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Research Article

MIXED CONVECTION FLOW OF A SECOND GRADE FLUID IN AN INCLINED POROUS CHANNEL

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ABSTRACT

A situation of laminar mixed convection flow of a visco elastic fluid of second grade through a porous medium in an inclined permeable channel has been examined in this paper. It is observed that as the Prandtl number increases, the velocity is found to be inversely proportional and also at times back flow is noticed. When the channel is held horizontal, the velocity profiles are parabolic. As the wall temperature increases, the velocity appears to be decreasing. Increase in the angle of inclination contributes to increase in velocity. However, nearly after 50% of the channel width, a reverse trend is observed. Such a pattern is found to be absent when the visco elasticity of the fluid is comparatively high. As the Darcy's parameter increases, more of back flow is noticed. In general, the velocity profiles are found to be parabolic in nature and the velocity is more when the channel is vertical. Further, the cross flow Reynold's number contributes to the velocity field significantly. It is seen that increase in cross flow Reynold's number causes the velocity field to decrease. The increase in visco elasticity for similar values of the Reynold's number contributes to more of a backward flow.

Key Words: *Visco Elasticity, Cross Flow Reynolds Number, Darcy Number, Second Grade Fluid*

INTRODUCTION

There is a fast growing belief that the many provocative experimental phenomena and dilemmas now have become a realistic possibility of being explained theoretically by Non-Newtonian Fluid Mechanics. An attempt is made in this paper to illustrate such an optimistic thought in advocating visco elastic effect that occurs in several industrial and biological applications. In recent years, considerable attention has been devoted to the study of boundary layer flow behaviour and heat transfer characteristics of a Newtonian fluid past a vertical plate embedded in a fluid saturated porous medium because of its extensive applications in engineering processes, especially in the enhanced recovery of petroleum resources and packed bed reactors. Considerable amount of interest had also been devoted to the study of transport properties in porous media subject to heat transfer which are subsequently characterized by highly non - linear coupled partial differential equations. In view of diversified applications in the fields of Physics, Chemistry and Chemical Technology and in situations demanding efficient transfer of mass over inclined beds, the viscous drainage in a channel of course horizontal/ vertical has been the subject of considerable interest to both theoretical and experimental investigators during the last several years.

In many chemical processing industries generally slurry adheres to the reactor vessels and gets consolidated. As a result of this, the chemical compounds within the reactor vessel percolates through the boundaries causing loss of production and then consuming more reaction time. The slurry thus formed inside the reactor vessel often acts as a porous boundary for the next cycle of chemical processing. Heat transfer in porous medium has several applications in the situations viz: nuclear waste disposal, geothermal energy extraction, fossil fuels detection, regenerator bed etc. Understanding the development of hydrodynamic and thermal boundary layer along with the heat transfer characteristics is the basic requirement to further investigate the problem extensively and more exhaustively.

It is known that liquids respond like elastic solids to impulses, which are very rapid compared to the time it takes for the molecular order associated with short range of intra molecular forces in the liquid enabling them to relax. After this, all liquids behave like viscous fluids with signals propagating by diffusion rather than by waves. For liquids with small molecules this time of relaxation is estimated around 10^{-13} or 10^{-10} seconds depending on the fluids. However, there exist few liquids which are known to have much longer times of relaxation. Polymers mixed in Newtonian solvents and in some cases polymers melted like molten plastics possessing high viscosity silicone oils are few such examples. These fluids are known as visco elastic fluids. The longest times of relaxation for these fluids are more of practical interest.

Such fluids have become important industrially. Specifically, in polymer processing applications as well as in chemical industry, one deals with flow of visco elastic fluids. A classical example of such a liquid is poly iso - butylene. With the development of general constitutive equations for visco elastic fluids, it has been a point of great concern for Non-Newtonian fluids. All such proposed constitutive equations should in principle lead to the definition of flow properties that need to be measured to define the rheology and also to the development of the equivalent Navier Stokes equations for the solution of all possible boundary value along with initial value problems that arises in several situations. The process is completed by solution of the appropriate equations, where the methods of computational fluid mechanics are required as a last resort. However, some of the analytical methods for complex flows of visco elastic fluids generally predict the nature of flow field and gives rise to more or less accurate solution though not a perfect solution. In all such situations, the methodology that is applied must be evolved and considered appropriately. It is pertinent to be quite specific about the experimental conditions applicable to the relevant phenomena. Generally, the flows are invariably complex and the experimental dilemmas clearly refer to complex flows, where the flow domain sometimes often involves abrupt changes in geometry, and where the flow strength is high enough to permit a terminology which majors on high Weissenburg and Deborah numbers. This is of course reasonable and but definitely not unrealistic, and it nevertheless need to be stated.

Therefore, now the question that often arises is to address the situation to know how elastic liquids behave in complex flows. It is immediately apparent that the answer must involve a consideration of how the same liquids behave in simple flows, so that obtaining rheometrical data on the test liquids is an essential part of the exercise. Such data, when available, serve more than one useful purpose. They certainly provide a foundation set of data, which must be accommodated in the associated mathematical model for the test liquids. That is to define a perfect constitutive equation, which is an essential ingredient in any theoretical resolution of the experimental dilemmas, and has to be consistent with the physical situation. Indeed, if the model cannot simulate behavior in simple flows, the chances of exploring new dimensions will be an all time open problem for analysis.

The model that has been considered here in this paper is of Second order fluid whose constitutive relation has been proposed by Noll. The relation involves visco elasticity and also covers the concept of cross viscosity. Flow through porous media has been the subject of considerable research activity in recent years because of its diversified applications notably in the flow of oil through porous rock, the extraction of geothermal energy from the deep interior of the earth to the shallow layers, the evaluation of the capability of heat removal from particulate nuclear fuel debris that may result from a hypothetical accident in a nuclear reactor, the filtration of solids from liquids, flow of liquids through ion-exchange beds, drug permeation through human skin, chemical reactor for economical separation or purification of mixtures and so on.

