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Research Article

ON MATHEMATICAL ANALYSIS FOR BIANCHI TYPE – V COSMOLOGICAL MODELS WITH TIME VARYING GRAVITATIONAL TERM ‘ G ’

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ABSTRACT
We investigate the Bianchi type –V space time with variable gravitational constant. Cosmological models have been obtained by assuming gravitational term G proportional to $R^{(1+3\omega)}$ (R is the scale factor). In this paper by using a law of variation for average scale factor we have been classify all the solutions according to the different values of ω. The behaviour of these models of the universe are also discussed.

Key Words: *Cosmology, Bianchi Type – V Universe, Variable Gravitational Constant*

INTRODUCTION
The Einstein’s field equations are a coupled system of highly nonlinear differential equations and we seek physical solutions to the field equations for applications in cosmology and astrophysics. In order to solve the field equations we normally assume a form for the matter content or suppose that space-time admits Killing vector symmetries Kramer and Schmutzer (1980). Solutions to the field equations may also be generated by applying a law of variation of the scale factor which was proposed by Pavon(1991) . The cosmological scale factor R(t) in the solution of Einstein’s field equations with Robertson – Walker line elements and Bianchi type models has been the subject of numerous studies. Theory with time – dependent G have also been proposed by Hoyle and Narlikar (1964), Rahman (1990), Biesiada and Malec (2004), Bisnovatyi-Kogan (2006), Chin and Stothers (1976), Dearborn and Schramm (1974). Singh et al. (2004) have studied higher-dimensional cosmological mdel with variable gravitational constant and bulk viscosity in Lyra geometry. The idea of a variable gravitational constant G in the framework of General Relativity was first proposed by Dirac (1938).
Dirac (1938) was probably the first to suggest the possibility that G might vary depending on the age of the universe, specifically proposed that G is a function of inverse time. This theory was developed decades later by Bran and Dicke (1961). Contrary to this, other propose that G increases with age of the universe Rahman (1990). At present, many authors conclude that measuring is a variation of G, but there is no consensus on sign nor the magnitude Garcia-Berro et al. (1995). Recently, Dubey and Tripathi (2011) have considered gravitational term proportional to R^{-n} in LRS Bianchi type model.
In this paper, we consider Bianchi type-V model with a gravitational term proportional to $R^{(1+3\omega)}$ to explain the relationship between the variation of G and expansion of the universe with. We apply the equation of state $p = \omega\rho$. First we present the basic equations of the model and the solutions. Then we discuss the models and present our results.

METRIC AND FIELD EQUATION
We consider the Bianchi type – V metric
$$ds^2 = -dt^2 + A^2dx^2 + e^{2x}\left(B^2dy^2 + C^2dz^2\right) \tag{1}$$
where A , B and C are functions of time t.
The distribution of matter in the space time consist of perfect fluid given by the energy momentum tensor
$$T_{ij} = (\rho + p)v_i v_i + p g_{ij} \tag{2}$$

Satisfying the equation of state

$$p = \omega \rho , \qquad 0 \le \omega \le 1 \tag{3}$$

where p and ρ are pressure and energy density respectively and v_i is the unit flow vector satisfying $v_i v^i = -1$

In comoving coordinates the field equations in case of perfect fluid with variable G are

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G T_{ij} \tag{4}$$

The Einstein field equations (4) for the metric (1) and matter distribution (2) give rise to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{1}{A^2} = -8\pi G p \tag{5}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = -8\pi G p \tag{6}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = -8\pi G p \tag{7}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} - \frac{3}{A^2} = -8\pi G \rho \tag{8}$$

$$2\frac{A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} = 0 \tag{9}$$

The usual energy conservation equation $T^{j}_{i;i} = 0$ yields

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \tag{10}$$

hear and elsewhere suffix " 4 " denotes ordinary differentiation with respect to t.

