Indian Journal of Fundamental and Applied Life Sciences ISSN: 2231-6345 (Online)
An Online International Journal Available at http://www.cibtech.org/jls.htm
2011 Vol. 1 (2) April – June, pp.98-106/Adhikari and Saha.
Research Article

Long Bones Are Not Just Props of the Structures Held By It

***Swapan Kumar Adhikari[1], Shibendra Kumar Saha[2]**
[1]Netaji Subhash Open University, 35/1, Krishnataran Naskar Lane, Ghusuri, Howrah, West Bengal, India – 711 107
[2]West Bengal Medical Education Service & Netaji Subhash Open University
****Author for Correspondence:*** E-mail: swapankumar.adhikari@gmail.com

ABSTRACT

This paper deals with interrelation of bone structures within long bones and their mathematical appreciation on the process of transmission of body-weight. We think that outcome of our paper is an insight into prosthesis i.e. Replacement material in cases of surgical interventions. Our analysis will also help to plan the physiotherapy in diseased conditions.

INTRODUCTION

Bone is a composite material consisting of collagen to provide tensile strength and hydroxyapatite mineral providing compressive strength. All matured bone is essentially lamellar; the cortex is relatively compact with high volume to surface ratio. Interspersed within the cortex are Haversian canals, each with a central blood vessel. Circumferential rings around the vessel form osteon. The physical properties of bone are governed by the micro-density of the material. In homogeneous material system, Compressive strength $\propto$ (density)^2. But bone is non-homogeneous. So it has weakest point in the area of least density. Strength of bone is determined by its geometrical pattern. The long bones of lower limbs take up the load of the body through pelvis and transmit it to the ground at rest (standing) and in locomotion. The long bones and the involved joints help and modify the body-load in many ways. Present paper is a mathematical attempt to understand physiological process.

Key Words: *Femur, Kinematics, Prosthesis, Tibia*

MATERIALS AND METHODS

Standard physiological and anatomical data of normal adult have been utilized in the process of mathematical analysis. No animal experiment has been undertaken for this paper. Higher mathematics has been avoided in the kinematics analysis for easy understanding.

RESULTS AND DISCUSSION

On standing and in locomotion body-weight is transmitted through the acetabular cavity (Fig. 1) to the head of femur. The area of the contact surface between the head of femur and acetabular cavity determines the quality of posture and locomotion e.g. standing, sprinting, marathon running etc but weight bearing surface line always keeps right angle to the line of body-weight transmission (Fig.5).

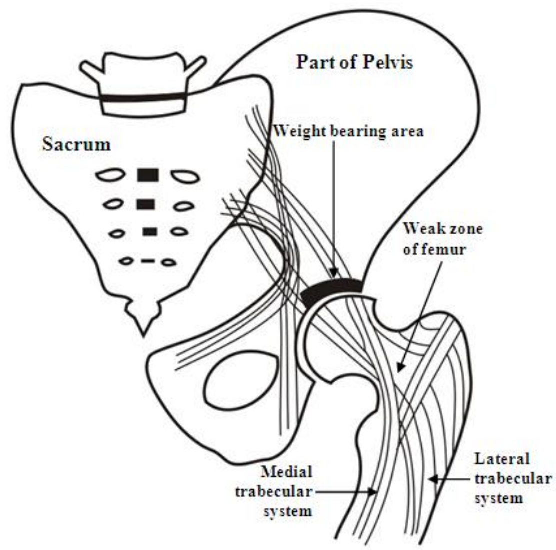

Fig. 1: Showing the lines of forces transmitting body-weight through the pelvis towards the femur.

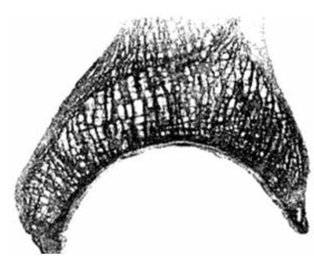

Fig. 2: Coronal section of pelvis (particularly pelvic cavity) showing perpendicular and tangential lamellae showing transmission and absorption of body-weight from upper part.

