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Research Article

GEOMETRICAL CONCEPTS WITHIN THE VERSES OF *BHĀSKARĀCĀRYA'S LĪLĀVATĪ* IN PRESENT SCHOOL MATHEMATICS

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ABSTRACT
On the geometrical concept Bhāskarāchārya in his *Līlāvatī* gave the names of different plane-geometric figures such as:

1) क्षेत्रम्(Kṣetram) – *field* – means plane geometric figure.

2) त्र्यस्रम(Tryasram) – *triangle* – it has three angles and three sides; it is also called त्रिभुजम् – *trilateral* – it has 3 sides.

a) ज्यात्यम्(Jātyam) –*right-angled triangle* – भुजकोटिकर्णाः(Bhujakoṭikarṇa) – it has base, an altitude (perpendicular to base) and a hypotenuse.

b) अन्तर्लम्बम्(Antarlambam) – *acute-angled triangle* – all perpendiculars are internal.

c) बहिर्लम्बम्(Bahirlambam)–*obtuse-angled triangle* – it has external perpendicular.

3) चतुरस्रम्(Caturasram) – *quadrilateral* – it has four sides as well as four angles.

a) समकर्ण(Samakarṇa) – *equal diagonals*

A. समभुज(Samabhuja)– *square* – equal sides.

B. आय़तंदीर्घचतुरश्रम्(Āy̤ataṅ dīrghacaturaśram)– *rectangle.*

C. आय़तसमलम्बम्(Āy̤atasamalambam) – *isosceles trapezium.*

b) बिसमकर्ण – *equal diagonals.*

A. बिसमभुज(Bisamabhuja) – *unequal sides.*

B. समलम्बचतुर्भुजम्(Samalambacaturbhujam) – *trapezium.*

4) पञ्चास्रम्(Pañcāsram) –*pentagon;* षड़ास्रम्(Ṣaḍāsram) – *hexagon.*

In this paper we are considering verses which assumes algebraic formulae in triangle, of course coplanar.

INTRODUCTION
प्रीतिंभक्तजनस्ययोजनय़तेविघ्नंविनिघ्नन्स्मृस्तंवृन्दारकवृन्दवन्दितपदंनत्वामतङ्गाननम्।
पाटींसद्गणितस्यवच्मिचतुरप्रीतिप्रदांप्रस्फुटांसङ्क्षिप्ताक्षरकोमलामलपदैर्लालित्यलीलावतीम्॥

Transliteration
Prītiṁ bhaktajanasya yo janay̤te vighnaṁ smṛstaṁ vṛndārakavṛndavanditapadaṁ natvā mataṅgānanam |
Pāṭīṁ sadgaṇitasya vacmi caturaprītipradāṁ prasphuṭāṁ saṅkṣiptākṣarakomalāmalapadairlālityalīlāvatīm ||

English Version
Having bowed to the God, whose head is like an elephant[1]; whose feet are adored by host of deities; who, when being remembered, relieves his votaries from embarrassment i.e. causes delight among the devotees by removing obstacles and bestows happiness on his worshippers; I[2] propound this easy process of computation[3] (arithmetic) to be demonstrated by expert mathematicians, delightful by its elegance[4], perspicuous with words concise, soft and correct and pleasing to the learned.

In the sixth chapter of *Līlāvatī*, Bhākarācārya delt with plane geometry & its properties whereas in seventh chapter he dealt with problems & mensuration.

We are trying to demonstrate the Ślokas with their symbolization with some examples.

MATERIALS AND METHODS

Methods

In sixth chapter of *Līlāvatī, Bhākarācārya* dealt with plane figures:
षष्ठोऽध्याय़ः
Ṣaṣṭha'dhyāyaḥ
अथक्षेत्रव्यवहारः
Atha kṣetravyavahāraḥ
Now, plane figures are dealt with.