To write metric functions explicity, we introduces the average scale factor R of Bianchi type – V space time defined by $R^3 = ABC$. From equations (5) – (7) and (9), we obtain

$$\frac{A_4}{A} = \frac{R_4}{R} \tag{11}$$

$$\frac{B_4}{B} = \frac{R_4}{R} - \frac{k_1}{R^3} \tag{12}$$

$$\frac{C_4}{C} = \frac{R_4}{R} + \frac{k_1}{R^3} \tag{13}$$

where k_1 is a constant of integration. Integrating equations (11) – (13), we obtain

$$A = m_1 R \tag{15}$$

$$B = m_2 \operatorname{Re}xp\left(-k_1 \int \tfrac{dt}{R^3}\right) \tag{16}$$

$$C = m_3 \operatorname{Re}xp\left(k_1 \int \tfrac{dt}{R^3}\right) \tag{17}$$

where m_1, m_2 and m_3 are constants of integration satisfying $m_1 m_2 m_3 = 1$.

The Hubble parameter H, Volume expansion θ, shear scalar σ, and deceleration parameter q are given by

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$$H = \frac{R_4}{R} \tag{18}$$

$$\theta = v^i_{;i} = \frac{3R_4}{R} = \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) \tag{19}$$

$$\sigma^2 = \frac{k_1^2}{3R^6} = \frac{1}{3}\left\{\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} - \frac{A_4 B_4}{AB} - \frac{B_4 C_4}{BC} - \frac{C_4 A_4}{CA}\right\} \tag{20}$$

$$q = -\frac{RR_{44}}{R_4^2} \tag{21}$$

Equations (5) - (8) and (10) can be written in terms of H , σ and q as

$$H^2(2q-1) - \sigma^2 + \frac{1}{R^2} = 8\pi G \mathrm{p} \tag{22}$$

$$3H^2 - \sigma^2 - \frac{3}{R^2} = 8\pi G\rho \tag{23}$$

$$\rho_4 + 3(\rho + p)\frac{R_4}{R} = 0 \tag{24}$$

Inserting equation (3) into equation (24) and then integrating, we obtain

$$\frac{\rho_4}{\rho} + 3(1+\omega)\frac{R_4}{R} = 0$$

on integration above equation, we get

$$\log\rho + 3(1+\omega)\log R = \log k_2$$

or

$$\rho = \frac{k_2}{R^{3(1+\omega)}} \tag{25}$$

where $k_2 > 0$ is a constant of integration.

SOLUTION TO THE FIELD EQUATION

The system of equations (3) and (5) – (8) supply only five equations in six unknowns A , B, C, ρ, p and G. One extra equation is needed to solve the system completely.

We consider $G(t) \propto \dfrac{1}{R^{-(1+3\omega)}}$

i.e.

$$G(t) = \frac{a}{R^{-(1+3\omega)}} \tag{26}$$

Subtracting equation (23) from equation (22) we obtain

$$H^2(2q-1) - 3H^2 + \frac{4}{R^2} = 8\pi G(\mathrm{p} - \rho) \tag{27}$$

Putting the value of H, σ and q from equations (18), (20) and (21) in (27) respectively we get

$$\frac{R_{44}}{R} + 2\left(\frac{R_4}{R}\right)^2 - \frac{4}{R^2} = 8\pi G(p-\rho) \tag{28}$$

Inserting the value of ρ from equation (25) and the value of G from equation (26) in equation (28) we get a differential equation

$$\frac{R_{44}}{R}+2(\frac{R_4}{R})^2-\frac{\{2+4\pi(1-\omega)ak_2\}}{R^2}=0 \tag{29}$$

Now we analyze for different values of ω

Cosmology for ω = 0 (Matter Dominated Solution)

For ω = 0, Equation (29) reduces to

$$\frac{R_{44}}{R}+2(\frac{R_4}{R})^2-\frac{\{2+4\pi ak_2\}}{R^2}=0 \tag{30}$$

To determine the time evolution of the Hubble parameter integrating equation (30), we get

$$H=\frac{R_4}{R}=\sqrt{\frac{(2+4\pi ak_2)}{2}}\left[\sqrt{\frac{(2+4\pi ak_2)}{2}}t+t_0\right]^{-1} \tag{31}$$

where the integration constant t_0 is related to the choice of origin of time. From the equation (31) we obtain the scale factor

$$\mathrm{R} \quad = \quad \left[\sqrt{\frac{(2+4\pi ak_2)}{2}}t+t_0\right] \tag{32}$$

where t_0 is a constant of integration.