Indian Journal of Fundamental and Applied Life Sciences ISSN: 2231-6345 (Online)
An Online International Journal Available at http://www.cibtech.org/jls.htm
2011 Vol. 1 (2) April – June, pp.98-106/Adhikari and Saha.
Research Article

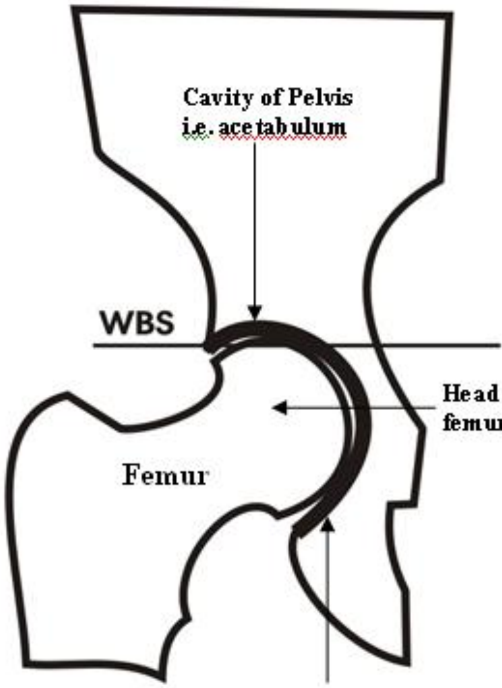

Fig. 3: Thick surface of acetabulum to resist friction due to gait and to accommodate body weight.

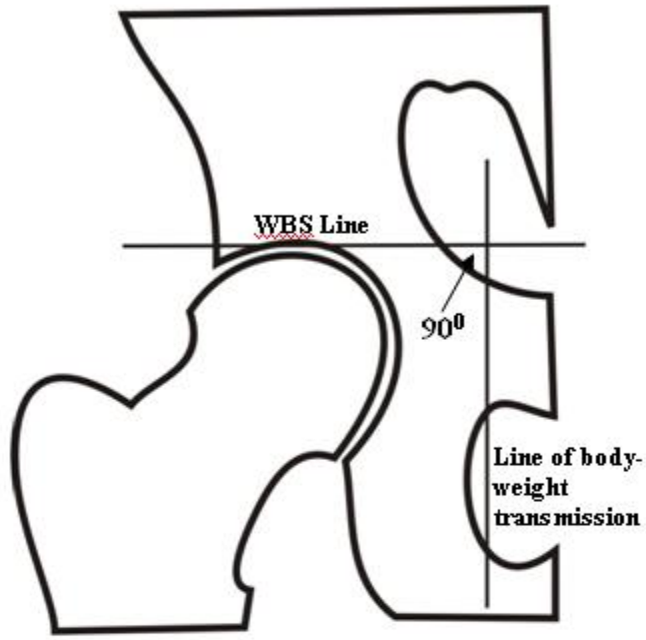

Fig. 5: Body-weight acts perpendicularly to the weight-bearing surface line.

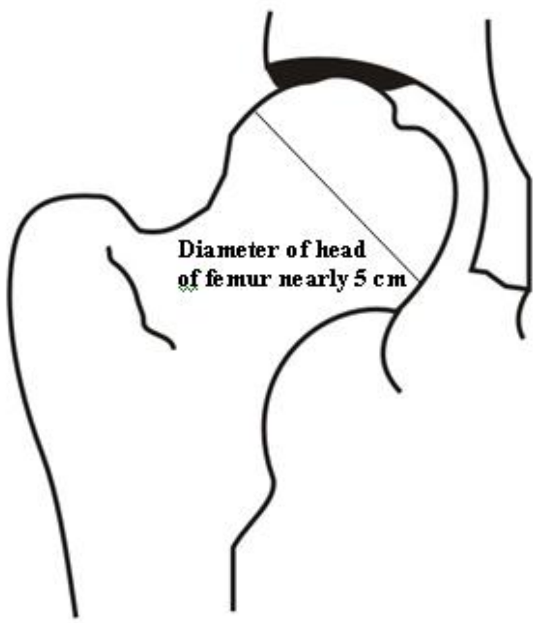

Fig. 4: Standard Diameter of head of femur has been considered to be 5cm.

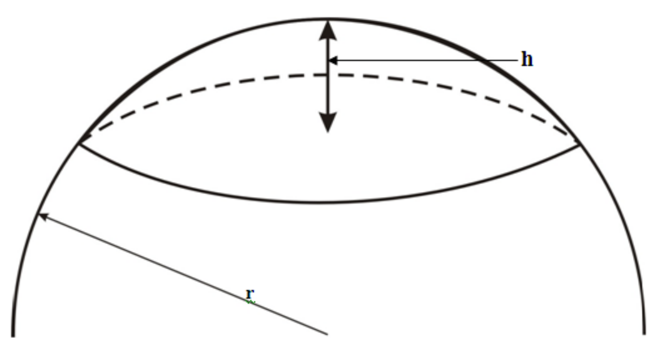

Fig. 6: Spherical contact surface area is nearly a geometrical figure.