Flow in a porous medium can be considered as an ordered flow in a disordered geometry. The transport process of fluid through a porous medium involves two substances, the fluid and the porous matrix, and therefore it will be characterized by specific properties of these two substances. A porous medium usually consists of a large number of interconnected pores each of which is saturated with the fluid. The exact form of the structure however, is highly complicated and differs from one medium to other medium. A porous medium may be either an aggregate of a large number of particles such as sand or gravel or solid containing many capillaries as seen in a porous rock (laterite stone). When the fluid percolates through a porous material, because of the complexity of microscopic flow in the pores, the actual path of an individual fluid particle cannot be followed analytically. In all such cases, one has to consider the gross effect of the phenomena represented by a macroscopic view applied to the masses of fluid, large compared to the dimensions of the pore structure of the medium. The process can be described in terms of equilibrium of forces. The driving force necessary to move a specific volume of fluid at a certain speed through a porous medium is in equilibrium with the resistive force generated by internal friction between the fluid and the pore structure. Such a resistive force is characterized by Darcy's (1856) semi - empirical law. The simplest model for flow through a porous medium is the one dimensional model derived by Darcy. From such empirical evidence, Darcy's law indicates that, for an incompressible fluid flowing through a channel filled with a fixed uniform and isotropic porous matrix, the flow speed varies linearly with longitudinal pressure variation. Several investigators have considered the non-Darcian model in the recent past to study the convection and heat transfer rates on bodies embedded in a porous medium for Newtonian fluids.

Although this problem is important in polymer processing applications. Benenati and Brosilow (1962) have shown that the permeability of a porous medium varies due to the variation of porosity from the wall to the interior of the porous medium. The problem of the exact solutions of two dimensional flows of a second order incompressible fluid has been examined by Pattabhi Ramacharyulu (1964) by considering rigid boundaries while Kaloni (1966) examined the fluctuating flow of a visco elastic fluid past an infinite porous plate subject to uniform suction. Thereafter, Merkin (1969) investigated the mixed convection boundary layer flow on a semi-infinite vertical flat plate when the buoyancy forces aid and oppose the development of the boundary layer. In this study two series solutions were obtained, one of which is valid near the leading edge and the other is valid asymptotically. In the regions where the series solutions are not valid, numerical solutions were obtained. Lloyd and Sparrow (1970), Oosthuizen and Hart (1973) and Wilks (1973) have carried out a numerical study of the combined forced and free convection flow over a vertical plate. Later, a linear analysis of the compressible boundary layer flow over a wall was presented by Lekoudis *et al.,* (1976) while, Shankar and Sinha (1976) studied the problem of Rayleigh for a wavy wall. Subsequently, Lessen and Gangwani (1976) examined the effect of small amplitude wall waviness on the stability of the laminar boundary layer. The problem of free convection heat transfer from a vertical plate embedded in a fluid saturated porous medium is studied by Cheng and Minkowycz (1977), who have obtained the similarity solutions for the problem considered.

Cheng (1978) has provided an extensive review of early works on free convection in porous media while, Mucoglu and Chen (1979) had examined the mixed convection flow over an inclined surface for both the assisting and the opposing buoyancy forces. The linearity between speed and pressure variation breaks down for large enough flow speed (a compilation of several experimental results) was presented by Mac Donald *et al.,* (1979). This was emphasized later by Joseph *et al* (1982) who stressed force modeled by the Frochheimer acts in a direction opposite to the velocity vector. Ramachandran *et al* (1987) have studied the mixed convection flow over vertical and inclined surfaces, theoretically as well as experimentally. Later, Knupp and Lage (1995) analyzed the theoretical generalization to the tensor permeability case (anisotropic medium) of the empirically obtained Frochheimer extended Darcy unidirectional flow model.

A numerical and experimental investigation of the effects of the presence of a solid boundary and initial forces on mass transfer in porous media was presented by Vafai and Tien (1982). Murthy *et al* (1977) had examined the dispersion effects due to a heated vertical flat plate. Subsequently, the problem of free convective heat transfer in a viscous incompressible fluid confined between vertical wavy wall and a flat wall was examined by Vajravelu and Shastri (1978). Merkin (1969) have obtained the similarity solution of the mixed convection flow over a vertical plate for the constant heat flux case. Tsuruno and Iguchi (1980) have investigated the effects of the surface mass transfer on the mixed convection flow on a permeable vertical surface. Plumb and Huenefeld (1981) have investigated non-Darcy natural convection from vertical isothermal surfaces in saturated porous media. Recently, Bejan and Poulikakos (1984) have used the model suggested by Forchheimer study vertical boundary layer natural convection in a porous medium. The steady flow of an incompressible second grade fluid past an infinite porous plate subject to suction or blowing was investigated by Rajagopal and Gupta (1984).

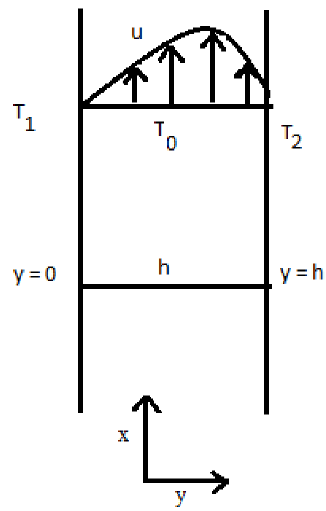
Chandrasekhar and Namboodiri (1985) have shown the effectiveness of variable permeability of the porous medium on velocity distribution and heat transfer. Hong *et al* (1987) have studied analytically the non-Darcian effects on a vertical plate natural convection in porous media. They used a combination of Rayleigh and Darcy numbers to describe the inertia and boundary terms and obtained similar solutions. They found that these effects decrease the velocity and reduce the heat transfer rate. Lai and Kulacki (1987) have used both Darcy and non- Darcy models (inertia effect only) to study mixed convection from horizontal and vertical surfaces embedded in saturated porous media. Nakayama and Koyama (1987) have obtained the similarity solution for the problem of free convection in the boundary layer adjacent to a vertical plate immersed in a thermally stratified porous medium. Kumari *et al*. (1990) have investigated the non-Darcian effects on forced convection heat transfer over a flat plate in a highly porous medium. Wickern (1991) has examined the influence of the inclination angle of the plate and the Prandtl number on the mixed convection flow over an inclined plate. Thereafter, Das and Ahmed (1992) had studied the effects of thermal dispersion and dissipation effects on non – Darcy mixed convection problems and established the trend of heat transfer rate convection from a vertical plate in porous medium and investigated the flow and temperature fields.