Putting the value of R from (32) in equations (15) , (16) and (17) we get

$$A(t)=m_1\left[\sqrt{\frac{(2+4\pi ak_2)}{2}}t+t_0\right] \tag{33}$$

$$B(t)=m_2\left[\sqrt{\frac{(2+4\pi ak_2)}{2}}t+t_0\right]\exp\left[\frac{-k_1}{2}\sqrt{\frac{2}{(2+4\pi ak_2)}}\left\{\sqrt{\frac{(2+4\pi ak_2)}{2}}t+t_0\right\}^{-2}\right] \tag{34}$$

$$C(t)=m_3\left[\sqrt{\frac{(2+4\pi ak_2)}{2}}t+t_0\right]\exp\left[\frac{k_1}{2}\sqrt{\frac{2}{(2+4\pi ak_2)}}\left\{\sqrt{\frac{(2+4\pi ak_2)}{2}}t+t_0\right\}^{-2}\right] \tag{35}$$

For this solution metric (1) assumes the form

$$\begin{aligned} ds^2 &= -dt^2+m_1^2\left[\sqrt{\frac{(2+4\pi ak_2)}{2}}t+t_0\right]^2 dx^2+e^{2x}\left(\sqrt{\frac{(2+4\pi ak_2)}{2}}t+t_0\right)^2 . \\ &\left[\begin{array}{l} m_2^2\exp 2\left\{\frac{k_1}{2}\sqrt{\frac{2}{(2+4\pi ak_2)}}\left(\sqrt{\frac{(2+4\pi ak_2)}{2}}t+t_0\right)^{-2}dy^2\right\} \\ +m_3^2\exp 2\left\{\frac{-k_1}{2}\sqrt{\frac{2}{(2+4\pi ak_2)}}\left(\sqrt{\frac{(2+4\pi ak_2)}{2}}t+t_0\right)^{-2}dz^2\right\} \end{array}\right] \end{aligned} \tag{36}$$

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For the model (36) cosmological energy density ρ and gravitational constant G are given by

$$\rho = \frac{k_2}{\left[\sqrt{\frac{(2+4\pi a k_2)}{2}}\,\mathrm{t} + t_0\right]^3} \tag{37}$$

$$G = a\left(\sqrt{\frac{(2+4\pi a k)_2}{2}}t + t_0\right)^4 \tag{38}$$

Expansion scalar θ and shear σ for this model are

$$\theta = \frac{3\sqrt{(2+4\pi a k_2)}}{\sqrt{2}\left[\sqrt{\frac{(2+4\pi a k_2)}{2}}\,\mathrm{t} + t_0\right]} \tag{39}$$

$$\sigma = \frac{k_1}{\sqrt{3}\left[\sqrt{\frac{(2+4\pi a k_2)}{2}}\,\mathrm{t} + t_0\right]^3} \tag{40}$$

The deceleration parameter q for the model (36) is zero

Discussion: In the model (36), we observe that The spatial volume is zero at t = $t = \frac{-t_0\sqrt{2}}{\sqrt{(2+4\pi a k_2)}} = t_1$ (say) and expansion scalar θ is infinite at t = t_1 which shows that the universe starts evolving with zero volume and infinite rate of expansion at t = t_1 . Initially at t = t_1 shear scalar σ and the energy density ρ are infinite. Average scale factors are zero at t = t_1 . As t tends to ∞, the spatial volume V becomes infinitely large. As t increases all the parameters p , ρ , θ, and σ are decreases and tend to zero asymptotically. Therefore, the model essentially gives an empty universe for large t. The gravitational constant G is zero at t = t_1 and it is increase with cosmic time t. The ratio $\frac{\sigma}{\theta} \to 0$ as $\mathrm{t} \to \infty$, which shows that the model approaches isotropy for the large value of t.