Body-weight create inverse cone at the centre of rotation of the femoral head (Fig.7). Weight-bearing contact surface area = S = $2\pi rh$ (Fig.6) where '**r**' is the radius of the spherical head of femur and '**h**' is the height of the contact surface area from the centre of the base of the surface. Acetabular contact area with femoral head differs for different individuals (Figs. 8-10). Here we consider that it subtends different vertical angles of the cone at the centre of rotation (Bombelli 1978) which determines the surface area and the weight-bearing density at the head of femur (Fig.7).

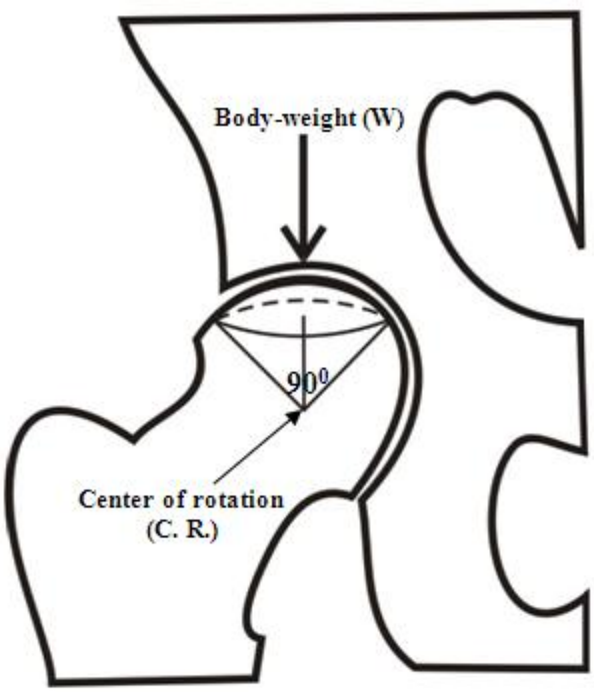

Fig. 7: Body-weight creating cone at the contact surface.

Now, $\cos 45^0 = \frac{CC'}{AC} = \frac{CC'}{5}$; {Diameter of head of femur = 5cm in Figure.4} or, $CC' = 5\times\cos 45^0 = \frac{5}{\sqrt{2}} = 3.54$ cm. $\therefore$ h[as per Fig. 6] = 5 − 3.54 = 1.46 cm. Contact surface area = S = $2\pi rh = 2\times3.14\times2.5\times1.46 = 22.92$ cm^2. So, Weight bearing density = $\frac{70}{22.92} = 3.05$ Kg. Wt./cm^2.

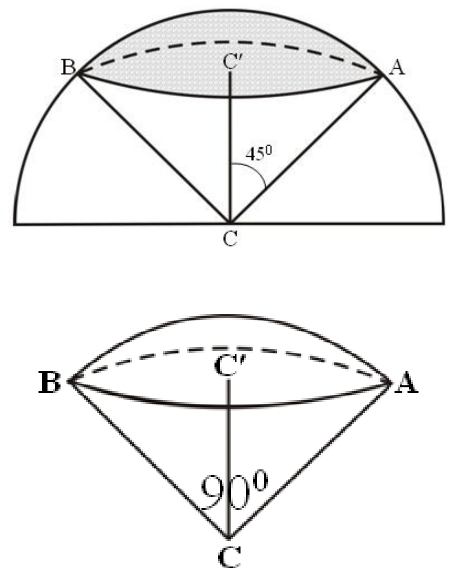

Fig. 8: Considering head of femur to be spherical structure contact surface between acetabulum and femur subtending 90^0 at the center of rotation of femur.

Similarly considering different contact surface area assumed by cone forming within the femur with different vertical angles at the center of rotation (Fig.8-10) we can show the following configurations.