Hsieh *et al.*, (1993) have obtained a non-similar solution for combined convection from vertical plates in porous media with variable surface temperatures or heat flux. Hung and Chen (1997) have studied non-Darcy free convection in a thermally stratified fluid saturated porous medium along a vertical plate with variable heat flux. It follows that, in multidimensional flow, the momentum equations for each velocity component derived by using the Frochheimer extended Darcy equation is at least speculative while Patidar and Purohit (1998) studied the free convective flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls. Later, Kuznetsov (2000) investigated the effect of transverse thermal dispersion on forced convection in porous media and identified the situations favorable to heat transfer under dispersion effects. Thereafter, Mohammadien and El-Amin (2000) studied the dispersion and radiation effects in fluid saturated porous medium on heat transfer rate for both Darcy and non-Darcy medium. An explicit analytical technique namely homotopy analysis to solve the non - Darcy natural convection over a horizontal plate with surface mass flux and thermal dispersion was studied by Wang *et al* (2003). Subsequently, Taneja and Jain (2004) had examined the problem of MHD flow with slip effects and temperature dependent heat source in a viscous incompressible fluid confined between a long vertical wall and a parallel flat plate. Recently, Ramana Murthy and Kulkarni (2005) examined the problem of elastic - viscous fluid of second order type by causing disturbances in the liquid which was initially at rest and the bounding surface was subjected to sinusoidal oscillations.

In this paper an attempt has been made to examine the momentum and heat transfer in the steady flow of a visco elastic fluid in a channel which is being either held horizontally or vertically. Flow of visco elastic fluid over vertical surfaces occurs in many industrial and technical applications which include nuclear reactors cooled during emergency shutdown, electronic devices cooled by fans, solar central receivers exposed to wind currents, and heat exchangers placed in a low velocity environment. In the study of fluid flow over heated surfaces, the buoyancy forces are generally neglected when the flow is horizontal. However, for vertical or inclined surfaces, the buoyancy forces exert strong influence on the flow field. Hence, it is not possible to neglect the effect of buoyancy forces for vertical or inclined surfaces. Also, these equations were solved by employing the local similarity and local non similarity methods. The aim of this investigation is to consider the effects of heating or cooling of certain portions of the surface on the steady laminar mixed convection flow over a permeable vertical plate. The magnetic field is applied normal to the surface.

Nonetheless, the inertia effects become all the more so important that in a sparsely packed porous medium and hence their effect on free convection problems needs to be investigated. The aim of the present investigation is, therefore, to study systematically the effect of inertial terms on combined free

and forced convective heat transfer past a semi-infinite vertical plate embedded in a saturated porous medium with variable permeability, porosity and thermal conductivity. The results obtained under limiting conditions agree well with the existing ones and thus verify the accuracy of the method used.
In all the above investigations, the fluid under consideration was viscous incompressible fluid and one of the bounding surfaces had a wavy character. The local volume averaging technique has been used to establish the governing equations. The numerical solution of the governing equations is used to investigate the mass concentration field inside a porous media close to an impermeable boundary. In conjunction with the numerical solution, a transient mass transfer experiment has been conducted to demonstrate the boundary and inertia effects on mass transfer. In all above analysis, such a concept was accomplished by measuring the time and space averaged mass flux through a porous medium. The results clearly indicate the presence of these effects on mass transfer through porous media.
We consider the laminar mixed convection flow of a visco elastic fluid through a porous medium in an inclined permeable channel. The plates are separated by a width h, as shown below.



Geometry of the flow field

It is assumed that the rate of injection at one wall is equal to the rate of suction at the other wall. A rectangular coordinate system (x, y) is chosen such that the x- axis is parallel to the gravitational acceleration vector g, but with opposite direction and the y- axis is transverse to the channel walls. The left wall (i.e. at $y = 0$) is maintained at constant temperature T_1 and the right wall (i.e. at $y = h$) is maintained at constant temperature T_2, where $T_1 > T_2$. The flow is assumed to be laminar, steady and is fully developed, i.e. the transverse velocity is zero. Then, the continuity equation drops to $\frac{\partial u}{\partial x} = 0$.
The fluid under consideration is assumed to be of Rivlin - Ericksen type whose constitutive equation is proposed as

$$S_{ij} = -P\delta_{ij} + \phi_1 E_{ij}^{(1)} + \phi_2 E_{ij}^{(2)} + \phi_3 E_{ij}^{(1)^2} \quad (1)$$

where
$$E_{ij}^{(1)} = U_{i,j} + U_{j,i} \quad (2)$$

and
$$E_{ij}^{(2)} = A_{i,j} + A_{j,i} + 2U_{m,i}U_{m,j} \quad (3)$$

In the above equations, S_{ij} is the stress-tensor, U_i , A_i are the components of velocity and acceleration in the direction of the i th co-ordinate X_i , P is indeterminate hydrostatic pressure and the coefficients ϕ_1, ϕ_2 and ϕ_3 are material constants.

The constitutive relation for general Rivlin-Ericksen fluids also reduces to equation (1) when the squares and higher orders of $E^{(2)}$ are neglected, the coefficients being constants. Also the non-Newtonian models considered by Reiner could be obtained from equation (1) when $\phi_2 = 0$, naming ϕ_3 as the coefficient of cross viscosity. With reference to the Rivlin – Ericksen fluids ϕ_2 may be called as the coefficient of viscosity. It has been reported that a solution of poly-iso-butylene in 4% of cetane behaves as a second order fluid and Markovitz determined the constants ϕ_1, ϕ_2 and ϕ_3.

The visco elastic fluids when modeled by Rivlin - Ericksen constitutive equation are termed as second grade fluids. It is assumed that:

$$\phi_1 \geq 0 \ , \phi_2 > 0 \text{ and } \phi_2 + \phi_3 = 0 \tag{4}$$

The basic equations of momentum and energy governing such a flow, subject to the Boussinesq approximation, are

$$\rho v_0 \frac{du}{dy} = -\frac{dp}{dx} + \phi_1 \frac{d^2u}{dy^2} + \phi_2 v_0 \frac{d^3u}{dy^3} - \frac{\phi_1}{k} u + \rho g \lambda (T - T_0) + g \sin\alpha \tag{5}$$

$$v_0 \frac{dT}{dy} = \psi \frac{d^2T}{dy^2} \tag{6}$$

where p is the pressure, ρ is the density, ϕ_1 is the dynamic viscosity of the fluid, g is acceleration due to gravity, λ coefficient of thermal expansion, ϕ_2 is the visco elastic parameter, k is the permeability of the porous medium and v_0 is the transpiration cross flow velocity, ψ is the thermal conductivity of the fluid medium. Further, here $\frac{dp}{dx}$ is a constant.

