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Research Article

NON-REACTIVE SOLUTE DISPERSION IN AQUIFERS SUBJECTED TO TEMPORALLY DEPENDENT SOURCE CONCENTRATION

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ABSTRACT
A two-dimensional advection-dispersion equation is solved analytically for longitudinal and lateral solute dispersion of non-reactive solutes in semi-infinite aquifers. The groundwater flow in the aquifer is considered unsteady. It is assumed that there is a temporally dependent source concentration at the origin of the aquifer. Initially, the aquifer is assumed to be polluted and it is represented by an exponentially decreasing spatial function. A dispersion parameter proportional to the square of velocity is used. Using new space variables, the two-dimensional advection-dispersion equation is reduced to a one-dimensional equation with constant coefficients. The Laplace Transform Technique is employed to derive the two-dimensional analytical solutions for Dirichlet type boundary conditions where the solution is prescribed along the boundary. The analytical solution is interpreted graphically and provides physical insights into pollutant dispersion.

Key Words: *Solute transport; Advection; Dispersion, Aquifers; Analytical solutions*

INTRODUCTION
Solute transport modeling is one of the most important tools to determine the distribution of contaminants in groundwater reservoir. There are different approaches to solute transport modeling, such as analytical approach, numerical approach, and statistical approach. The present discussion focuses on the analytical approach to solute transport modeling. There has been a considerable attention on contaminant transport in porous media, because of public concern and widespread disposal, movement and fate of contaminants in natural subsurface systems. Solute transport in subsurface regions, such as aquifers, is often driven by highly transient flow. The complexity of transient flow and transport phenomena poses a significant obstacle to attaining reliable numerical solutions and involves an extensive search for suitable analytical methods. For well-defined, ideal aquifers, analytical solute transport models are frequently employed. Furthermore, analytical models are often used for verifying the accuracy of numerical solutions to complex solute transport models. It has always been of great importance to analyze the relationship between the dispersion parameter $D\left[L^2T^{-1}\right]$ and the seepage velocity u $\left[LT^{-1}\right]$ occurring in the advection-dispersion equation. Taylor (1953) obtained D proportional to u^2, whereas Bear and Todd (1960) suggested D as proportional to u. Freeze and Cherry (1979) observed that D is proportional to a power *n* of velocity u which ranges between 1 and 2. Scheidegger (1957) summarized his analysis of two possible relationships: (1) $D=\alpha u^2$, where α is a constant of the porous medium, and (2) $D=\beta u$, where β is another constant of the porous medium. In the Indian context, Ghosh and Sharma (2006) experimentally observed the same, i.e., D is proportional to a power *n* of velocity u which ranges between 1 and 1.6. There is a considerable body of literature in groundwater hydrology, surface water hydraulics, fluid mechanics, chemical engineering, and environmental engineering on the dispersion of pollutants in general and the use of advection-dispersion equation in particular (Marino 1978;

Latinopoulos *et al.*, 1988; Aral and Tang 1992; Aral and Liao 1996; Chen *et al.*, 1996; Kumar and Kumar 2002). Rasmuson (1981) derived a solution to a two-dimensional differential equation for dispersion from a disk source, coupled with a differential equation for diffusion and sorption through the successive use of Laplace and Hankel transforms with the result in the form of an infinite double integral. These solutions are useful for analyzing how the spread of poor quality water can be mitigated by flow of fresh water.