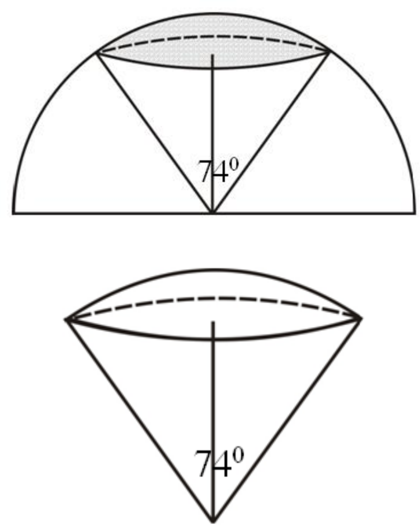

Fig. 9: Contact surface between acetabulum and femur subtending 74^0 at the center of rotation of femur. Contact Surface Area = 16.21 cm^2 and Weight-Bearing Density = 4.32 Kg.wt./cm^2/femur

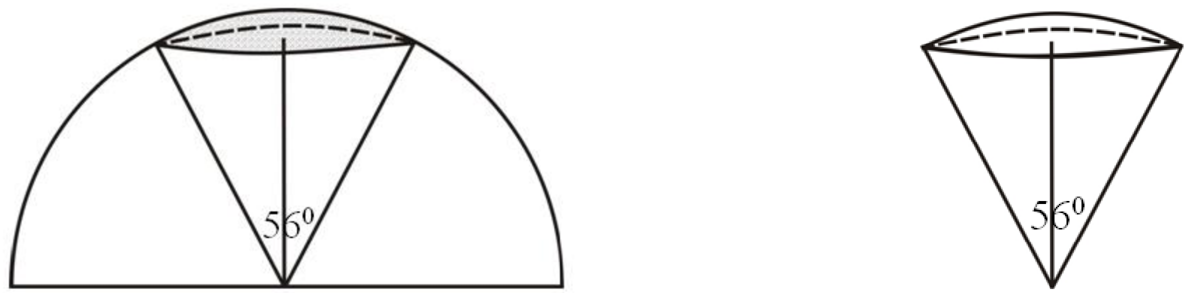

Fig. 10: Contact surface between acetabulum and femur subtending 56^0 at the center of rotation of femur. Contact Surface Area = 9.41 cm^2 and Weight-Bearing Density = 7.44 Kg.wt./cm^2/femur.

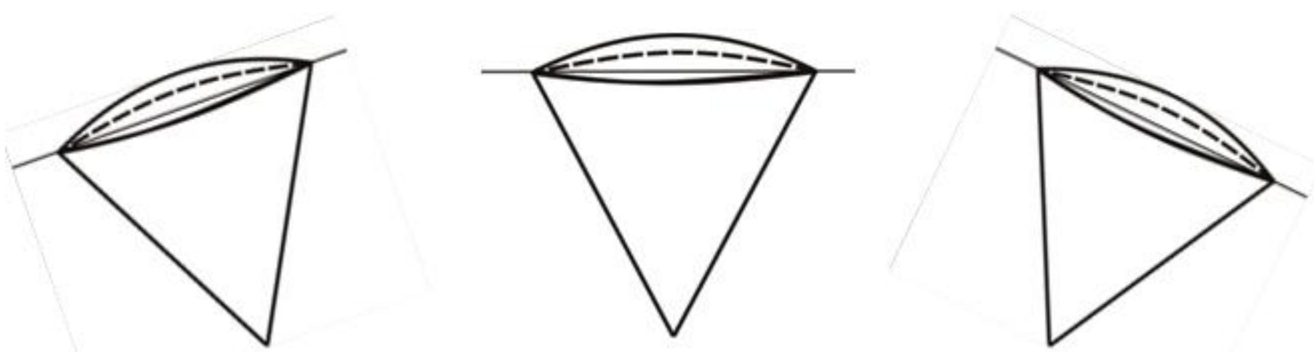

Fig. 11: Throughout the gaits cone forming at the surface of contact between acetabulum and head of femur remains same but weight-bearing cone within the femur turns along with the movements of femur.

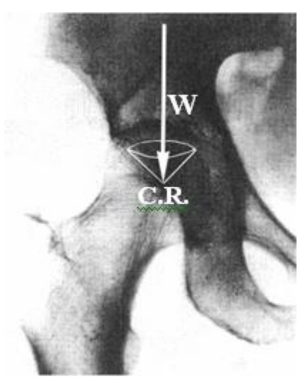

Fig. 12a: Weight Bearing Surface (WBS) is less with fewer angulations at the centre of rotation i.e. with the smaller vertical angle of the cone.

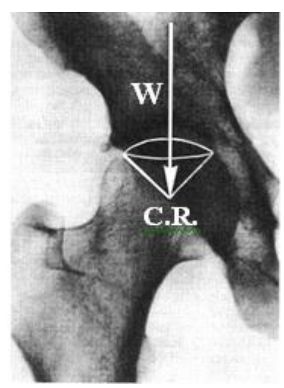

Fig. 12b: Weight Bearing Surface (WBS) is more with more angulations at the center of rotation (CR) i.e. with the greater vertical angle of the cone.