क्षेत्र - Kṣetra[5] means plain geometric figures. Best commentaries on Līlāvatīhad been provided in *Buddhivilāsinī* of *Gaṇeśadaivajña* and according to him geometrical plane figures considered by *Bhāskara-II* were of four fold: triangle[6], quadrangle[7], circle[8] and bow[9].

तत्रभुजकोटिकर्ण[10] नामन्यतमाभ्यामन्यतमानय़नाय़करणसूत्रंवृत्तद्वय़म्।१३३

Transliteration

Tatra bhujakoṭikarṇanāmanyatamābhyāmanyatamānayanāya karaṇasūtraṁ vṛttadvayam |

English Version

Here, side, altitude and diagonals[11] are considered and dealt with base, altitude and hypotenuse.

इष्टाद्वाहोर्यःस्यात्तत्स्पर्धिन्यां[12]दिशीतरोवाहुः।
त्र्यस्रेचतुस्रेवासाकोटिः[13] कीर्त्तितातज्ज्ञैः[14]॥१३३

Transliteration

Iṣṭādvāhoryaḥ syāttattspardhinyāṁ diśītaro vāhuḥ |
Tryasre catusrevāsakoṭiḥ kīrtitā tajjñaiḥ ||133

English Version

In a triangle or quadrilateral, with a desire side, a perpendicular to the side will the other side and it is called altitude; it can only guess by learned.

तत्कृत्योर्योगपदंकर्णोदोःकर्णवर्गयोर्विवरात्।
मूलंकोटिःकोटिश्रुतिकृत्योरन्तरात्पदंबाहुः॥१३४

Transliteration

Tatkṛtyoryogapadaṁ karṇo doḥkarṇavargayorvivarāt|
Mūlaṁ koṭiśrutikṛtyorantarātpadaṁ bāhuḥ||134

English Version

The square-root of the sum[15] of the squares of those legs[16] in the diagonal. The square-root, extracted from the difference of the squares[17] of the diagonal and side, is the upright; and that, extracted from the difference of the squares of the diagonal and upright, is the side[18].

Algebraically, we consider, in a right-angled triangle, sides containing right-angle are a and b whereas hypotenuse is c. Then, as per above citation:

1. $\sqrt{a^2+b^2}=c$
2. $\sqrt{c^2-a^2}=b$ or, $\sqrt{c^2-b^2}=a$

It has been expressed in Euclid's Proposition-47 of Book-I[19].

The Pythagorean theorem was demonstrated and proved in *Baudhyāyan Sulba Sūtra* (800 BCE) as:

दीर्घचतुरश्रस्याक्ष्णया रज्जु: पार्श्र्वमानी तिर्यग् मानी च यत् पृथग् भूते कुरूतस्तदुभयं करोति ॥

Transliteration

Dīrghachaturaśrasyākṣaṇayā rajjuḥ pārśvamānī tiryagmānī cha yatpṛthagbhūte kurutastadubhayaṁ karoti ||

English Version

A rope stretched along the length of the diagonal produces an area which the vertical and horizontal sides make together.

It means *The diagonal of a rectangle produces by itself both (the areas) produced separately by its two sides.*
If this refers to a rectangle, it is the earliest recorded statement of the Pythagorean theorem.
अथप्रकारान्तरेणतज्ज्ञानाय़करणसूत्रंसार्घवृत्तम्।

Transliteration
Atha prakārāntareṇa tajjñānāy̱a karaṇasūtraṁ sārdhavṛttam |

English Version
Now, algebraic method on right-angled triangle for the learned given below in two and half stanza.
राश्योरन्तरर्गेण[20]द्विघ्नेघाते[21]युतेतय़ोः।
वर्गयोगोभवेदेवंतय़ोर्योगान्तराहतिः[22]।
वर्गान्तरंभवेदेवंज्ञोय़ंसर्वत्रधीमता॥१३५

Transliteration
Rāśyorantarargeṇa dvighne ghāte yute tay̱oḥ|
Vargantaraṁyogo bhavedevaṁ tay̱oryogāntarāhati|
Vargantaraṁ bhavedevaṁ jñoy̱aṁ sarvatra dhimatā||135

English Version
Twice the product of two quantities, added to the square of their difference, will be the sum of their squares. The product of their sum and difference will be the difference of their squares: as must be everywhere understood by the intelligent calculators.