A generalized two-dimensional analytical solute transport model in bounded media for flux-type finite multiple sources was developed by Batu (1993). Two-dimensional advective transport in groundwater flow parameter estimation was presented by Anderman *et al.*, (1996). Zoppou and Knight (1999) provided analytical solutions for two- and three-dimensional advection-diffusion equations with spatially variable velocity and diffusion coefficients. They assumed that the velocity component was proportional to the distance from a source and that the diffusion coefficient was proportional to the square of the local velocity component. Tartakovasky (2000) obtained an analytical solution for two-dimensional steady-state transport of a conservative contaminant between injection and pumping wells by considering flow and transport in the vertical cross-section. Analytical solutions of contaminant transport from finite one-, two-, and three-dimensional sources in a finite thickness aquifer were derived by Park and Zhan (2001). Applying the Laplace-transformed power series technique, Chen *et al.*, (2003) analytically solved the two-dimensional advection-diffusion equation in cylindrical coordinates for non-axisymmetric solute transport in a radially convergent flow field. Sander and Braddock (2005) presented analytical solutions for transient unsaturated transport of water and contaminants through two-dimensional porous media. Three-dimensional analytical models for solute transport in finite and semi-infinite porous media have been discussed by Goltz and Roberts (1986), Leij *et al.*, (1991), Chrysikopoulos (1991,1995), Sim and Chrysikopoulos (1998), among others.

The present study assumes that D is proportional to u^2, as suggested by Scheidegger (1957), along both perpendicular directions of the horizontal semi-infinite porous medium. The porous medium is assumed to be polluted initially and is represented by an exponentially decreasing function of space parameters. A time-dependent source contaminant concentration is introduced at the origin of the porous medium, instead of uniform point source as considered by Jaiswal *et al.*, (2011), and the contaminant concentration gradient at an infinite distance in both the directions away from the source is supposed to be zero at all times. The Laplace transform technique is used to obtain analytical solutions.

MATHEMATICAL FORMULATION AND ANALYTICAL SOLUTION

The advection-dispersion equation for a two-dimensional homogeneous porous medium can be written as

$$\frac{\partial C}{\partial t}=\frac{\partial}{\partial x}\left\{D_x\left(x,t\right)\frac{\partial C}{\partial x}-u\left(x,t\right)C\right\}+\frac{\partial}{\partial y}\left\{D_y\left(y,t\right)\frac{\partial C}{\partial y}-v\left(y,t\right)C\right\} \qquad (1)$$

where C is the solute concentration with dimension $\left[ML^{-3}\right]$ at the position $\left(x,y\right)$ each of dimension $\left[L\right]$ of the plane at time $t\left[T\right]$; u,v are referred to as the longitudinal and lateral velocity components, respectively, each of dimension $\left[LT^{-1}\right]$; and coefficients D_x, D_y are referred to as the longitudinal and lateral dispersion coefficients, respectively, each of dimension $\left[L^2T^{-1}\right]$.

Now the solute dispersion components in both longitudinal and lateral directions are considered proportional to the square of respective velocity components and the flow domain is considered temporally dependent. Therefore, we can express

$$u\left(x,t\right)=u_0 f\left(mt\right); \qquad v\left(y,t\right)=v_0 f\left(mt\right) \qquad (2)$$

and

$$D_x(x,t) = D_{x0} f^2(mt); \qquad D_y(y,t) = D_{y0} f^2(mt) \tag{3}$$

where u_0, v_0 occurring in Eq. (2) can be referred to as the initial longitudinal and lateral velocity components, respectively, each of dimension $\left[LT^{-1}\right]$, and coefficients D_{x0}, D_{y0} occurring in Eq. (3) can be referred to as the initial longitudinal and lateral dispersion coefficients, respectively, each of dimension $\left[L^2T^{-1}\right]$. Also m is a flow resistance coefficient whose dimension is inverse of the dimension of time t, i.e., of dimension $\left[T^{-1}\right]$. Here, $f(mt)$ is a non-dimensional expression chosen as in the sinusoidal and exponential forms: $f(mt) = 1 - sin(mt)$ and $f(mt) = \exp(mt)$, respectively, in such a way, that $f(mt) = 1$ for $m = 0$ or $t = 0$.