Indian Journal of Fundamental and Applied Life Sciences ISSN: 2231-6345 (Online)
An Online International Journal Available at http://www.cibtech.org/jls.htm
2011 Vol. 1 (2) April – June, pp.98-106/Adhikari and Saha.
Research Article

Actual X-ray is showing Weight-bearing surface (Figs. 12a -12b). Here cone with the centre of rotation (C.R.) of the head of femur can be made out easily. A simple straight view of pelvis can help us to forecast the sports performance of a person i.e., ability of sprinting, marathon running, weight-lifting etc. For example a smaller angulation indicates more mobility as sprinters whereas a greater angulation indicates more stability may turn to weight-lifter. The angle of inclination of femur is the angle formed by the axes of the head and neck with the shaft (Kapandji 1968) and it is about 125^0 (Fig.13). The angle of inclination varies among the individuals and between sexes (Kapandji 1968). In women, the angle is somewhat smaller than it is in men owing to the greater width of pelvis. All these can be made out from simple straight X-ray of the body parts. The angle of torsion (nearly 15^0) (Fig.15) of femur can be best viewed by looking down the length of femur from the top to bottom. It is the angle which the axis of femoral head and neck makes with the axis of femoral condyles. The angle of torsion is known to be Ante-version when angle of torsion > 15^0 and Retroversion when angle of torsion < 15^0.

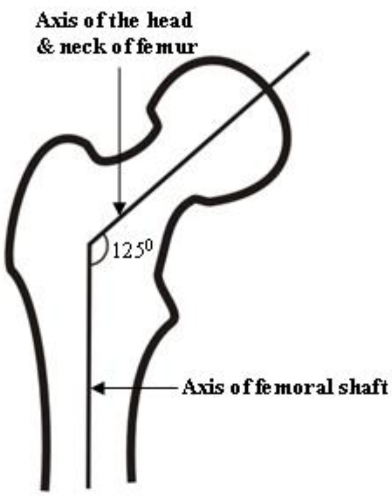

Fig. 13: Axes of femur.

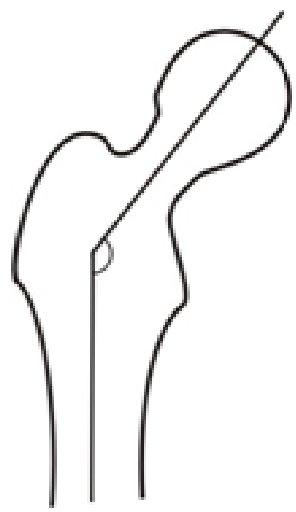

Fig. 14a: When inclination > 125^0 it is known as Coxa Valga state.

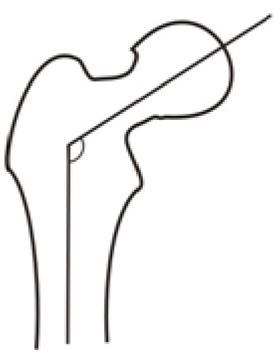

Fig. 14b: When inclination < 125^0 it is known as Coxa Vara state.

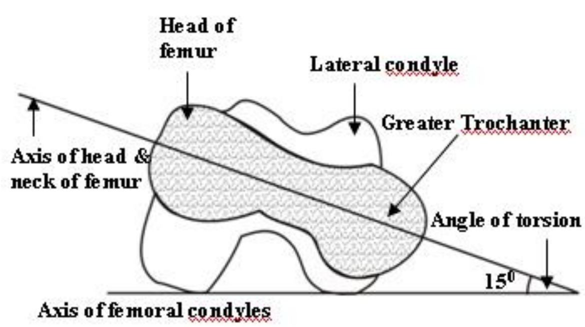

Fig. 15: Showing top view of femur.

Inference

With head having greater inclination > 125^0 (*i.e.*, Coxa Valga) (Fig.14a) and angle of torsion > 15^0 (Ante-version) the shaft is slender and the pelvis is small and high slung. Such configuration favours maximum range of movement at the joint i.e. Adaptation of speed.With head having less inclination < 125^0 (i.e., Coxa Vara) (Fig.14b) and angle of torsion < 15^0 (i.e. Retroversion) the shaft is thicker and pelvis is large and heavy. That favours weight-bearing by reducing mobility at the contact surface.

