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Research Article

ACCURATE DETERMINATION OF ACCELERATION DUE TO GRAVITY, g IN SHILLONG USING ELECTRONIC TIMER

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ABSTRACT
The only value of g known in Shillong is the one obtained from International Gravity Formula, no first hand and accurate determination of g is carried out till date. We designed a simple but accurate experiment to get the value of g in our laboratory which is located at 25° 26′ 28.2″N, 92° 11′ 59.4″E. The value that we obtained is therefore the first measured value of g. This experiment also illustrates the use of statistical method in physics experiment.

Keywords: *Acceleration, Least Square Fitting, LINEST, Standard Deviation*

INTRODUCTION

The acceleration due to gravity is the most fundamental parameter in Physics. The motion of object on the earth's surface, atmosphere, etc. is governed by the earth's gravity. Earth's gravity is the force with which the earth attracts the object towards its centre. If the object is free to move under the effect of this force, it continues to move with certain value of acceleration, called the acceleration due to gravity and symbolized by the letter 'g'. In recent years, people developed very precise instruments like gravimeters with resolution of 1 in 10^6 for accurate determination of local gravity (Cook, 1957). The underlying principle on all these sophisticated instruments is either the principle of pendulum or time of fall of an object. But all parameters are measured precisely. The main objective is to minimize all the errors that are usually encounter in all scientific experiment. Another way of minimizing errors, is to repeat the number of observations to suitable till a pattern of stability of data distribution and then draw conclusion from these data by using standard laws of statistics. Precise and greater resolutions instruments cannot be easily set up in all Undergraduate Physics Laboratory because of the financial constraints of the institutions. Therefore, the feasible approach towards accurate measurement of any quantity is to repeat the experiment many times under optimum condition and tediously analyze the data mechanically. The process of analyzing the data are no more time taking nowadays because the computer software or programmed are always there for whatever purpose so desired. Since the value of acceleration due to gravity in our place is not found in any standard literature and for all purpose its theoretical is only the one calculated by using the international formula, we feel that accurate measurement of this physical constant g that we come across in our day to day Physics classes should be undertaken now and if there is any other scholar who want to improve or re-investigate its value they should not feel surprise that such work is already done. Moreover the detail analysis of the experimental data in Microsoft Excel 2007 should be appreciated by all students of physics at undergraduate degree level of Indian Universities especially in the Northeastern Parts of India where rigorous analysis of data in undergraduate level is never carried out to estimate the uncertainty level or accuracy level. In all experiments, estimation of errors is important; otherwise the whole work of reporting the result is incomplete (Taylor, 1997).

This experiment that we designed is not to be compared with those of high level of research at university level; rather it is our humble step of improving the laboratory work, but with an aim to determine the accurate value of the constant we use so often in our topical deliberations. The experiment also introduced the application of statistical methods in analyzing the experimental data.

In principle, g can be determined by measuring the time t for an object to fall through a known vertical distance y by applying simple formula

$$y = ut + \frac{1}{2}gt^2 \tag{1}$$

Where u is the velocity of the object at the time which we start the clock. If we start the clock exactly at the time the ball just falls from its stationary position, then $u = 0$. Then the equation (1) becomes,

$$y = \frac{1}{2}gt^2 \qquad (2)$$

The greatest challenge in all free fall experiment is the accurate measurement of time of flight and the validity of equation (1) is questionable (Garg *et al.*, 2007). There is no means of measuring "start time" with sufficient accuracy by using stop clock at one hand and eyes focused on the falling object (Wick and Ruddick,). Again there is a problem if the stop time is exactly at the instant when the object falls through a height h. Thus a scale and clock method is not an accurate way of doing the experiment. To be precise in the measurement of time, we rely on electronic device which can switch the timer at the correct time and stop the same at the correct time. We employed an electronic timer in our experiment. But even then, any instrument has its own limitation. It is because of this, we need ways of minimizing the error that arises from the limitation of the instrument we used. The limitation of the electronic timer in our experiment is considered and care has been taken to minimize the error that arises from its limitations. Since the start time and the stop time play a crucial role in the determination of g, we have to take care of it. In this experiment we determine the error in the time due to off start and off stop of the electronic timer. More-over the formula (1) or (2) is strictly true when the object falls in the presence of gravity alone. But it is not possible to remove atmospheric air in the laboratory we perform the experiment, so we have to take care of the buoyancy and viscous drag of atmospheric air (Lindemuth,).