Using Eqs. (2) and (3) along with these assumptions, Eq. (1) can now be written as:

$$\frac{1}{f(mt)}\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left\{D_{x0} f(mt)\frac{\partial C}{\partial x} - u_0 C\right\} + \frac{\partial}{\partial y}\left\{D_{y0} f(mt)\frac{\partial C}{\partial y} - v_0 C\right\} \tag{4}$$

Now the aquifer is assumed to be polluted initially, i.e., some initial contaminant exists in the aquifer at $t = 0$. Also a time-dependent source contaminant concentration is injected at the origin of the aquifer and the contaminant concentration gradient at an infinite distance in both the directions away from the source is assumed to be zero at all times. The aquifer is assumed to be of semi-infinite extent along both directions.

The set of initial and boundary conditions stated as above can now be expressed as:

$$C(x,y,t) = C_i \exp\{-\gamma(x+y)\}; \qquad x \geq 0, y \geq 0, t = 0 \tag{5}$$

$$C(x,y,t) = \frac{C_0}{2}\{1 + \exp(-qt)\}; \qquad x = 0, y = 0, t > 0 \tag{6}$$

$$\frac{\partial C}{\partial x} = 0, \frac{\partial C}{\partial y} = 0; \qquad x \to \infty, y \to \infty, t \geq 0 \tag{7}$$

where C_i is the initial solute concentration $\left[ML^{-3}\right]$ included with an exponentially decreasing function of space describing the distribution of concentration at all points in both directions of the flow domain, i.e., at $t = 0$, and γ is a constant coefficient whose dimension is the inverse of the space variable, i.e., $\left[L^{-1}\right]$. Here, C_0 is the solute concentration $\left[ML^{-3}\right]$, and q is the decay rate coefficient $\left[T^{-1}\right]$.

Introducing new independent variables, X and Y, using the transformations (Jaiswal *et al.*, 2011):

$$X = \int \frac{dx}{f(mt)} = \frac{x}{f(mt)} \tag{8}$$

and

$$Y = \int \frac{dy}{f(mt)} = \frac{y}{f(mt)} \tag{9}$$

Eq. (4) can now be written as

$$\frac{\partial C}{\partial t} = D_{x0}\frac{\partial^2 C}{\partial X^2} + D_{y0}\frac{\partial^2 C}{\partial Y^2} - u_0\frac{\partial C}{\partial X} - v_0\frac{\partial C}{\partial Y} \tag{10}$$

Also, the initial and boundary conditions, given in Eqs. (5) - (7), are transformed as follows:

$$C(X,Y,t) = C_i \exp\{-\gamma(X+Y)\}; \qquad X \geq 0, Y \geq 0, t = 0 \tag{11}$$

$$C(X,Y,t)=\frac{C_0}{2}\left\{1+\exp(-qt)\right\}; \qquad X=0, Y=0, t>0 \tag{12}$$

$$\frac{\partial C}{\partial X}=0, \frac{\partial C}{\partial Y}=0 \qquad X\to\infty, Y\to\infty, t\geq 0 \tag{13}$$

Again, introducing another transformation

$$\eta = X+Y \tag{14}$$

which reduces Eq. (10) to a one-dimensional equation as:

$$\frac{\partial C}{\partial t}=D_0\frac{\partial^2 C}{\partial \eta^2}-U_0\frac{\partial C}{\partial \eta} \tag{15}$$

where $D_0 = D_{x0}+D_{y0}$ and $U_0 = u_0 + v_0$.

In addition, the initial and boundary conditions in Eqs. (11) - (13) become

$$C(\eta,t)=C_i\exp(-\gamma\eta); \qquad 0\leq\eta<\infty, t=0 \tag{16}$$

$$C(\eta,t)=\frac{C_0}{2}\left\{1+\exp(-qt)\right\}; \qquad \eta=0, t>0 \tag{17}$$

$$\frac{\partial C}{\partial \eta}=0; \qquad \eta\to\infty, t\geq 0 \tag{18}$$

Further, a transformation is considered as

$$C(\eta,t)=k(\eta,t)\exp\left(\frac{U_0}{2D_0}\eta-\frac{U_0^2}{4D_0}t\right) \tag{19}$$