Mathematical Appreciation of the Functions of Femur

The torsion angle at the shaft of the femur has a role to play in distribution of weight of body. Haversian system of bone structure and the cylindrical spring-like arrangement of Haversian units add to the efficiency of weight transmission in various postures during locomotion. Mathematically we know that helical distribution of rods in pillars can provide maximum strength with minimum number of rods.

Considering shaft to be more or less cylindrical; mathematically we can consider it as circular helix i.e. Curve traced on the surface of a cylinder of radius a and the curve make a constant angle α with the generator (Fig. 17).

Then position vector $r = a(\cos\theta, \sin\theta, \tan\alpha)$,
Curvature = $\kappa = \frac{1}{a}.\sin\alpha = \text{constant}$; Torsion = τ = $\frac{1}{a}.\sin\alpha.\cos\alpha = \text{constant}.$ (Bell 1965)

It obviously follows that the curvature and torsion are both constant, therefore their ratio will be constant, the principal normal will intersect the axis of the cylindrical helix orthogonally and the tangent and binormal will be inclined at constant angles to the direction of generators. Therefore we can deduce that:

- Due to constant value of curvature all the grains/osteoblasts are capable of distributing body-weight (i.e. Stress) in a spring-like fashion.
- Due to constant torsion the bone will tolerate strains or shearing forces to its maximum ability.
- When the principal normal will intersect the axis of the cylinder normally then all the forces due to weight will be distributed through the thick wall of the bone to assume its maximum capacity.
- When tangent and binormal are inclined at constant angles to the fixed direction of the generator, then it will indicate symmetrical distribution of forces towards the base.

We know that geodesic of a circular helix is a helix and geodesic is the shortest path of transmitting force-vector.

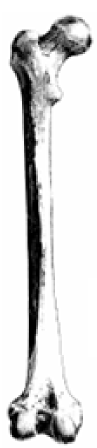

Fig. 16: Femur

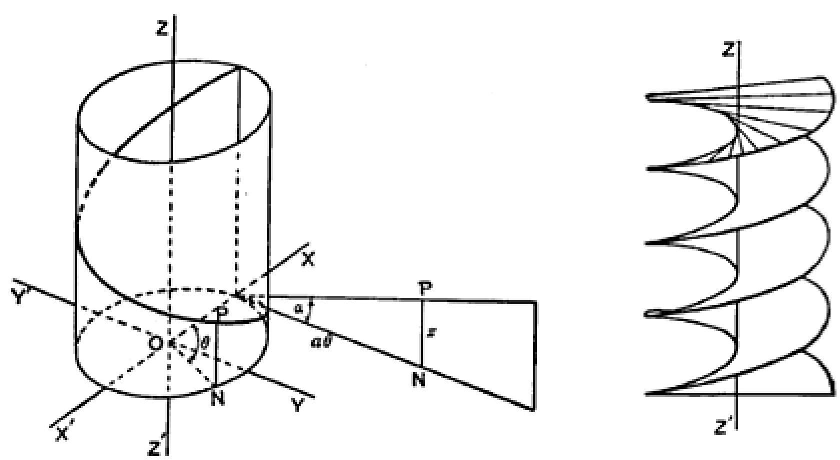

Fig. 17: Mathematical configuration of trabecular structure of bones within the femur.

