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Research Article

UNSTEADY MHD BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER A STRETCHING SURFACE

***Anuj Kumar Jhankal**
Department of Applied Mathematics, Birla Institute of Technology, Mesra, Ranchi, Extension Center-Jaipur, Jaipur-302 017 (INDIA)
**Author for Correspondence*

ABSTRACT
This paper considers the problem of unsteady boundary layer flow and heat transfer over a stretching surface in the presence of transverse magnetic field. The governing boundary layer equations for fluid flow and energy are reduced into ordinary differential equations by means of similarity transformations. Numerical solutions of the resulting similarity equations are obtained and the effects of various parameters are presented and discussed.

Key Words: *MHD Boundary Layer Flow, Heat Transfer, Stretching Surface, Numerical Study*

Nomenclature

A	Unsteadiness Parameter
B	Constant Applied Magnetic Field
M	Magnetic Parameter
c_p	Specific Heat of the Fluid
f	Dimensionless Stream Function
Pr	Prandtl Number
t	Time
T	Temperature of the Fluid
T_w	Temperature at the Wall
T_∞	Free Stream Temperature
u, v	Velocity Component of the Fluid along the x and y Directions, Respectively
x, y	Cartesian Coordinates along the Surface and Normal to it, respectively

Greek symbols

ρ	Density of the Fluid
μ	Viscosity of the Fluid
σ_e	Electrical Conductivity
η	Dimensionless Similarity Variable
κ	Thermal Conductivity
υ	Kinematic Viscosity
Ψ	Stream Function
θ	Dimensionless Temperature

Superscript

$'$	Derivative With Respect To η

Subscripts

w	Properties at the Plate
∞	Free Stream Condition

INTRODUCTION
The flow and heat transfer of an incompressible viscous fluid over a stretching sheet has wide important applications in several manufacturing process from industry such as the extrusion of polymers, the

cooling of metallic plates, the aerodynamic extrusion of plastic sheets, etc. The study of heat transfer and flow field is necessary for determining the quality of the final products of such processes. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products. In recent years, MHD flow problems have become more important industrially. Indeed, MHD laminar boundary layer behaviour over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. Crane (1970) studied the flow over a linearly stretching sheet in an ambient fluid and gave a similarity solution in closed analytical form for the steady two-dimensional problem. Gupta and Gupta (1977), Carragher and Crane (1982), Dutta *et al.*, (1985), Chiam (1994), Magyari and Keller (1999, 2000) and more recently Mahapatra and Gupta (2002, 2004) studied the heat transfer in the steady two-dimensional stagnation-point flow of a viscous, and incompressible Newtonian and viscoelastic fluids over a horizontal stretching sheet considering the case of constant surface temperature. As many natural phenomena and engineering problems are worth being subjected to MHD analysis, the effect of transverse magnetic field on the laminar flow over a stretching surface was studied by a number of researchers Jhankal and Kumar (2013), Pavlov (1974), Chakrabarthi and Gupta (1979), Chima (1993), Noor *et al.*, (2010) etc.

Motivated by works mentioned above and practical applications, the main concern of the present paper is to study the problem of unsteady boundary layer flow and heat transfer over a stretching surface in the presence of transverse magnetic field.

Formulation of the Problem

Let us consider two-dimensional unsteady boundary layer flow over a continuously stretching plate in an incompressible electrically conducting fluid, when t=0, the plate is impulsively stretched with the velocity U_w, where x-axis is along the sheet and y-axis perpendicular to it, the applied magnetic field B is transversely to x-axis. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. Under the usual boundary layer approximations, the governing equation of continuity, momentum and energy under the influence of externally imposed transverse magnetic field are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_e B^2}{\rho}u \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Along with the boundary conditions are:

$$y=0: u=U_w, v=0, T=T_w$$
$$y\to\infty: u\to 0, T\to T_\infty \tag{4}$$

Here, we assume that the stretching velocity $U_w(x,t)$ and the surface temperature $T_w(x,t)$ are of the form:

$$U_w(x,t)=\frac{ax}{1-ct},\ T_w(x,t)=T_w+\frac{bx}{1-ct} \tag{5}$$

Where a, b and c are constants with $a>0, b\geq 0$ and $c\geq 0$ (with $ct<1$).