Using Eq. (19), the initial and boundary value problem defined by Eqs. (15) - (18) become

$$\frac{\partial k}{\partial t}=D_0\frac{\partial^2 k}{\partial \eta^2} \tag{20}$$

and

$$k(\eta,t)=C_i\exp\left\{-\left(\frac{U_0}{2D_0}+\gamma\right)\eta\right\}; \qquad \eta\geq 0, t=0 \tag{21}$$

$$k(\eta,t)=\frac{C_0}{2}\left\{2\exp\left(\frac{U_0^2}{4D_0}t\right)-qt\exp\left(\frac{U_0^2}{4D_0}t\right)\right\}; \qquad \eta=0, t>0 \tag{22}$$

$$\frac{\partial k}{\partial \eta}=-\frac{U_0}{2D_0}k; \qquad \eta\to\infty, t\geq 0 \tag{23}$$

Now, applying the Laplace transform (Sneddon, 1974) to Eqs. (20) to (23), one can get:

$$\frac{d^2\bar{k}}{d\eta^2}-\frac{p}{D_0}\bar{k}=-\frac{C_i}{D_0}\exp(-\xi\eta) \tag{24}$$

$$\bar{k}(\eta,p)=\frac{C_0}{2}\left\{\frac{2}{(p-\alpha^2)}-\frac{q}{(p-\alpha^2)^2}\right\}; \qquad \eta=0, t>0 \tag{25}$$

$$\frac{d\bar{k}}{d\eta}=-\frac{U_0}{2D_0}\bar{k}; \qquad \eta\to\infty, t\geq 0 \tag{26}$$

where $\xi = \left(\frac{U_0}{2D_0} + \gamma \right)$, $\alpha^2 = \frac{U_0^2}{4D_0}$ and $\overline{K}(\eta, p) = \int_0^\infty K(\eta, t)\exp(-pt)\,dt.$

Solving Eq. (24), we obtain

Complementary Function= $C_1 \exp\left(-\eta\sqrt{\frac{p}{D_0}}\right) + C_2 \exp\left(\eta\sqrt{\frac{p}{D_0}}\right)$

and Particular Integral= $\frac{C_i}{p - D_0\xi^2}\exp(-\xi\eta)$

The general solution of Eq. (24) can now be written as follows:

$$\overline{k}(\eta, p) = C_1 \exp\left(-\eta\sqrt{\frac{p}{D_0}}\right) + C_2 \exp\left(\eta\sqrt{\frac{p}{D_0}}\right) + \frac{C_i}{p - D_0\xi^2}\exp(-\xi\eta) \tag{27}$$

Putting $\eta = 0$ in the above and using the condition in Eq. (25), we obtain:

$$C_1 + C_2 = \frac{C_0}{2}\left\{ \frac{2}{(p-\alpha^2)} - \frac{q}{(p-\alpha^2)^2} \right\} - \frac{C_i}{p - D_0\xi^2} \tag{28}$$

Again, using the condition from Eq. (26) in Eq. (27), we obtain $C_2 = 0$, and then putting this value of C_2 in Eq. (28), we obtain the value of C_1 as follows:

$$C_1 = \frac{C_0}{2}\left\{ \frac{2}{(p-\alpha^2)} - \frac{q}{(p-\alpha^2)^2} \right\} - \frac{C_i}{p - D_0\xi^2}$$

Putting these values of C_1 and C_2 in Eq. (27), we can get the solution as follows:

$$\overline{k}(\eta, p) = \left[\frac{C_0}{2}\left\{ \frac{2}{(p-\alpha^2)} - \frac{q}{(p-\alpha^2)^2} \right\} - \frac{C_i}{p - D_0\xi^2} \right]\exp\left(-\eta\sqrt{\frac{p}{D_0}}\right) + \frac{C_i}{p - D_0\xi^2}\exp(-\xi\eta)$$