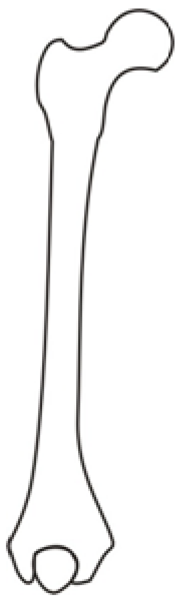

Fig. 18a: Unloaded Femur

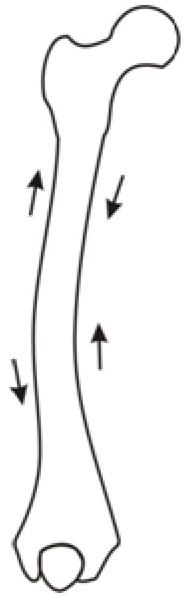

Fig. 18b: Loaded femur

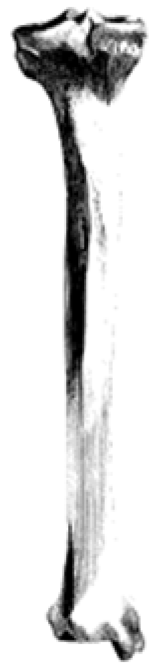

Fig. 19a: Tibia

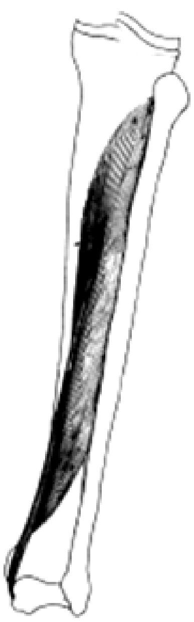

Fig. 19b: tibia attached with fibula

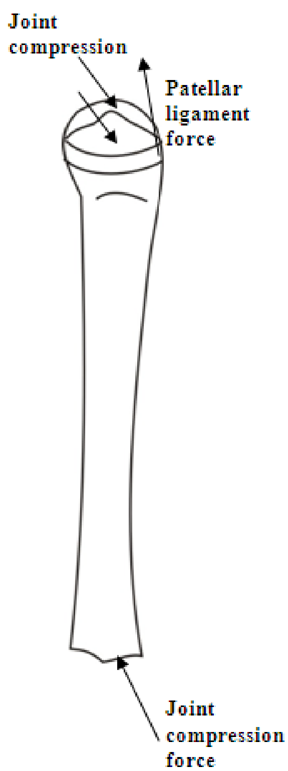

Fig. 20a: Forces acting on tibia to keep it in equilibrium

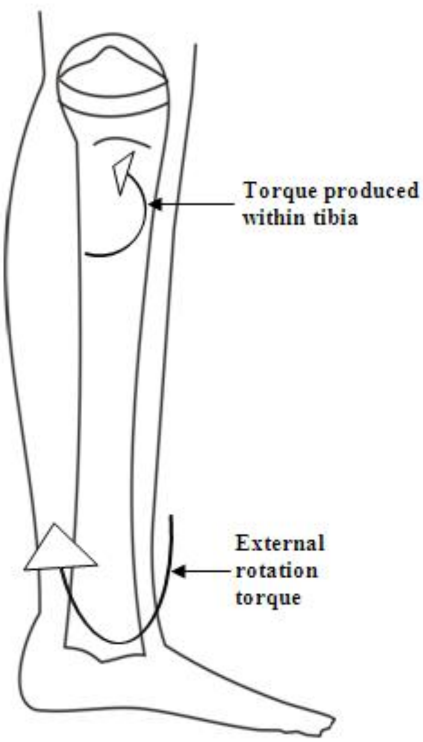

Fig. 20b: External rotational forces on tibia.

Indian Journal of Fundamental and Applied Life Sciences ISSN: 2231-6345 (Online)
An Online International Journal Available at http://www.cibtech.org/jls.htm
2011 Vol. 1 (2) April – June, pp.98-106/Adhikari and Saha.
Research Article

This arrangement naturally occurs within the long bone. Both tensile and compressive stresses and strains are created when a structure such as long bone is subjected to bending moments (Fig.18a). Tensile stress and strain develop on the convex side whereas compressive stress and strain develop at the concave side of the long axis of the bone (Fig.18b).

This can only be possible due to helical structure of long bone. Tibia is mainly used to transmit ground-reaction force to the knee-joint. It also plays a role in transmitting body-weight to the talus by jointly acting with fibula (Fig.19a).

Flexibility of muscle-structure between tibia and fibula permits the tolerance of plantar movements in different postures of gait (Fig.19b). Forces and moments applied across tibia to maintain equilibrium state are distributed on the surface (Fig.20a). External rotation of foot is resisted within the tibia by a torque generated by forces acting parallel to the transverse plane of the tibia and these forces produce shear stress on the transverse cross section (Fig. 20b).

Stress and strain borne by the long bone is dependent of its density of grains (Table 1). Mathematically speaking Stress $\propto$ (Density)^2 and Elasticity $\propto$ (Density)^3.

Table 1: Table of comparison of stress and modulus of elasticity of femur and tibia.

Item	Yield Stress	Ultimate stress	Elastic Modulus
Femur	108 mpa	126 mpa	16 gpa
Tibia	124 mpa	147 mpa	20 gpa

(***Abbrev:*** Mpa - Mega-Pascal (10^6) per unit area; gpa - Giga-Pascal (10^9) per unit area)

Above table (Table 1) comprising stress and modulus of elasticity shows tibia has capacity of absorbing stress is more than that of femur.

Forces and torsions arising during gait:

During the stance phase of gait, the femur applies both shear force and a valgus moment to the proximal tibia to balance the effect of the medial component of ground reaction force (Fig.21).