MATERIALS AND METHODS

Description of Apparatus

The photographs of our experimental set up are shown in figure 1 & 2.

Figure 1: Experimental Set-up

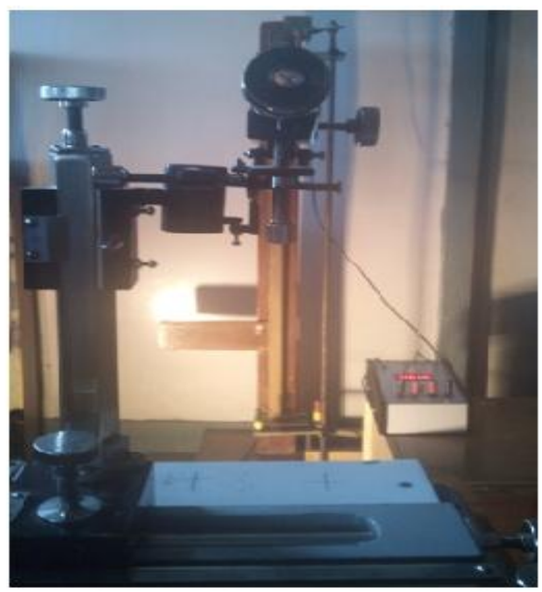

Figure 2: Experimental Set-up with modified telescope for scale reading

The main component of the apparatus consists of the electronic timer. Since the least count of the timer is only 0.01 s, we feel that a scale of least count 0.001 m is sufficient to measure a vertical height through which the steel ball falls. The ball release mechanism consists of an electromagnet and a switch. When the switch is on the catch position, the timer is set to zero. The ball is held in its position by the action of electromagnet. The ball fall immediately when the circuit is broken, or the electromagnet lose its magnetism. The timer start immediately once the "catch" switch is turn to the "release" switch. As long as the circuit of the gate is not broken the timer reading continues to record the time. When the ball hits the Flap below, which complete the gate circuit, the circuit is broken and the timer stop. The time of flight is thus the duration of time from the time of release of the ball by the electromagnet to the time the gate circuit is broken. But it is advisable to investigate that the timer record the exact time when the ball falls and the flap detach from the gate circuit. But it is an inherent property of the timer to start ahead or later of the time when the ball is actually released, or stop earlier or later of the time the ball actually hits the flap placed at scale reading 0.0 cm. To eliminate these offset times of the timer; we plot a graph of height (y) versus the square of time (t^2). The graph is extrapolated to find the offset time, here of course, we find the Square of the offset time. Vertical alignment of the apparatus is also taken care of by using plumb line. The distance of the lowermost portion of the spherical steel ball from the flap is measured by seeing the alignment through a low power telescope attached to the carriage which we modified on the travelling microscope. The parallax is taken care of while measuring the distance.

In order to prevent the steel ball from hitting the hard floor, we use a saw dust filled tray to catch the ball.

Theory

The equation of motion of an object falling through a fluid is

$$m\frac{d^2y}{dt^2} = \left(m - \frac{4}{3}\pi r^3 \rho_{air}\right)g - k\left(\frac{dy}{dt}\right)^2 \qquad (3)$$

where

$m =$ mass of falling object

$r =$ radius of the sphere

$k =$ propotionality constant for viscous drag .

In our experiment, the effect of buoyancy may be neglected since we are using a dense spherical ball, the mass of air displace is negligible compared to mass m of the ball. Thus the equation (3) becomes

$$m\frac{d^2y}{dt^2} = mg - k\left(\frac{dy}{dt}\right)^2 \qquad (4)$$

The quantity k is given by

$$k = \frac{1}{2}\mathrm{C_d}\rho_{\mathrm{air}}\mathrm{A}$$

Where

$C_d =$ drag coefficient

$A =$ Cross-sectional area of the fallingball $= \pi r^2$

The drag coefficient depends on shape, nature of surface and Reynolds's number, R_e (Landau and Litshift, 1987).