Taking the inverse Laplace Transform (Bateman, 1954), we obtain:

$$\begin{aligned} k(\eta, t) = {} & \frac{C_0}{2}\left\{ \exp\left(\frac{U_0^2}{4D_0}t - \frac{U_0}{2D_0}\eta\right) erfc\left(\frac{\eta - U_0 t}{2\sqrt{D_0 t}}\right) + \exp\left(\frac{U_0^2}{4D_0}t + \frac{U_0}{2D_0}\eta\right) erfc\left(\frac{\eta + U_0 t}{2\sqrt{D_0 t}}\right) \right\} \\ & - \frac{qC_0}{2}\left\{ \frac{1}{2U_0}(U_0 t - \eta)\exp\left(\frac{U_0^2}{4D_0}t - \frac{U_0}{2D_0}\eta\right) erfc\left(\frac{\eta - U_0 t}{2\sqrt{D_0 t}}\right) \right. \\ & \left. + \frac{1}{2U_0}(U_0 t + \eta)\exp\left(\frac{U_0^2}{4D_0}t + \frac{U_0}{2D_0}\eta\right) erfc\left(\frac{\eta + U_0 t}{2\sqrt{D_0 t}}\right) \right\} + C_i \exp\left(D_0\xi^2 t - \xi\eta\right) \\ & - \frac{C_i}{2}\exp\left(D_0\xi^2 t - \xi\eta\right)\left\{ erfc\left(\frac{\eta - 2D_0\xi t}{2\sqrt{D_0 t}}\right) + \exp(2\xi\eta)\, erfc\left(\frac{\eta + 2D_0\xi t}{2\sqrt{D_0 t}}\right) \right\} \end{aligned}$$

Using the transformation given in Eq. (19), we obtain the solution as:

$$C(\eta,t)=\frac{C_0}{2}\left\{erfc\left(\frac{\eta-U_0t}{2\sqrt{D_0t}}\right)+\exp\left(\frac{U_0}{D_0}\eta\right)erfc\left(\frac{\eta+U_0t}{2\sqrt{D_0t}}\right)\right\}+C_i\exp\left\{\left(\gamma^2D_0+\gamma U_0\right)t-\gamma\eta\right\}$$
$$-\frac{qC_0}{4U_0}\left\{(U_0t-\eta)erfc\left(\frac{\eta-U_0t}{2\sqrt{D_0t}}\right)+(U_0t+\eta)\exp\left(\frac{U_0}{D_0}\eta\right)erfc\left(\frac{\eta+U_0t}{2\sqrt{D_0t}}\right)\right\}$$
$$-\frac{C_i}{2}\exp\left\{\left(\gamma^2D_0+\gamma U_0\right)t-\gamma\eta\right\}\left[erfc\left\{\frac{\eta-(U_0+2D_0\gamma)t}{2\sqrt{D_0t}}\right\}+\exp\left(\frac{U_0}{D_0}+2\gamma\right)\eta\right.$$
$$\left.\times erfc\left\{\frac{\eta+(U_0+2D_0\gamma)t}{2\sqrt{D_0t}}\right\}\right]$$

Again, using the transformations given in Eq. (14), (9) and (8), we obtain the solution as:

$$C(\eta,t)=F(\eta,t)-G(\eta,t)-H(\eta,t)+I(\eta,t) \tag{29}$$

where

$$F(\eta,t)=\frac{C_0}{2}\left\{erfc\left(\frac{\eta-U_0t}{2\sqrt{D_0t}}\right)+\exp\left(\frac{U_0}{D_0}\eta\right)erfc\left(\frac{\eta+U_0t}{2\sqrt{D_0t}}\right)\right\} \tag{30a}$$

$$G(\eta,t)=\frac{qC_0}{4U_0}\left\{(U_0t-\eta)erfc\left(\frac{\eta-U_0t}{2\sqrt{D_0t}}\right)+(U_0t+\eta)\exp\left(\frac{U_0}{D_0}\eta\right)erfc\left(\frac{\eta+U_0t}{2\sqrt{D_0t}}\right)\right\} \tag{30b}$$