1. $2ab + (a-b)^2 = a^2 + b^2$
2. $(a+b)(a-b) = a^2 - b^2$

Geometrical interpretation of the above algebraic formulae was given by Euclid in his Propositions 5[23] & 9[24] of Book-II.
अथत्र्यस्रजात्येकरणसुत्रंवृत्तद्वय़म्।

Transliteration
Atha tryasrajātye karaṇasutraṁ vṛttadvay̱am |

English Version
Now rule for generating right-angled triangles expressed in two stanzas.
इष्टोभुजोऽस्माद्दिगुणेष्टनिघ्नादिष्टस्यकृत्यैकवियुक्तय़ाप्तम्।
कोटिःपृथक्सेष्टगुणाभुजोनाकर्णोभवेत्र्यस्रमिदंतुज्यात्यम्॥१३९

Transliteration
Iṣṭo bhujo’smāddviguṇeṣṭanighnādiṣṭasya kṛtyaikaviyuktay̱āptam |
Koṭiḥ pṛthak seṣṭaguṇa bhujonā karṇo bhavettryasramidaṁ tu jyātyam ||139

English Version
Let us take a base (side). Assume a number. Multiplying the side by twice of assumed number and dividing by one less than the assumed number altitude is obtained. Multiply altitude by assumed number and subtract the side from the product is hypotenuse. Such triangle is termed as right-angled triangle.
Let a denote the base / side and assumed number is n. Then, proceeding as per rule altitude $h = \frac{2an}{n^2-1}$ and then hypotenuse $b = \frac{2an}{n^2-1} \times n - a = a \times \frac{n^2+1}{n^2-1}$
Now, verify it as:

$$a^2 + \left(\frac{2an}{n^2-1}\right)^2 = \frac{a^2}{(n^2-1)^2}\{(n^2-1)^2 + 4n^2\} = \frac{a^2(n^2+1)^2}{(n^2-1)^2} = \left[a\frac{n^2+1}{n^2-1}\right]^2$$

The quantities, $2n,\ n^2-1,\ n^2+1$ represent base, altitude and hypotenuse respectively as because $(2n)^2 + (n^2-1)^2 = (n^2+1)^2$.
Let us consider another right-angled triangle similar to above with side a. Then, for similar triangle, sides of the two triangles are proportional. So, altitude of second triangle $= \frac{2an}{n^2-1}$.
As n ×altitude of above triangle = its side (base) + its hypotenuse.

Then n ×altitude of present triangle = a + hypotenuse

∴ Hypotenuse $= n \times \frac{2an}{n^2-1} - a = \frac{2an^2}{n^2-1} - a = a\frac{n^2+1}{n^2-1}$.

This is the process, how expression of hypotenuse was obtained by Bhāskara-II.

Therefore, we find two sets Pythagorean Triples $[2n,\ n^2-1,\ n^2+1]$ and $\left[a,\ \frac{2an}{n^2-1},\ a\frac{n^2+1}{n^2-1}\right]$. These triples are generalised for $n = 2,\ 3,\ 4,\ .\ .\ .\ .\ .$

इष्टभुजस्तत्कृतिरिष्टभक्ताद्विःस्थापितेष्टोनयुतार्द्धितावा।
तौकोटिकर्णाविवितिकोटितोवाबाहुश्रती[25]वाऽकरणीगते[26]स्तः॥१४०

Transliteration

Iṣṭa bhujastatkṛtiriṣṭbhaktā dviḥ sthāpiteṣṭonayutārdvitā vā |
Tau koṭikarṇāviti koṭito vā bahuśratī vā’karaṇīgate staḥ || 140

English Version

Let us consider a side and an arbitrary assumed number. Square the side, divide by the assumed number, is set down in two places, and the assumed number being added and subtracted; then, sum and difference is halved, the results are hypotenuse and altitude. Or, in like manner, the side and hypotenuse may be deducted from the altitude. Both results are rational quantities.