The boundary conditions are given by $u(0) = u(h) = 0$, $T(0) = T_1$ and $T(h) = T_2$ (7)

The constitutive relation that has been proposed for the fluid under consideration needs to be solved in conjunction with the stress equations of motion and the equation of continuity and then to predict and explain the experimental phenomena and dilemmas. Analytic solutions are out of the question so far as complex flows are concerned and computational Rheology is now established, if fairly recent science is made more analytical which seeks theoretical answers to provocative experiments and phenomena. Clearly, the choice of constitutive equation is central to the whole operation and this choice is far from trivial or obvious. Indeed, a constitutive model which satisfies the dual constraints of tractability and quantitative (or even semi quantitative) prediction may not exist. However, it should not and does not prevent a search for such a missing link. But it is wise to be aware of the possibility of disappointment.

Introducing the following non -dimensional variables $\bar{y} = \frac{y}{h}$, $\bar{u} = \frac{u}{h^2}$ and $\theta = \frac{T_1 - T_0}{T_2 - T_0}$ into the equations (5) and (6), the governing equations for momentum and energy are transformed into:

$$\beta R \frac{d^3u}{dy^3} + \frac{d^2u}{dy^2} - R\frac{du}{dy} - \frac{1}{Da} u + \frac{Gr}{\mathrm{Re}} \theta + A + G Sin\alpha = 0 \tag{8}$$

$$\frac{d^2\theta}{dy^2} - R\Pr\frac{d\theta}{dy} = 0 \tag{9}$$

where $\beta = \frac{\phi_2}{\rho h^2}$ is the visco elastic parameter, $R = \frac{\rho v_0 h}{\phi_1}$ is the cross flow Reynolds number, $Gr = \frac{g\lambda(T_2 - T_1)h^3}{\nu^2}$ is the Grashoff number, $\mathrm{Re} = \frac{\rho u_0 h}{\mu}$ is the Reynolds number, $\Pr = \frac{\nu}{\psi}$ is the Prandtl number, $r_T = \frac{T_1 - T_0}{T_2 - T_0}$ is the wall temperature parameter and $A = -(\frac{dp}{dx})\frac{u_0\nu}{h^2}$ is the constant pressure gradient, $\nu = \frac{\phi_1}{\rho}$ is the momentum diffusivity.

The corresponding dimensionless boundary conditions are given by

$$u(0) = u(1) = 0\,, \theta(0) = r_T \text{ and } \theta(1) = 1 \tag{10}$$

SOLUTION

The first – order perturbation solution of the BVP (4) – (6) for small values of β are consider due to the reason that, the constitute equation (1) has been derived up to only the first – order of smallness of β, therefore, the perturbation solution obtained by retaining the terms up to the same order of smallness of β must be quite logical and reasonable. Therefore, it is reasonable to assume that:

$$u = u_0 + \beta u_1 \tag{11}$$

and

$$\theta = \theta_0 + \beta\theta_1 \tag{12}$$

Substituting equations (11) and (12) into equations (8) and (9) and boundary conditions (10) and then equating the like powers of β, the following set of equations are obtained:

Zeroth-order system (β^0)

$$\frac{d^2u_0}{dy^2} - R\frac{du_0}{dy} - \frac{1}{Da}u_0 = -\frac{Gr}{\mathrm{Re}}\theta_0 - A - G\sin\alpha \tag{13}$$

$$\frac{d^2\theta_0}{dy^2} - R\Pr\frac{d\theta_0}{dy} = 0 \tag{14}$$

together with boundary conditions $u_0(0) = u_0(1) = 0\ ,\theta_0(0) = r_T$ and $\theta_0(1) = 1$ (15)

First-order system (β^1)

$$\frac{d^2u_1}{dy^2} - R\frac{du_1}{dy} - \frac{1}{Da}u_1 = -R\frac{d^3u_0}{dy^3} - \frac{Gr}{\mathrm{Re}}\theta_1 \tag{16}$$

$$\frac{d^2\theta_1}{dy^2} - R\Pr\frac{d\theta_1}{dy} = 0 \tag{17}$$

together with boundary conditions $u_1(0) = u_1(1) = 0$ and $\theta_1(0) = \theta_1(1) = 0$ (18)

Zeroth-order solution (or Solution for a Newtonian fluid)

Solving equations (13) and (14) using the boundary conditions (15), we get

$$\theta_0 = \frac{(1 - r_T e^{R\Pr}) + (r_T - 1)e^{R\Pr y}}{(1 - e^{R\Pr})} \tag{19}$$

$$u_0 = c_1 e^{ay} + c_2 e^{by} + \frac{Gr}{\mathrm{Re}}(f_1 - f_2 e^{R\Pr y}) + ADa + Da\,GSin\alpha \tag{20}$$

where

$$a = \frac{R + \sqrt{R^2 + 4/Da}}{2}, b = \frac{R - \sqrt{R^2 + 4/Da}}{2}, \ f_1 = \frac{(1 - r_T e^{R\Pr})Da}{(1 - e^{R\Pr})},$$

$$f_2 = \frac{(r_T - 1)}{(1 - e^{R\Pr})(R^2 \Pr^2 - R^2 \Pr - 1/Da)}, \ f_3 = \frac{Gr}{\mathrm{Re}}(f_1 - f_2) + ADa + DaG\sin\alpha,$$

$$f_4 = \frac{Gr}{\mathrm{Re}}(f_1 - f_2 e^{R\Pr}) + ADa + DaG\sin\alpha, \ c_1 = \frac{f_4 - f_3 e^b}{e^b - e^a}, c_2 = \frac{f_3 e^a - f_4}{e^b - e^a}. \tag{21}$$

First-order solution (or Solution for a second-grade fluid)

Solving equation (17) with corresponding boundary conditions, we obtain $\theta_1 = 0$ (22)

Substituting the equations (20) and (22) into the Eq. (16) and then solving the resulting equation with the corresponding conditions, we get

$$u_1 = c_3 e^{ay} + c_4 e^{by} - f_6 y e^{ay} - f_7 y e^{by} + f_5 e^{R\Pr y} \tag{23}$$

where $$f_5 = \frac{Gr}{\mathrm{Re}} \frac{f_2 R^4 \Pr^3}{(R^2 \Pr^2 - R^2 \Pr - 1/Da)}, \ f_6 = \frac{Rc_1 a^3}{2a - R} \ f_7 = \frac{Rc_2 b^3}{2b - R},$$

$$f_8 = f_5 e^{R\Pr} - f_6 e^a - f_7 e^b, \quad c_3 = \frac{f_8 - f_5 e^b}{e^b - e^a}, \ c_4 = \frac{f_5 e^a - f_8}{e^b - e^a}. \tag{24}$$

The complete solution is given by $u = u_0 + \beta u_1$ and $\theta = \theta_0 + \beta\theta_1$ (25)

It can be verified that, when $\beta = 0$, $R = 0$ and $Da \to \infty$ our results reduces to those given by Aung and Worku (1986).