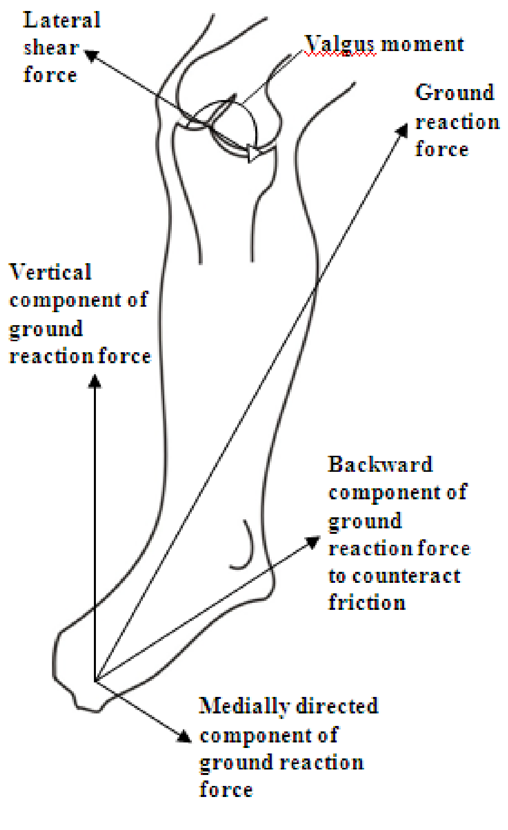

Fig. 21: Forces during gaits.

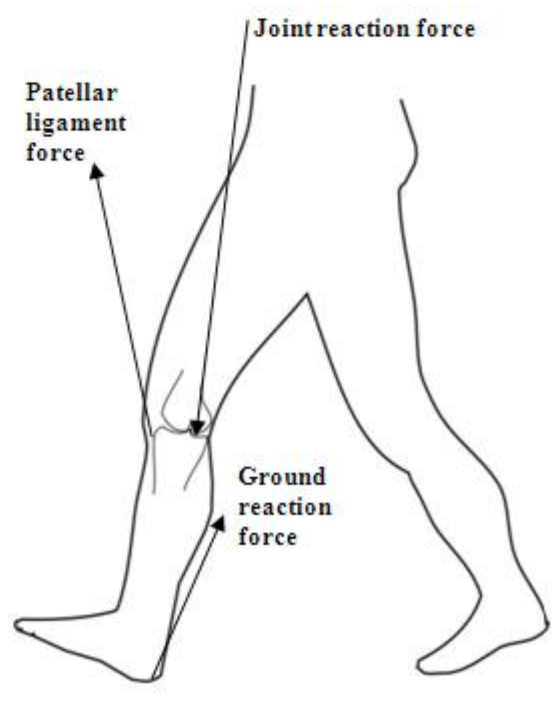

Fig. 22: Nominal forces acting on tibia during heel strike.

Body weight is ultimately transmitted to the ground through arches of foot (Burstein *et al.*, 1994). The medial and lateral arches are made of tarsal bones and its joints. The anterior arch of foot is made by metatarsals. All these arches play very important role in standing and in locomotion. Ground reaction force counter balances the resultant of the component forces along with moments produced at the joints (Fig. 22).

In this paper we see that body-weight of the upper part are transmitted to the lower by spiralic path. Though it has not proved that the force generated due to body weight travel along geodesic i.e. the shortest curvilinear path, but it is fact. So, long bones are not just props for transmitting weight of the parts. Mathematical analysis of the anatomy and physiology allows deep insight into the marvel of the NATURE.

REFERENCES

Adhikari SK (2003). The role of mathematics on human structure. (Dipali Publication, Howrah) **711** 107.

Albright JA and Brand RA (1979). The scientific basis of orthopaedics; (Appleton-Century-Crofts, New York, USA).

Bell JT (1965). An elementary treatise on co-ordinate geometry of three dimension, (Macmillan & Co., London).

Bombelli R (1978). Structure and function in normal and abnormal hips; 3rd edn, (Springer Verlag, USA).

Burstein AH and Wright TM (1994). Fundamentals of orthopaedic biamechanics; (Williams & Wilkins, Philadelphia, USA).

Cailiat R (1995). Ankle pain. (Jaypee Brothers, New Delhi, India).

Hollingshead WH (1969). Functional anatomy of the lims and back; (W B Saunders Company, London).

Kapandji IA (1968). The physiology of joints, Vol **2** (E & S Livingstone, London).