Let a denote the side and n the assumed number. Then, by rule$\frac{1}{2}\left(\frac{a^2}{n} + n\right)$ = hypotenuse, $\frac{1}{2}\left(\frac{a^2}{n} - n\right)$ = altitude of the right-angled triangle.

Let us verify it:

$$a^2 + \left\{\frac{1}{2}\left(\frac{a^2}{n} - n\right)\right\}^2 = \frac{1}{4}\frac{(a^2-n^2)^2}{n^2} = \frac{4a^2n^2 + (a^2-n^2)^2}{4n^2} = \frac{(a^2+n^2)^2}{4n^2} = \left\{\frac{1}{2}\left(\frac{a^2}{n} + n\right)\right\}^2$$

Thus, the triple $a,\ \frac{1}{2}\left(\frac{a^2}{n} - n\right),\ \frac{1}{2}\left(\frac{a^2}{n} + n\right)$ are base, altitude and hypotenuse of a right-angled triangle. For any integer $n > 0$ generates as many rational triangles as required.

अथेष्टकर्णात्कोटिभुजानय़नेकरणसुत्रंवृत्तम्।

Transliteration

Atheṣṭakarṇātkoṭibhujānaẏane karaṇasutraṁ vṛttam |

English Version

Now, a rule to find base and altitude of a right-angled triangle with given hypotenuse.

इष्टेननिघ्नाद्विगुणाच्चकर्णादिष्टस्यकृत्यैकयुजा[27]यदाप्तम्।
कोटिर्भवेत्सापृथगिष्टनिघ्नीतत्कर्णय़ोरन्तरमत्रबाहुः॥१४२

Transliteration

Iṣṭena nighnād dviguṇācca karṇādiṣṭasya kṛtyaikayujā yadāptam |
Koṭirbhavetsā pṛthagiṣṭnighnī tatkarṇaẏorantaramatra bāhuḥ ||142

English Version

Twice the hypotenuse taken into an arbitrary assumed number, being divided by the square of the assumed number added with one, the quotient is the altitude. This quotient multiplied by the assumed number and subtracting the hypotenuse is the side or base.

Algebraically, let b is the hypotenuse and n is the arbitrary assumed number. Then, as per rule altitude $= \frac{2bn}{n^2+1}$ and side or base $\frac{2bn^2}{n^2+1} - b = b\frac{n^2-1}{n^2+1}$.

For verification, we see:

$$\left(\frac{2bn}{n^2+1}\right)^2 + \left(b\frac{n^2-1}{n^2+1}\right)^2 = b^2$$

प्रकारान्तरेणतत्करणसुत्रंवृत्तम्।

Transliteration

Prakārāntareṇa tatkaraṇasutraṁ vṛttam |

English Version

On the other way of finding sides with the help of hypotenuse of a right-angled triangle.

इष्टवर्गेणसैकेनद्विघ्नःकर्णोऽथवाहृतः।

फलोनःश्रावणःकोटिःफलमिष्टगुणंभुजः॥१४४

Transliteration

Iṣṭavargeṇa saikena dvighnaḥ karṇo'tha vā hṛtaḥ |
Phalonaḥ śrāvaṇaḥ koṭiḥ phalamiṣṭaguṇṁ bhujaḥ ||144

English Version

Hypotenuse is doubled and divided by the square of an assumed number added to one. Hypotenuse less than that quotient is altitude. The same quotient multiplied by the assumed number is the side.

Algebraically may be expressed as: Let us take hypotenuse $= b$; then, altitude $= b - \frac{2b}{n^2+1} = b\frac{n^2-1}{n^2+1}$; and side $= \frac{2bn}{n^2+1}$.