$$H(\eta,t)=\frac{1}{2}I(\eta,t)\left[erfc\left\{\frac{\eta-(U_0+2D_0\gamma)t}{2\sqrt{D_0t}}\right\}+\exp\left(\frac{U_0}{D_0}+2\gamma\right)\eta\times erfc\left\{\frac{\eta+(U_0+2D_0\gamma)t}{2\sqrt{D_0t}}\right\}\right] \tag{30c}$$

$$I(\eta,t)=C_i\exp\left\{\left(\gamma^2D_0+\gamma U_0\right)t-\gamma\eta\right\} \tag{30d}$$

$$\eta=\frac{x+y}{f(mt)} \tag{30e}$$

$$D_0=\left(D_{x0}+D_{y0}\right) \tag{30f}$$

$$U_0=\left(u_0+v_0\right) \tag{30g}$$

PARTICULAR CASE

If we put $q=0$ and $\gamma=0$ in Eq. (29), then the solution can be written as:

$$C(\eta,t)=(C_0-C_i)F(\eta,t)+C_i \tag{31}$$

where

$$F(\eta,t)=\frac{1}{2}\left\{erfc\left(\frac{\eta-U_0t}{2\sqrt{D_0t}}\right)+\exp\left(\frac{U_0}{D_0}\eta\right)erfc\left(\frac{\eta+U_0t}{2\sqrt{D_0t}}\right)\right\} \tag{32a}$$

$$\eta=\frac{x+y}{f(mt)} \tag{32b}$$

$$D_0 = \left(D_{x0} + D_{y0}\right) \tag{32c}$$

$$U_0 = \left(u_0 + v_0\right) \tag{32d}$$

This represents the solution of temporally dependent dispersion along transient flow with uniform source concentration. This has already been obtained by Jaiswal *et al.,* (2011) for pulse-type input concentration which is the particular case of the present problem and also validates the solution of the problem.

RESULTS AND DISCUSSION

The two-dimensional contaminant concentration dispersion is computed by for the analytical solution of Eq. (29) for an input with $C_i = 0.01$, $C_0 = 1.0$, $u_0 = 0.2\left(km/year\right)$, $v_0 = 0.02\left(km/year\right)$, $D_{x0} = 0.25\left(km^2/year\right)$, $D_{y0} = 0.025\left(km^2/year\right)$, $m = 0.1\left(/year\right)$, $\gamma = 0.001\left(/km\right)$, and $q = 0.5\left(/year\right)$ along transient groundwater flow in the finite domain $0 \le x \le 1(km)$ and $0 \le y \le 1(km)$ along longitudinal and transverse directions, respectively, and time domain $0.4 \le t \le 1.6(years)$ with the sinusoidal form of temporally dependent dispersion, $f(mt) = 1 - sin(mt)$, and the exponential form of temporally dependent dispersion, $f(mt) = \exp(mt)$, and is shown in Fig.1 with solid line and dotted lines, respectively. The transverse components of velocity and dispersion coefficients are considered around one-tenth of the respective longitudinal components because transverse mixing is worth taking into account in shallow aquifers and surface water bodies, otherwise it is neglected in comparison of the respective longitudinal components of velocity in water bodies of substantial depth. The concentration value increases with time but decreases with distance for both forms (sinusoidal and exponential) of temporally dependent dispersion. One can easily observe a variation at the initial stage due to the introduction of time-dependent input source contaminant concentration at the origin. The concentration values at each position are higher in the case of exponential form of dispersion than sinusoidal form of dispersion. This comparative representation of concentration distribution patterns is observed from Fig.1.

If we remove the decay rate coefficient, i.e., $q = 0$, and constant coefficient parameter, i.e., $\gamma = 0$ in the analytical solution given by Eq. (29), then the problem is converted to a uniform source of input concentration C_0 for which the analytical solution is given by Eq.(31) and the concentration distribution patterns are depicted in Fig. 2. In this case one can find only slight changes in the concentration values because of small values of decay rate coefficient and constant coefficient but the concentration distribution pattern remains the same. Also comparative representations of concentration distribution patterns for both forms (sinusoidal and exponential) of temporally dependent dispersion are shown in Fig. 2 with solid line and dotted lines, respectively.