Cosmology for ω=1(Zidovichfluid Distribution)

It corresponds to the equation of state p = ρ, has been widely used in general relativity to obtain stellar and cosmological models for utterly dense matter

For ω = 1 equation (29) reduces

$$\frac{R_{44}}{R} + 2\left(\frac{R_4}{R}\right)^2 - \frac{2}{R^2} = 0 \tag{41}$$

To determine the time evolution of the Hubble parameter integrating equation (41) we get

$$H = \frac{R_4}{R} = \left[t + t_0\right]^{-1} \tag{42}$$

Where the integration constant t_0 is related to the choice of origine of time. From equation (42) we obtain

$$\mathrm{R} = \left(\mathrm{t} + t_0\right) \tag{43}$$

Putting the value of R from (43) in equations (15) , (16) and (17), we get

$$\mathrm{A} = m_1(\mathrm{t} + t_0) \tag{44}$$

$$B(t) = m_2(t + t_0)\exp\left[\frac{k_1}{2}(t + t_0)^{-2}\right] \tag{45}$$

$$C(t) = m_3(t + t_0)\exp\left[\frac{-k_1}{2}(t + t_0)^{-2}\right] \tag{48}$$

For this solution metric (1) assumes the form

$$ds^2 = -dt^2 + m_1^2 (t + t_0)^2 dx^2 + e^{2x}(t + t_0)^2 ..$$
$$\left[m_2^2 \exp 2\left\{\frac{k_1}{2}(t + t_0)^{-2} dy^2\right\} + m_3^2 \exp 2\left\{\frac{-k_1}{2}(t + t_0)^{-2} dz^2\right\}\right] \tag{49}$$

For the model (49) cosmological energy density ρ , pressure p, gravitational constant G are given by

$$p = \rho = \frac{k_2}{[\ \mathrm{t} + t_0]^6} \tag{50}$$

$$G = (at + t_0)^4 \tag{51}$$

Expansion scalar θ and shear σ for this model are

$$\theta = 3(t + t_0)^{-1} \tag{52}$$

The deceleration parameter q for the model (49) is zero (q = 0)

Discussion: In the model (49), we observe that The gravitational constant G is increases with cosmic time t and G $\to \infty$ as t $\to \infty$. It becomes zero at t = t_0 (say). As t increases all the parameters p , ρ , θ, and σ are decreases and tend to zero asymptotically. Therefore, the model essentially gives an empty universe for large t. At t = t_0 all the parameters p , ρ , θ, and σ becomes infinite. The spatial volume V is zero and expansion scalar θ is infinite at t = t_0 , which shows that the universe starts evolving with zero volume and infinite rate of expansion. The scale factors also vanish at t = t_0 and hence the model has a point type singularity at initial epoch. The ratio $\frac{\sigma}{\theta} \to 0$ as $\mathrm{t} \to \infty$, which shows that the model approaches isotropy for the large value of t.

Cosmology for $\omega = \frac{1}{3}$ ***, (Radiation Dominated Solution)***

In this case (For ω = 1) equation (29) reduces to

$$\frac{R_{44}}{R} + 2\left(\frac{R_4}{R}\right)^2 - \frac{\{6 + 8\pi a k_2\}}{3R^2} = 0 \tag{53}$$

To determine the time evolution of the Hubble parameter integrating equation (53), we get

$$H = \frac{R_4}{R} = \sqrt{\frac{(6 + 8\pi a k_2)}{6}}\left[\sqrt{\frac{(6 + 8\pi a k_2)}{6}}t + t_0\right]^{-1} \tag{54}$$

where the integration constant t_0 is related to the choice of origin of time. From the equation (54) we obtain the scale factor

$$R = \left[\sqrt{\frac{(6+8\pi a k_2)}{6}}t + t_0\right] \tag{55}$$

where t_0 is a constant of integration.
Putting the value of R from (55) in equations (15) , (16) and (17) we get

$$A(t) = m_1\left[\sqrt{\frac{(6+8\pi a k_2)}{6}}t + t_0\right] \tag{56}$$

$$B(t) = m_2\left[\sqrt{\frac{(6+8\pi a k_2)}{6}}t + t_0\right]\exp\left[\frac{-k_1}{2}\sqrt{\frac{6}{(6+8\pi a k_2)}}\left\{\sqrt{\frac{(6+8\pi a k_2)}{6}}t + t_0\right\}^{-2}\right] \tag{57}$$

$$C(t) = m_3\left[\sqrt{\frac{(6+8\pi a k_2)}{6}}t + t_0\right]\exp\left[\frac{k_1}{2}\sqrt{\frac{6}{(6+8\pi a k_2)}}\left\{\sqrt{\frac{(6+8\pi a k_2)}{6}}t + t_0\right\}^{-2}\right] \tag{58}$$