The continuity equation (1) is satisfied by introducing a stream function Ψ such that $u=\frac{\partial\Psi}{\partial y}$ and $v=-\frac{\partial\Psi}{\partial x}$. (6)

The momentum and energy equations can be transformed into the corresponding ordinary nonlinear differential equations by using the following transformations:

$$\eta=\left(\frac{U_w}{\nu x}\right)^{1/2}y,\ f(\eta)=\frac{\Psi}{(U_w \nu x)^{1/2}},\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\ \text{and}\ B=\frac{B_0}{\sqrt{1-ct}} \tag{7}$$

Then, the transformed non-linear differential equations are:

$$f'''+ff''-f'^2-Mf'-A\left(f'+\frac{1}{2}\eta f''\right)=0 \tag{8}$$

$$\frac{1}{Pr}\theta'' + f\theta' - f'\theta - A\left(\theta + \frac{1}{2}\eta\theta'\right) = 0 \quad (9)$$

The transformed boundary conditions are:

$$\eta = 0: f = 0, f' = 1, \theta = 1$$
$$\eta \to \infty: f' = 0, \theta = 0. \quad (10)$$

Where prime denotes differentiation with respect to η , $M = \frac{\sigma_e B_0^2}{a\rho}$ is the magnetic parameter, $Pr = \frac{\mu c_p}{\kappa}$ is the Prandtl number, and $A = \frac{c}{a}$ is the unsteadiness parameter.

Numerical Solution and Discussion

The non-linear differential equations (8) and (9) subject to the boundary conditions (10) is solved numerically using Runge-Kutta-Fehlberg Forth-Fifth order method. To solve this equation we adopted symbolic algebra software Maple. Maple uses the well known Runge-Kutta-Feulberg Forth-Fifth (RKF45) order method to generate the numerical solution of boundary value problem.

It is shown in Figures 1 and 2 the velocity gradient at the surface increases with the increasing values of magnetic parameter M and unsteadiness parameter A, respectively when the other parameter is fixed. The plots of temperature profiles $\theta(\eta)$ against η are shown in Figures 3 for various values of magnetic parameter M when other parameters are fixed. It is observed that, temperature gradient at the surface decreases in small amount, with an increase values in magnetic parameter M.

Figure 4 is plotted for the We observe that the skin friction coefficient $|f''(0)|$ strongly depending on M and A, is found to increase with M or A also, it is observed that, because of velocity boundary layer is caused solely on the stretching plate, therefore we found the negative values of $f''(0)$.

Figure 5, which is a representation of the local dimensionless coefficient of heat transfer $-\theta'(0)$, knows as the Nusselt number for the different values of unsteadiness parameter A versus magnetic parameter M. It is noted that for increasing value of A, the Nusselt number increases but it decreases in small amount, with the increasing value of M. This is because, when A increases, the thermal diffusivity decreases and thus the heat is diffused away from the heated surface more slowly and in consequence increase the temperature gradient at the surface.

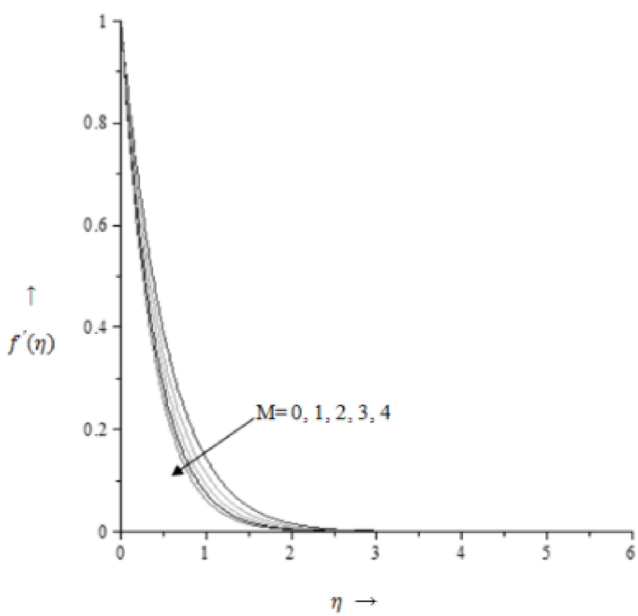

Figure 1: Velocity distribution for various values of M, when A=2.0.

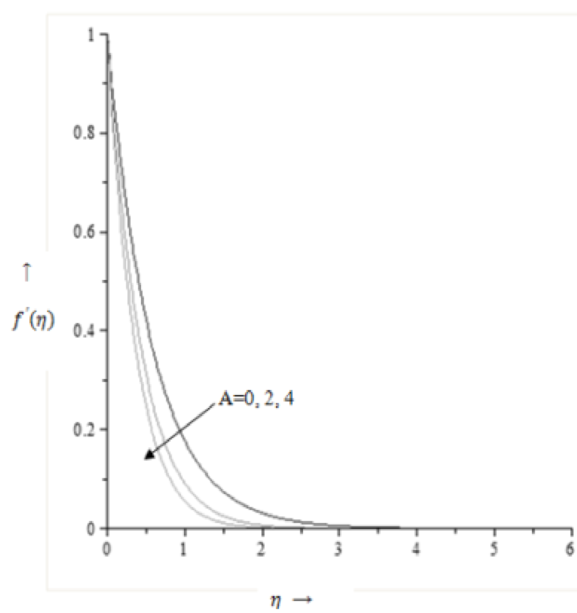

Figure 2: Velocity distribution for various values of A, when M=2.0.