It can be verified as:

$$\left(\frac{2bn}{n^2+1}\right)^2 + \left(b\frac{n^2-1}{n^2+1}\right)^2 = b^2$$

On the basis of above triples we may take $2n, n^2-1, n^2+1$ as Pythagorean triples as: $(2n)^2 + (n^2-1)^2 = (n^2+1)^2$ [As per verse-139].

अथेष्टाभ्यांभुजकोटिकर्णानय़नेकरणसुत्रंवृत्तम्।

Transliteration

Atheṣṭānhyāṁ bhujakoṭikarṇānaya̤ne karaṇsutraṁ vṛttam |

English Version

Now, having side and altitude, hypotenuse can be determined.

इष्टय़ोराहतिर्दिघ्नीकोटिर्वर्गान्तरंभुजः।

कृतियोगस्तय़ोरेवंकर्णश्चाकरणीगतः॥१४५

Transliteration

Iṣṭa̤yorāhatirdighnī koṭirvargāntaraṁ bhujaḥ |
Kṛtiyogastaya̤orevaṁ karṇaścākaraṇīgataḥ ||145

English Version

Let twice the product of two assumed numbers will be altitude and the difference of their squares, the side: the sum of their squares will be the hypotenuse, and a rational number.

Algebraically, let two assumed number are a and b; then $2ab =$ altitude where side $= a^2 - b^2$. Therefore, hypotenuse $= \sqrt{(2ab)^2 + (a^2-b^2)^2} = a^2 + b^2$.

RESULTS AND DISCUSSION

Result

Pythagoras triple was known to Indian from ancient times and its used in different forms were within the verses in Debanāgari Scripts.

Conclusion

It is Indian heritage that *Śulvasūtram* refers to Pythagoras Theorem, in particular, through it stands for any of the geometric rules of *Baudhāyana, Mānava, Āpasthamba* and *Kātyāyana Śulvasūtras.* Where *Śulvasūtram* contains Pythagoras Theorem in three forms: If a = altitude, b = base and c = hypotenuse, then $\sqrt{a^2+b^2} = c$, $\sqrt{c^2-b^2} = a$ and $\sqrt{c^2-a^2} = b$. From the know-how of above verses by *Bhāskarācarya* we may conclude that different processes of determining base or altitude or hypotenuse of a triangle-triangle had been introduced then in his *Līlāvatī*. More over the expression we find that Pythagorean triples used then were not only integers but also fractions. These shows advance thinking was then for the use of Pythagorean Triples. In most of our school mathematics we use triples of whole numbers but it should be generalized. This types of geometrical concepts is a part of *Mathematics Education.*