Conclusions

The Laplace Transform Technique is used to obtained analytical solutions for two-dimensional non reactive solute transport along transient groundwater flow in a semi-infinite aquifer. The solutions describe the nature of the contaminant concentration with respect to space and time. The results obtained for two expressions of temporally dependent dispersion, such as sinusoidally and exponentially increasing forms, are significant, because the time-dependent input concentration is considered at the source. A smaller value of the decay rate coefficient q and constant coefficient parameter γ approaches the case of time-dependent form towards the uniform source of input concentration by considering q =0 and $\gamma = 0$. This is the particular case in the present work which was obtained by Jaiswal *et al.,* (2011) for pulse type boundary conditions with uniform input concentration. This validates the solution of the problem in the present work. For the validation, we have taken the same set of input values except the decay rate coefficient q and constant coefficient parameter γ. The time-dependent source of input concentration is more significant than uniform source of input concentration in ground water resource management.

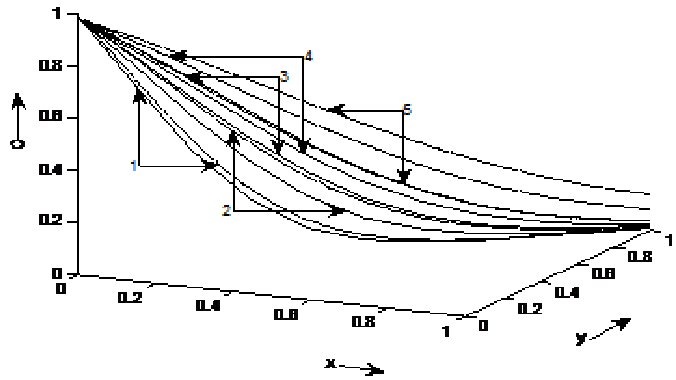

Figure 1: Concentration distribution pattern for exponential form of velocity (dotted line), and sinusoidal form of velocity (solid line) for input values: $C_i = 0.01$, $C_0 = 1.0$, $u_0 = 0.2$, $v_0 = 0.02$, $D_{x0} = 0.25$, $D_{y0} = 0.025$, $m = 0.1$, $\gamma = 0.001$, $q = 0.5$

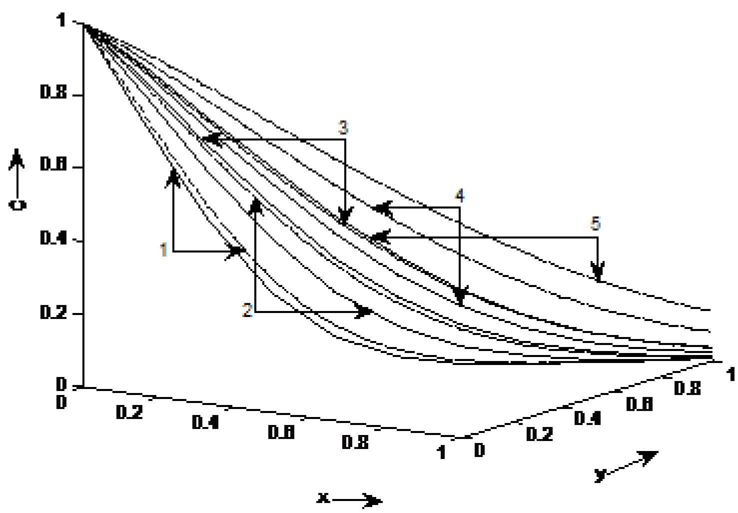

Figure 2: Concentration distribution pattern for exponential form of velocity (dotted line), and sinusoidal form of velocity (solid line) for input values: $C_i = 0.01$, $C_0 = 1.0$, $u_0 = 0.2$, $v_0 = 0.02$, $D_{x0} = 0.25$, $D_{y0} = 0.025$, $m = 0.1$, $\gamma = 0.0$, $q = 0.0$

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