For this solution metric (1) assumes the form

$$ds^2 = -dt^2 + m_1^2\left[\sqrt{\frac{(6+8\pi a k_2)}{6}}t + t_0\right]^2 dx^2 + e^{2x}\left(\sqrt{\frac{(6+8\pi a k_2)}{6}}t + t_0\right)^2.$$

$$\left[\begin{array}{l} m_2^2 \exp 2\left\{\frac{k_1}{2}\sqrt{\frac{6}{(6+8\pi a k_2)}}\left(\sqrt{\frac{(6+8\pi a k_2)}{6}}t + t_0\right)^{-2} dy^2\right\} \\ + m_3^2 \exp 2\left\{\frac{-k_1}{2}\sqrt{\frac{6}{(6+8\pi a k_2)}}\left(\sqrt{\frac{(6+8\pi a k_2)}{6}}t + t_0\right)^{-2} dz^2\right\}\end{array}\right] \tag{59}$$

For the model (59) cosmological energy density ρ and gravitational constant G are given by

$$\rho = \frac{k_2}{\left[\sqrt{\frac{(6+8\pi a k_2)}{6}}\,\mathrm{t} + t_0\right]^4} \tag{60}$$

$$G = a\left(\sqrt{\frac{(6+8\pi a k_2)}{6}}t + t_0\right)^2 \tag{61}$$

Expansion scalar θ and shear σ for this model are

$$\theta = \frac{3\sqrt{(6+8\pi a k_2)}}{\sqrt{6}\left[\sqrt{\frac{(6+8\pi a k_2)}{6}}\,\mathrm{t} + t_0\right]} \tag{62}$$

$$\sigma = \frac{k_1}{\sqrt{3}\left[\sqrt{\frac{(6+8\pi a k_2)}{6}}\,\mathrm{t} + t_0\right]^3} \tag{63}$$

The deceleration parameter q for the model (59) is zero

Discussion: In this section, we observe that The gravitational constant G is increases with cosmic time t and tends to infinite as t→ ∞. It becomes zero at t $= \frac{-t_0\sqrt{6}}{\sqrt{6+8\pi a k_2}} = t_3$ (say). As t increases all the parameters p , ρ , θ, and σ are decreases and tend to zero asymptotically. Therefore, the model essentially gives an empty universe for large t. At t = $\frac{-t_0\sqrt{6}}{\sqrt{6+8\pi a k_2}} = t_3$, all the parameters p , ρ , θ, and σ becomes infinite. The spatial volume V is zero and expansion scalar θ is infinite at t = t_3, which shows that the universe starts evolving with zero volume and infinite rate of expansion. The scale factors also vanish at $t = t_3$, and hence the model has a point type singularity at initial epoch. The ratio $\frac{\sigma}{\theta} \to 0$ as t → ∞, which shows that the model approaches isotropy for the large value of t.

CONCLUSION

We have studied a spatially homogeneous and isotropic Bianchi type-V space time with the variable Gravitational constant G(t). Einstein's field equations have been solved exactly by using a law of variation of scale factor with a variable Gravitational term. i.e. a Gravitational term that scales as $G(t) \propto \frac{1}{R^{-(1+3\omega)}}$ (where R is the scale factor).

Expressions for some important cosmological parameters have been obtained and physical behaviour of the models are discussed in detail, clearly the model represent shearing, non-rotating and expanding models with a big-bang start. The models have point type singularity at the initial epoch and approach isotropy at late times. It is interesting that the proposed variation law provides an alternative approach to obtain an exact solution of Einstein's field equations. For all cases gravitational constant G is increasing function of cosmic time t. The value of Deceleration Parameter (q = 0) shows that model expands with constant velocity.

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