The value of C_d in our experiment is calculated to be $C_d \approx 0.5$ (Landau and Litshift, 1987). Therefore for the radius of ball (Table I) used in our experiment,

$$k \approx \frac{1}{2} \times 0.5 \times 1.225 \times \frac{22}{7} \times \left(\frac{0.7156}{100}\right)^2$$

$$k \approx 4.929 \times 10^{-5}\, Ns^2 m^{-2}$$

The solution of Eq. (4) is

$$y(t) = \frac{V_T^2}{g}\ln\left(\cosh\left(\frac{gt}{V_T}\right)\right) \qquad (5)$$

Where

V_T = terminal velocity of the ball,

$$V_T = \sqrt{\frac{mg}{k}} \tag{6}$$

Expanding Eq. (5)

$$y(t) = \frac{V_T^2}{g} \ln\left(1 + \frac{1}{2!}\left(\frac{gt}{V_T}\right)^2 + \frac{1}{4!}\left(\frac{gt}{V_T}\right)^4 + \ldots\right) \tag{7}$$

We estimate the maximum possible value of the term $\frac{1}{4!}\left(\frac{gt}{V_T}\right)^4$ and found it to be insignificant as compare to the term $\frac{1}{2!}\left(\frac{gt}{V_T}\right)^2$ keeping in consideration of the degree of precision of our experimental set up. Thus we neglect the term, $\frac{1}{4!}\left(\frac{gt}{V_T}\right)^4$.

Hence Eq. (7) reduces to

$$y(t) = \frac{V_T^2}{g} \ln\left(1 + \frac{1}{2!}\left(\frac{gt}{V_T}\right)^2\right). \tag{8}$$

Substituting the value of V_T from Eq.(6), we rewrite Eq. (8),

$$y(t) = \frac{\frac{mg}{k}}{g} \ln\left(1 + \frac{1}{2!}\left(\frac{gt}{\sqrt{\frac{mg}{k}}}\right)^2\right)$$

$$y(t) = \frac{m}{k} \ln\left(1 + \frac{1}{2!}\left(\sqrt{\frac{gk}{m}}\,t\right)^2\right).$$

$$y(t) = \frac{m}{k} \ln\left(1 + \frac{1}{2}\frac{gk}{m}t^2\right). \tag{9}$$

This gives the height that the object, starting from rest would fall in time, t.
Eq. (9) may be further simplified,

$$y = \frac{m}{k}\left[\frac{1}{2}\frac{k}{m}gt^2 - \frac{1}{2}\left(\frac{1}{2}\frac{k}{m}gt^2\right)^2 + \ldots\right]$$

$$y = \frac{1}{2}gt^2 - \frac{1}{4}\frac{k}{m}g^2t^4$$

The value of $\frac{1}{4}\frac{k}{m} \approx 1.04\times10^{-3}$, this term is insignificant as compared to the first term, if the vertical distance is less than 1m.

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Data Collection and Analysis

The diameter of the sphere is measured by using a screw gauge of least count $0.001\ cm$ and the mass of the ball measured using a digital balance of least count $0.1\ g$. The values are recorded in Table I

$r = 0.7156 \pm 0.000418\ cm$

$m = 11.9 g$

The experimental set up is aligned properly; the ball is allowed to fall through height starting from a height of 10.0 cm till 90.0 cm increasing by a step of 10.0 cm. For each height, y, 20 observations of time of fall are taken. The observations are directly recoded in the Microsoft Excel spread sheet. The mean of 20 observations gives the observed time of fall, t' for a given height is calculated and its standard deviation is taken, Table II. The graph is plotted with vertical height in vertical axis and square of time t' in vertical axis, Figure 3. Then the graph is extrapolated to meet the time axis at $(0, t_0^2)$. The time t_0 is the error in time due to inaccuracy in start and stop time of the timer. This quantity t_0^2 is to be subtracted from the square of the average time, t'^2 of various heights. Then another column is inserted for true value of square of average time of fall for a given height. Using the square of the true time, t^2 a graph of y vs t^2 is plotted and using the curve fitting method in Excel, the graph is fitted linearly and the coefficients are determined from these curve fittings. The values of the coefficients are indicated in the graph 2. (Figure 3) the value of g is obtained from the graph.

Using LINEST function (Morrison F A) the standard deviation of the slope of the linear trend line is found to be $0.072 ms^{-2}$ [13]

The value of g is compared with the one obtained from the theoretical calculation based on the international gravity formula [11],

$$g = 9.780327\left[1 + 0.0053024\sin^2 L - 0.000058\sin^2 2L\right] - 3.8\times10^{-5} H \quad ms^{-2}$$

Where the latitude L of Shillong College in radians is 25° 26′ 28.2″ radian and H= altitude of the place from Sea-level, H=1475 m (GPSMAP 78 Series, GARMIN Make)

RESULTS AND DISCUSSION

The value of g is obtained as $9.782 ms^{-2} \pm 0.072 ms^{-2}$.