RESULTS AND CONCLUSIONS

1. The influence of Prandtl number (in case of an inclined channel) on the velocity profiles has been illustrated in fig. 1, fig. 2, fig. 3 and fig. 4. It is observed that as the Prandtl number increases, the velocity decreases and also at times back flow is seen. Increase in the Prandtl number appears to influence the intra molecular forces to be more coherent and strong so as to prevent the forward motion. In a situation where the visco elasticity of the fluid and all other parameters are similar as the angle of inclination increases, the fluid velocity decreases. For relatively smaller values of the visco elasticity completely back flow is noticed and as the visco elasticity increases, the motion appears to be in the forward direction.

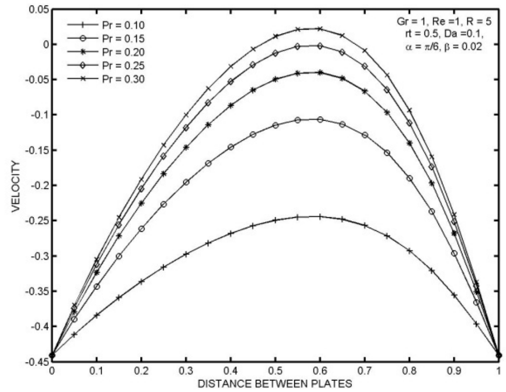


Fig. 1: Velocity profiles for different Prandtl no. when β = 0.02 and α = π/6.

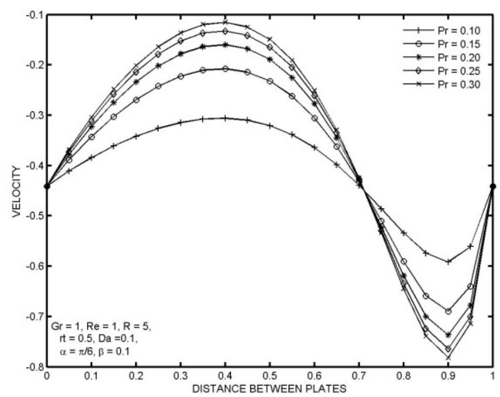


Fig. 2: Influence of Prandtl no. on velocity profiles when β = 0.1 and α = π/6.

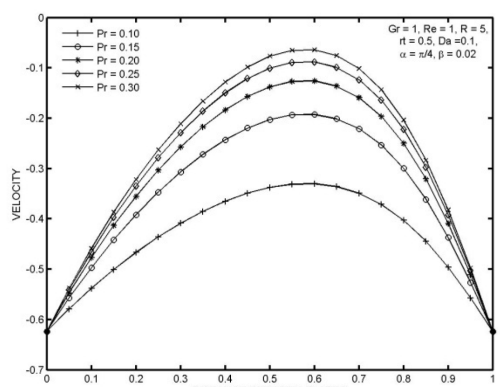


Fig. 3: Effect of Prandtl no. on velocity profiles when β = 0.02 and α = π/4.

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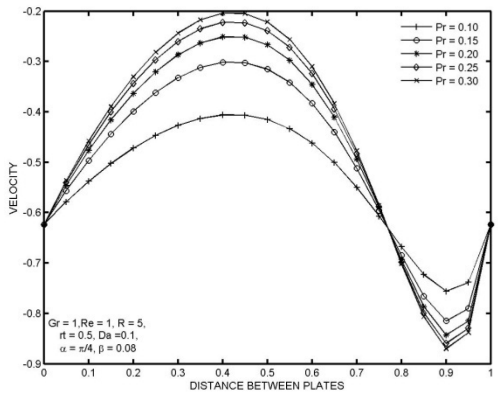

Fig. 4: Characterisation of velocity profiles under the influence of Prandtl no. for β = 0.08 and α = π/4.

2. The effect of the Darcy's number on the velocity field is shown in fig. 5, fig.6 and fig. 7. In each of these cases, it is seen that as the Darcy's parameter (the pore size of the medium) increases, more of backward flow is seen. This is in anticipation of the realistic situation that as the pore size increases, the fluid almost percolates and the situation is more seen when the channel is inclined. From fig. 5 and fig. 6, it is seen that as the visco elasticity decreases, more of backward flow is noticed.

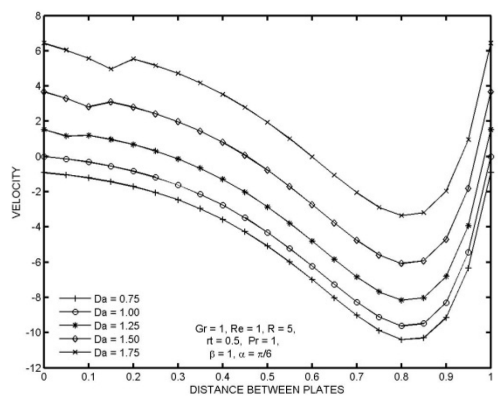

Fig. 5: Profiles for velocity under the influence of Darcy no. when β = 1 and α = π/6.

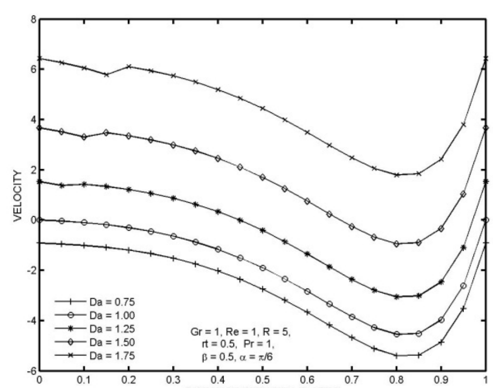

Fig. 6: Effect of Darcy no. when β = 0.5 and α = π/6 on velocity profiles.