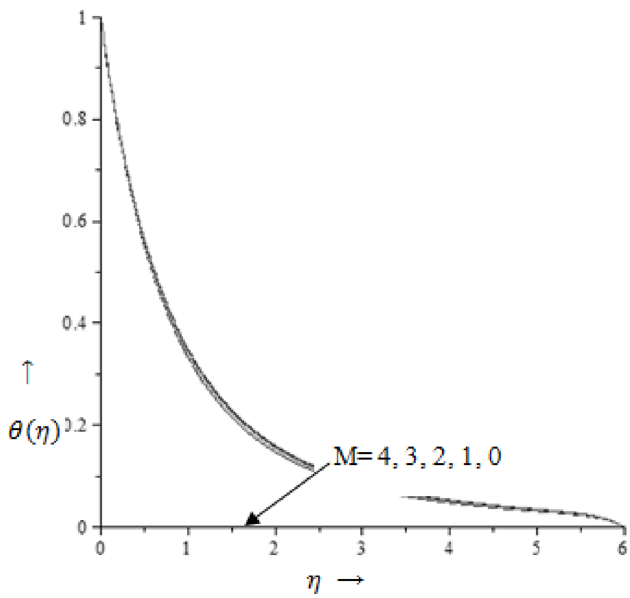

Figure 3: Temperature distribution for various values of M, when A=2.0 and Pr=0.71.

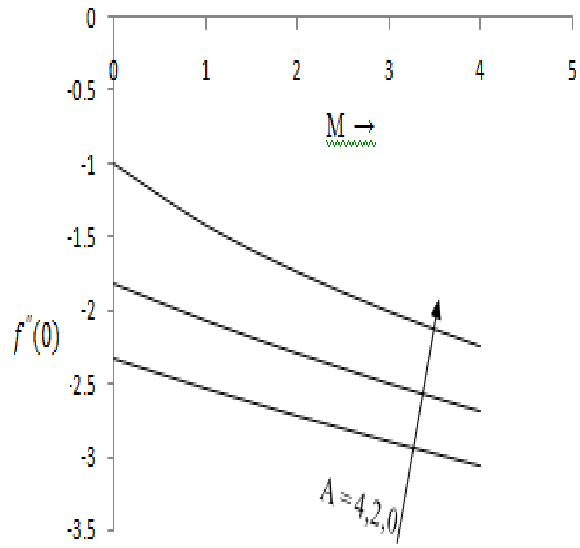

Figure 4: Skin friction coefficient $f''(0)$ for different values of unsteadiness parameter A versus magnetic parameter M.

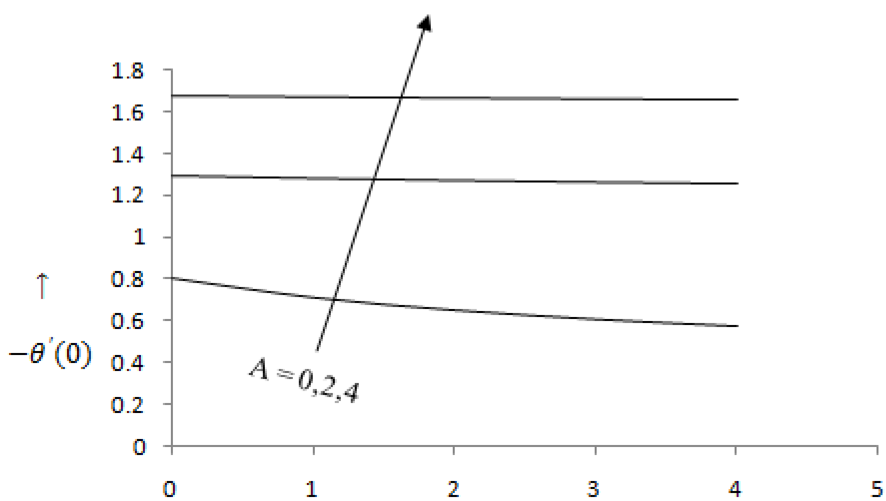

Figure 5: Nusselt number for the different values of unsteadiness parameter A versus magnetic parameter M.

CONCLUSION

A mathematical model has been presented for the unsteady boundary layer flow and heat transfer over a stretching surface in the presence of transverse magnetic field. From the study, following conclusions can be drawn:

1. It is observed that the velocity gradient at the surface increases with the increasing values of magnetic parameter M and unsteadiness parameter A.

2. The temperature gradient at the surface (in magnitude) decreases in small amount, with the increasing values of magnetic parameter M.

3. We observe that the skin friction coefficient $|f''(0)|$ strongly depending on M and A, is found to increase with M or A also, it is observed that, because of velocity boundary layer is caused solely on the stretching plate, therefore we found the negative values of $f''(0)$.

4. It is noted that for increasing value of A, the Nusselt number increases but it decreases in small amount, with the increasing value of M. This is because, when A increases, the thermal diffusivity decreases and thus the heat is diffused away from the heated surface more slowly and in consequence increase the temperature gradient at the surface.

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