The theoretical value of g obtained from this formula is 9.78528 ms^{-2}

Our value is slightly less than the theoretical value.

The value of g thus obtained from experiment is in good agreement with the one obtained from the theoretical calculation. There is a scope for improving the experimental result with instruments of greater resolution and larger degree of precision. The vertical scale that we used may be replaced by a 100 cm long Vernier scale. The electronic timer of resolution 0.01 s may be replaced by another timer of least count 0.1 ms. The accuracy of the results may be easily reproduced at any other laboratory. T

Table 1

						mean (cm)	**stdev (cm)**
Diameter, D'	1.421	1.422	1.422	1.423	1.423	1.4222	
Corrected D	1.430	1.431	1.431	1.432	1.432	1.4312	0.000837
radius, r	0.715	0.7155	0.7155	0.716	0.716	0.7156	0.000418
mass	11.9	11.9	11.9	11.9	11.9	11.9	0

Table 2

y (m)	**observed time** (t'), (s)	t'^2 (s^2)	(t^2), (s^2)	**standard deviation in t'**, (s)	**Standard Deviation in t^2**, (s^2)
0.1	0.142	0.02	0.017	0.007	0.01
0.2	0.217	0.047	0.044	0.014	0.03
0.3	0.257	0.066	0.063	0.009	0.02
0.4	0.289	0.084	0.081	0.012	0.02
0.5	0.321	0.103	0.103	0.013	0.03
0.6	0.36	0.13	0.127	0.008	0.02
0.7	0.381	0.145	0.142	0.010	0.02
0.8	0.409	0.167	0.164	0.009	0.02
0.9	0.43	0.184	0.181	0.006	0.01

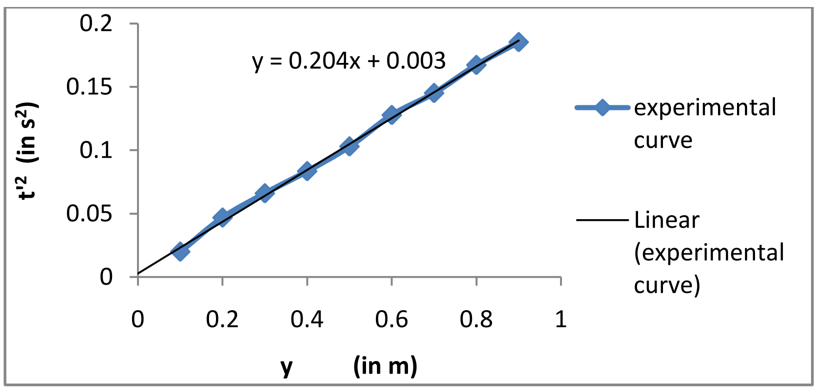

Figure 3: Graph 1. A plot of t'^2 versus vertical height, y. From the graph, the error time, t_0^2 is determined

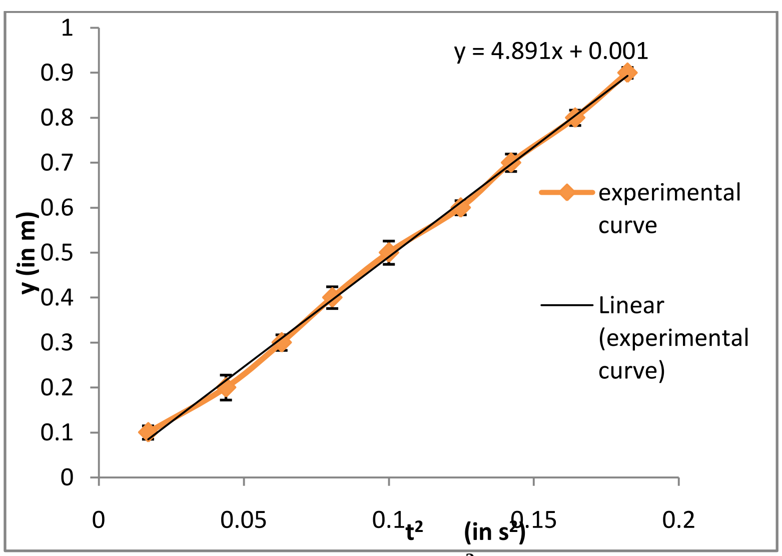

Figure 4: Graph.2 A plot of corrected square of time t^2 versus vertical height, y. From the graph, value of g is determined

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