Therefore, it can be converted into a current mode circuit by (i) interchanging the output and the inverting terminals of the OA, (ii) connecting a current source current mode circuit.
The voltage transfer function of the circuit shown in Figure 2 can be expressed as

$$T(s) = T_S(s)(1+m) - m \quad \rightarrow (-m)\{1 - T(s)\} + T(s). \tag{33}$$

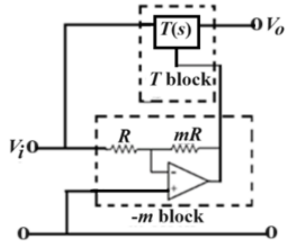


Figure 8: Circuit after interchanging -*m* and *T*$_S$(*s*) blocks

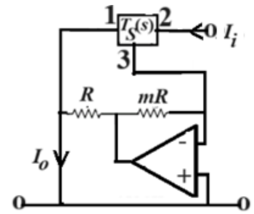
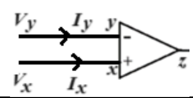
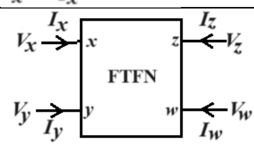
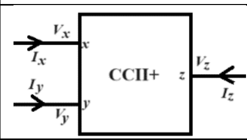
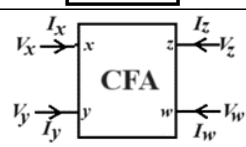


Figure 9: Current mode circuit

Table 1: Various devices, their symbols and characteristics

Device	Symbol	Terminal characteristics
OA		$V_x = V_y$, $I_x = I_y = 0$
FTFN		$V_x = V_y$, $I_x = I_y = 0$, $I_z = \pm I_w$.
CCII		$V_x = V_y$, $I_y = 0$, $I_z = I_x$.
CFA		$V_x = V_y$, $I_x = I_z$, $I_y = 0$, $V_z = V_w$

From this relation, we see that (-*m*) and $T(s)$ blocks can be interchanged. Thus, we get an alternative circuit shown in Figure 8. This voltage mode circuit has a virtual ground. Therefore, it can be converted into a current mode circuit (Rathore *et al.,* 2007) by (i) interchanging the output and the inverting terminals of the OA, (ii) connecting a current source I_i where a voltage output was taken, and I_o will be the output current where the input voltage source was connected in the voltage mode circuit. This current mode circuit so obtained is shown in Figure 9. It can be verified that its current transfer function is the same as that of the voltage mode circuit of Figure 8.

V. CONCLUSION

It is shown that a third order voltage transfer function requires two conditions to be satisfied to act as a BRF. A general second order filter circuit is derived from the corresponding passive circuit which can realize BPF, BRF APF just by changing a resistor and some connections. These circuits require smaller numbers of passive elements, and are cascadable. A current mode circuit is derived from a circuit in which the device has a one input terminal grounded by voltage mode to current mode transformation.

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