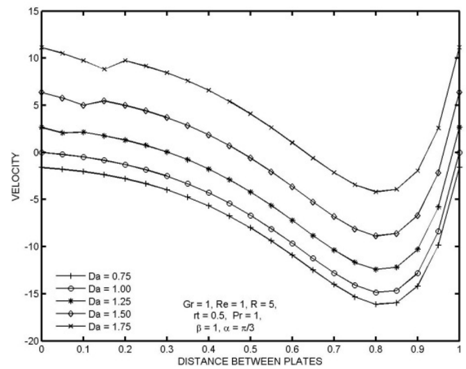

Fig. 7: Influence of Darcy no. on velocity profiles when β = 1 and α= π/3.

3. In a situation of a horizontal flow, the effect of wall temperature on the velocity field has been studied in fig. 8, fig. 9, fig. 10 and also when the channel is inclined in fig. 11. In a case where the channel is held horizontal, the velocity profiles are found to be parabolic and as the wall temperature increases, the velocity decreases. On comparing the fig. 8, fig. 9 and fig. 10 it is noticed, the visco elasticity of the fluid contributes significantly to the velocity. Increase in visco elasticity causes the fluid velocity to decrease. In a situation where all other parameters are held constant including the visco elasticity of the fluid, as the channel is more inclined the fluid velocity is more proportional.

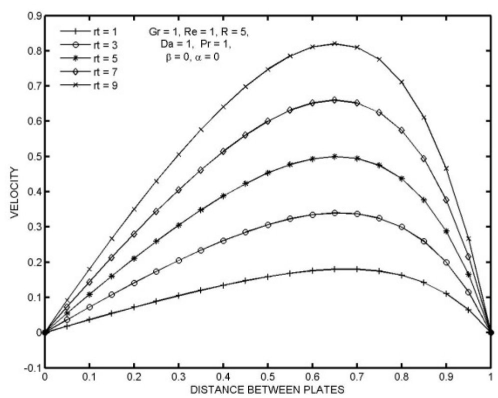

Fig. 8: Characterisation of velocity profiles under the influence of wall temperature when the channel is horizontal.

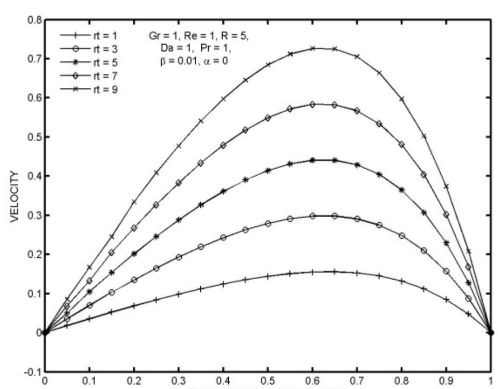

Fig. 9: Effect of wall temperature on velocity profiles when the channel is horizontal and visco elasticity is 0.01.

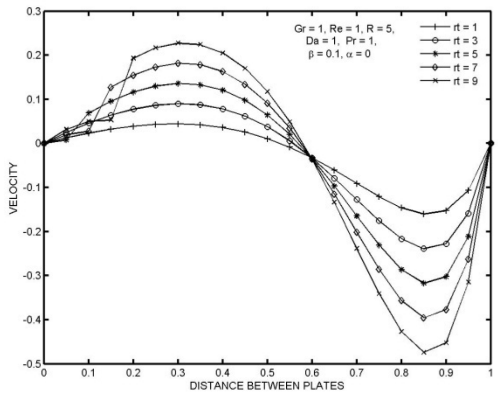

Fig. 10: Velocity profiles under the influence of room temperature when β = 0.1 and α= 0.

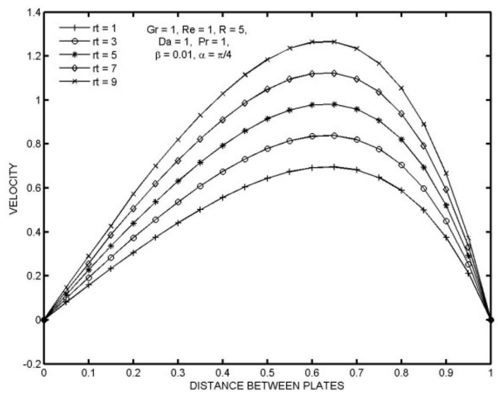

Fig. 11: Influence of room temperature on velocity profiles when β = 0.01 and α= π/4.

4. The effect of Grashof number on the velocity field has been illustrated in fig. 12, fig. 13, fig. 14 and fig. 15. A consolidated review of all such illustrations shows that as the visco elasticity increases, the fluid velocity decreases. Further, in each of these cases it is noticed that increase in Grashof number contributes to increase in the velocity.

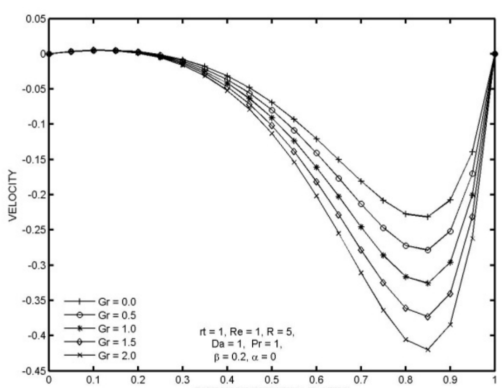

Fig. 12: Profiles for velocity for different Grashof numbers when β = 0.2 and α = 0.

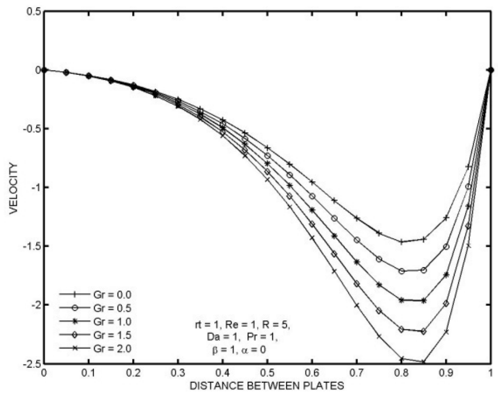

Fig. 13: Characterisation of the profiles of velocity under the influence of Grashof number with β = 1 and α = 0.

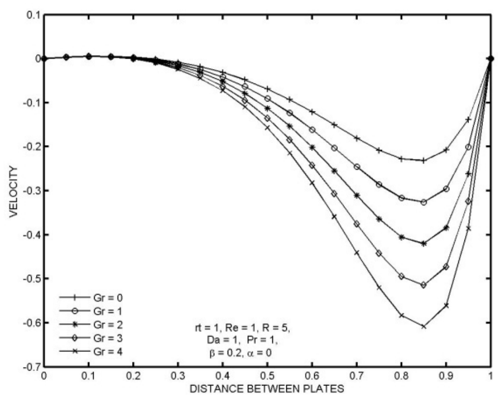

Fig. 14: Effect of Grashof number on velocity profiles when the channel is horizontal and β = 0.2.