Notes

1. *Ganeśa*, represented with elephant's head & human body.
2. *Bhāskarācārya*
3. सद्गणितस्यपाटींवच्मि–describe arithmetic demonstrated by expert mathematicians. *Pāṭīgaṇita* where *pāṭī* means *paripāṭī* i.e. methodical or *Vyaktagaṇita* i.e. arithmetic. चतुरप्रीतिप्रदां – which satiates the ingenious intellect.
4. प्रस्फुटांलालित्यलीलावतीम् - lucid beautifully sportive *Līlāvatī* where it means delightful. सङ्क्षिप्ताक्षर-कोलामल-पदैः – by beautiful and lucid words characterized by brevity.
5. It signifies plane surface bounded by straight-lines or curved-line.
6. त्र्यस्र – *Tryasra (*त्रिकोण – *trikoṇa,* त्रिभुज – *tribhuja)* contains three (अस्र–asraor कोण - koṇa) angles and consisting of as many (भुज– bhuja, वाहु – Bāhu) sides and word had been invented from human arm.
7. चतुरस्र– Caturasra (चतुष्कोण - catuṣkoṇa, चतुर्भुज – caturbhuja) is quadrilateral or tetrahedron is consisting of (चतुर) four (अस्र–asraor कोण- koṇa) angles or (भुज– bhuja) sides.
8. वर्तुल–*Vartula.*
9. चाप – *Cāpa*: Arc.
10. कर्ण- hypotenuse. A diagonal or oblique line connecting two extremities of two sides is the *karṇa* also termed as *śruti, śravaṇa* literally means ear. It is diagonal of tetragon / parallelogram wherefore triangle is half of it and in case of right-angled triangle it is hypotenuse.
11. The other side in the rival direction is called the *upright.* Which proceedsin the opposite direction at right angles .e.iand it is called *koṭi, Ucchrāya, Ucchriti* or by any other terms signifying as upright or elevated. Here both are alike sides of right-angled triangle or of tetrahedron differing only assumptions.
12. तत्स्पर्घिन्यांदिशि- in the direction perpendicular to the side.
13. कोटिः- altitude / ordinate.
14. तज्ज्ञैः-by the learned.
15. योगपदं- square-root of sum
16. कृत्योः- squares of two sides
17. मूलंविवरात्- square-root of difference
18. The proof, both algebraically & geometrically was given by Ganeśa, (flourished in 1507 CE) in his *Commentary on Līlāvatī,* as उपपत्तिअव्यक्तकृय़या (Upapatti avyakta kṛyayā) – Algebraical proof; क्षेत्रगतोपपत्ति (Kṣetragatopapatti– Geometrical proof; Bhāskara-II also gave its proof in his *Vijagaṇitam* §146.
19. In right-angled triangles, the square on the side subtending the right angle is equal to the (sum of the) squares on sides containing the right-angle.
20. अन्तर्वर्गेण- square of the difference of quantities.
21. द्विघ्नेघाते- double the product of two quantities.
22. योगान्तराहतिः- product of sum and difference of two quantities.
23. If a straight-line is cut into equal and unequal (pieces) then the rectangle contained by the unequal pieces of the whole (straight-line), plus the square on the (difference) between the (equal and unequal) pieces, is equal to the square on half (of the straight-line). $ab + \left(\frac{a+b}{2} - b\right)^2 = \left(\frac{a+b}{2}\right)^2$ or, $4ab + (a-b)^2 = (a+b)^2$; or $4ab + (a-b)^2 = a^2 + b^2 + 2ab$; $\therefore 2ab + (a-b)^2 = a^2 + b^2$.

24. If a straight line is cut into equal and unequal (pieces) then the (sum of the squares) of unequal pieces of the whole (straight-line) is double the (sum of the squares) on half (the straight-line) and (the square) on the (difference) between the (equal and unequal) pieces. $a^2+b^2=2\left[\left(\frac{a+b}{2}\right)^2+\left(\frac{a+b}{2}-\right.\right.$ *b2=2a+b22+a−b22=22a2+b24.*

25. बाहुश्रुती – base and hypotenuse.
26. अकरणीगते – rational.
27. कृत्यैकयुजा – one plus the square.

Transcription / Transliteration

अ	A, a	आ	Ā, ā	इ	I, i	ई	Ī, ī
उ	U, u	ऊ	Ū, ū	ऋ	Ṛ, ṛ	ए	E, e
ऐ	ai	ओ	o	औ	ou	क	K, k
ख	Kh, kh	ग	G, g	घ	Gh. gh	ङ	ṅ
च	C, c	छ	Ch, ch	ज	J, j	झ	Jh, jh
ञ	ñ	ट	Ṭ ṭ	ठ	Ṭh, ṭh	ड	Ḍ, ḍ
ढ	Ḍh, ḍh	ण	ṇ	त	T, t	थ	Th, th
द	D, d	ध	Dh, dh	न	N, n	प	P, p
फ	Ph, ph	ब	B, b	भ	Bh, bh	म	M, m
य	y	र	R, r	ल	L, l	व	V, v
श	Ś, ś	ष	Ṣ, ṣ	स	S, s	ह	H, h
ड़	ḋ	ढ़	ḋh	य़	ỵ	ऽ	’
ं	ṁ	ः	ḥ	ऌ	Ḷ, ḷ		

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