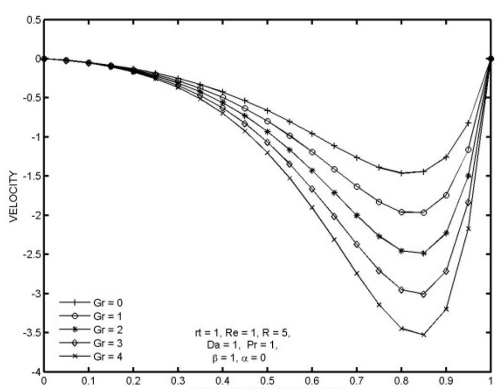

Fig. 15: Influence of Grashof number with β = 1 and α = 0 on velocity profiles.

5. Relatively for smaller values of visco elasticity and when the channel is held horizontal, the contribution of the Grashof number is shown in fig 16 and fig. 17. In both the illustrations, it is seen that as the visco elasticity increases, apart from the forward motion, a backward motion is also noticed. And a point of inflection is seen nearly at 70% of the channel width.

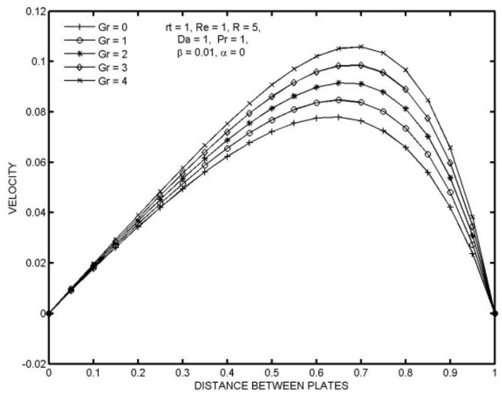

Fig. 16: Velocity profiles for different Grashof numbers under the influence of visco elasticity.

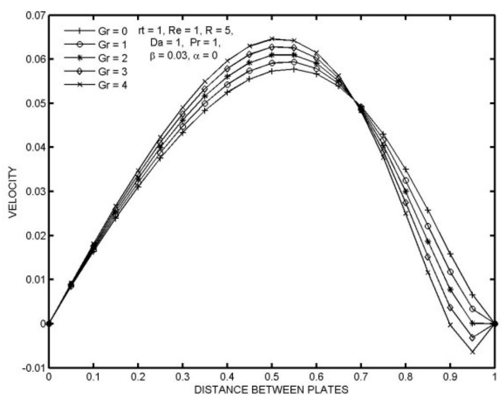

Fig. 17: Characterisation of the influence of Grashof number and β = 0.03 and α = 0 on velocity profiles.

6. The contribution of the inclination of the channel for different visco elastic parameters has been illustrated in fig. 18, fig. 19 and fig. 20. A general trend that can be seen from fig. 18, fig. 19 and fig. 20 is that the increase in the visco elasticity contributes to the more of the back flow at the other boundary of the channel. However, in each of these situations, increase in the angle of inclination contributes to the increase in the velocity. However, nearly after the 50% of the channel width, a reverse trend is observed which is found to be absent when the visco elasticity of the fluid is comparatively high.

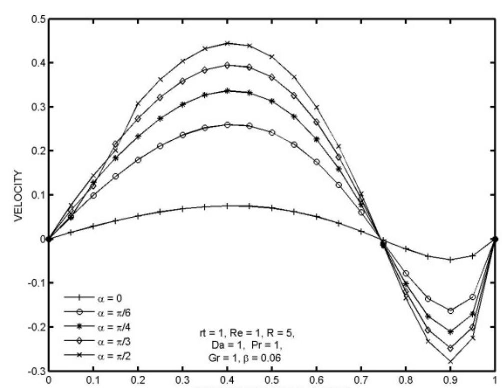

Fig. 18: Effect of angle of inclination on velocity profiles with β = 0.06.

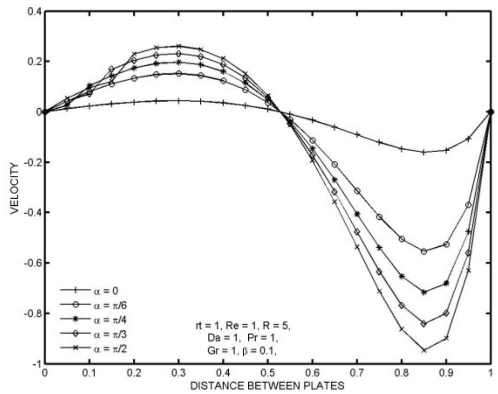

Fig. 19: Influence of angle of inclination with β = 0.1 on velocity profiles.

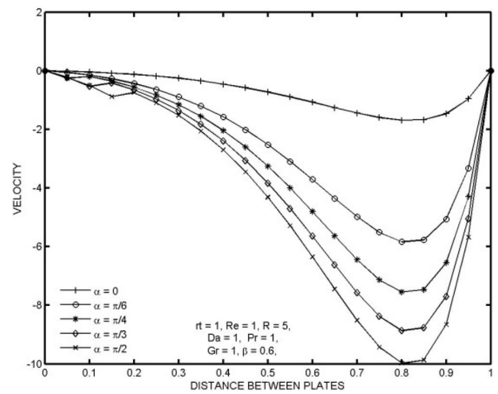

Fig. 20: Velocity profiles for different angles of inclination with the effect of visco elasticity.

7. The influence of cross flow Reynold's number in a situation where the channel is held perfectly vertical fig. 21 and horizontal fig. 22 has been depicted. However might be the situation, in general the velocity profiles are found to be parabolic and the velocity is more in a situation when the channel is held vertical. In both the cases, increase in the cross flow Reynold's number contributes to the decrease in the velocity field.

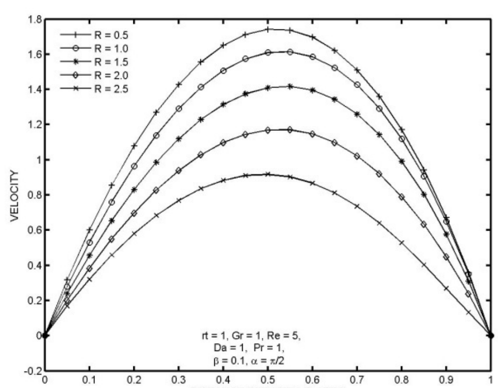

Fig. 21: Characterisation of cross flow Reynold's no. on velocity profiles when β = 0.1 and α = π.

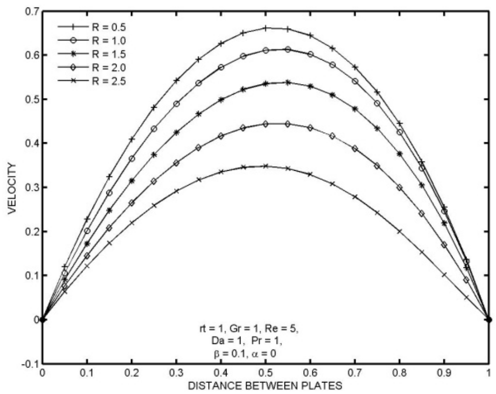

Fig. 22: Velocity profiles for cross flow Reynold's no. when the channel is held horizontal and β = 0.1.

8. The effect of cross flow Reynold's number with respect to the angle of inclination has been depicted in fig. 23 and fig. 24. In general it is observed that, as the channel is more inclined, the velocity appears to be proportional. Further, the influence of the cross flow Reynold's number does not qualitatively alter the characteristic features as was seen above.

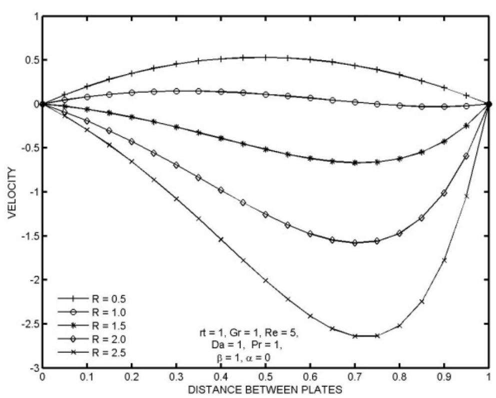

Fig. 23: Effect of cross flow Reynold's no. on velocity profiles when β = 1 and α = 0.

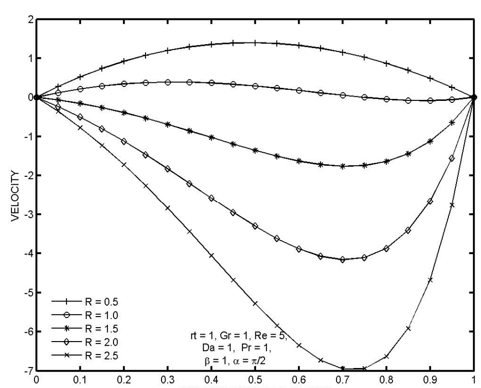

Fig. 24: Influence of cross flow Reynold's no. on velocity profiles when β = 1 and α = π/2.

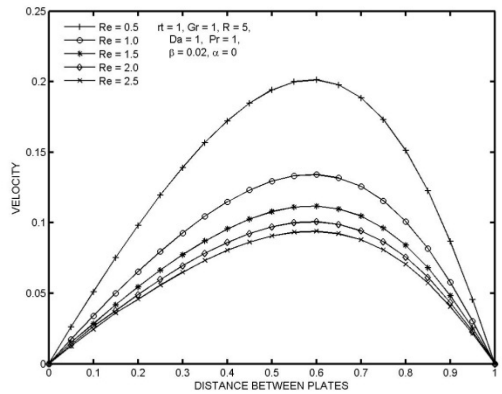

Fig. 25: Characterisation of Reynold's no. on velocity profiles when the channel is held horizontal and β = 0.02.

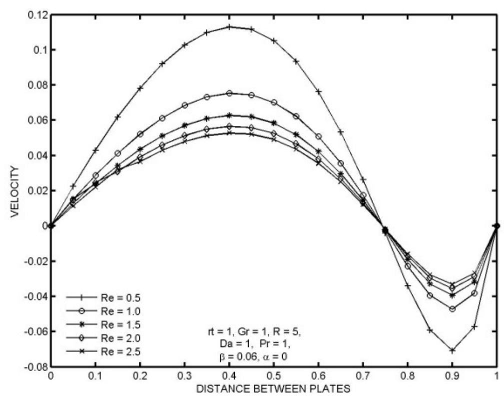

Fig. 26: Velocity profiles for Reynold's number when β = 0.06 and α = 0.

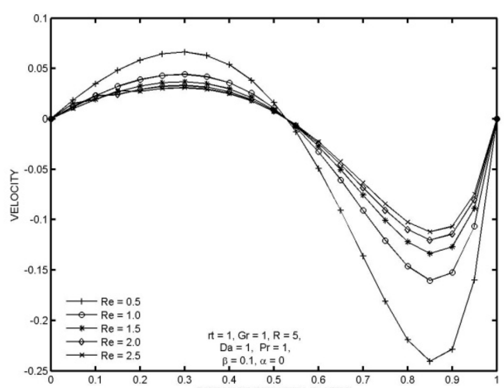

Fig. 27: Effect of Reynold's no. on flow velocity in a horizontal channel with visco elasticity 0.1.

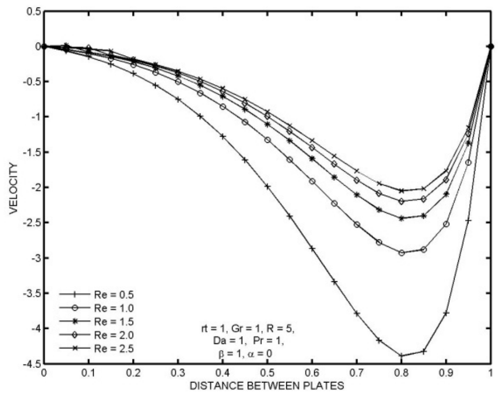

Fig. 28: Velocity profiles under the influence of different Reynold's number, and visco elasticity.

9. From fig. 25, fig. 26, fig. 27 and fig. 28 illustrates the effect of Reynold's number when the channel is held perfectly horizontal. A consolidated review shows that, for relatively smaller values of visco elasticity, the velocity profiles are perfectly parabolic in nature while a point of inflection has been noticed nearly at 70% of the channel width and a backward flow is observed. The increase in the visco elasticity, for similar values of the Reynold's number contributes to more of a backward flow.

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