International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2019 Vol. 9 (4) October-December, pp. 30-33/Mathur and Parihar
Research Article

MAGNETOHYDRODYAMIC FLOW AND IMPORTANCE OF DIMENSIONAL HOMOGENEITY WITH NON-DIMEMSIONAL PARAMETERS

[1]Pankaj Mathur and [2]P. R. Parihar*
[1]Department of Mathematics, Government College, Tonk (Raj.)
[2]Department of Mathematics, S.P.C. Government College, Ajmer (Raj)
**Author for Correspondence: prparihar@gmail.com*

ABSTRACT
In this paper we have discussed Magnetohydrodynamic Flow and the electrodynamic field equations. Law of dimensional homogeneity and non-dimensional parameters also presented to explain their importance in study of fluids and their flow.

Keywords: *Magnetohydrodynamic, homogeneity and non-dimensional parameters*

1.1 INTRODUCTION

The study Magnetohydrodynamic Flow deals with the flow of electromagnetic fluids in the presence of magnetic field. This is type of flows are modelled with the help of fundamental electrodynamic field equations. Law of dimensional homogeneity and non-dimensional parameters are of utmost importance in study of fluid flows the same is discussed and presented to explain their importance in such phenomenon.

1.2 MAGNETOHYDRODYNAMIC (MHD) FLOW

The flow of an electrically conducting fluid in the presence of magnetic field is defined as magnetofluiddynamic (MFD) flow. A particular case when fluid is taken to be electrically conducting incompressible then the flow is termed as magnetohydrodynamic (MHD) flow instead of MFD flow.
In MHD flow the equation of state reduces to

$$\rho = \text{constant.}$$

Moreover, the other fluid properties, such as viscosity μ, thermal conductivity k and electrical conductivity σ are also nearly constant in such a flow. The MFD or MHD flow can be defined completely by sixteen electromagnetic field equations along with six fluid dynamics field equations. The electromagnetic field equation are as follows:

(i) Charge conservation equation (one).
(ii) Maxwell's equation (six).
(iii) Constitutive equation (six).
(iv) Generalized Ohm's law (three).

(i) Charge conservation equation (Current continuity Equation)

Current continuity equation is defined as

$$\frac{\partial \rho_e}{\partial t} + div\,\vec{J} = 0 \qquad ...(1.2.1)$$

where $\vec{J}$ is current density and ρ_e is charge density. If ρ_e is independent of time then we have

$$div\,\vec{J} = 0$$

which implies $\vec{J}$ is solenoidal and all current in steady state must flow in closed circuits.

(ii) Maxwell's equations

$$Curl\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad ...(1.2.2)$$

$$Curl\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad ...(1.2.3)$$

$div\ \vec{D} = \rho_e$ (Charge continuity equations) ...(1.2.4)

$div\ \vec{B} = 0$ ...(1.2.5)

where $\vec{E}$ is electrostatic field, $\vec{B}$ is magnetic field, $\vec{D}$ is displacement vector, $\vec{J}$ and ρ_e are as defined above.

(iii) Constitutive equations

$\vec{D} = \in \vec{E}$ ...(1.2.6)

$\vec{B} = \mu_e \vec{H}$ ...(1.2.7)

where $\in$ is the electrical permittivity or dielectric constant of the medium and μ_e is the magnetic permeability of the medium (fluid). Ordinarily, we may assume that both $\in$ and μ_e are constant for given isotopic material.

(iv) Generalized Ohm's law

$\vec{J} = \sigma\left(\vec{E} + \vec{v} \times \vec{B}\right) + \rho_e \vec{v}$ ...(1.2.8)

where σ is the electrical conductivity of the medium.

1.3 LAW OF DIMENSIONAL HOMOGENEITY

This states that a physical phenomenon represented by an analytically derived equation should be valid for all system of units. If a group of quantities has a dimensional representation most simply of unity when multiplied together, the group is called a dimensionless group. Buckingham's π-theorem defines that how many dimensionless parameters can be formed from a group of variables known to be involved in a physical phenomenon. Buckingham's π-theorem states that the number of independent dimensionless groups that may be employed to describe a phenomenon known to involve n variable is equal to the number $n - r$, where r is the number of basic dimensions needed to express the variable dimensionally. In most fluid phenomenon, where heat transfer can be neglected, change in pressure p, length L, viscosity μ, surface tension T, velocity of sound c, acceleration of gravity g, mass density ρ, velocity v are the eight important variables. Since three basic dimensions are needed to describe the variables so by Buckingham's π-theorem there are 8−3=5 independent dimensionless groups. If at least all but one of the dimensionless groups are duplicated for geometrically similar flows, the flow will probably be dynamically similar. This fact introduces the possibility of testing a model of some proposed apparatus to study, less expensively, full-scale performance and possible design variations.

In the next section some non-dimensional parameters are presented.

1.4 NON-DIMENSIONAL PARAMETERS

(i) Reynolds number - Re

Ratio of the inertia force to the friction force, usually defined as

$Re = \frac{\rho V^2/L}{\mu V/L^2} = \frac{\rho VL}{\mu}$ ...(1.4.1)

is called Reynolds number and is the characteristic parameter of flow, which determine the nature of the fluid flow. Beyond the critical value of the Reynolds number, i.e. Re>2300, the fluid flow becomes turbulent flow.

(ii) Euler number - E

Ratio of the pressure force to the inertia force is called Euler number and define as

$E = \frac{\triangle p/L}{\rho V^2/L} = \frac{\triangle p}{\rho V^2}$ ...(1.4.2)

In practical testing work Euler number is used in terms of pressure coefficient. The pressure coefficient is equal to twice the Euler number.

(iii) Prandtl number - Pr

This dimensionless parameter is the ratio of the Kinematic viscosity υ, and thermal diffusivity α of the fluid and is represented mathematically by

$$\mathrm{Pr} = \frac{\upsilon}{\alpha} = \frac{\mu C_p}{\kappa} \quad ...(1.4.3)$$

where κ the thermal conductivity, and C_p the specific heat at constant pressure. It is the measure of relative importance of heat conduction and viscosity of the fluid. This number is like viscosity and thermal conduction is a material property and it thus varies from fluid to fluid, for air Pr=0.7 and for water (at 60^0 F) Pr = 7.0 etc. This determines how thick the boundary layers will be for a given external flow field.

(iv) Grashoff number - Gr

In free convection flow system the ratio of buoyancy boundary force to viscous force define a non dimensional parameter called Grashoff number, represented by

$$Gr = \frac{g\beta(T_\omega - T_\infty)\chi^2}{\upsilon^2} \quad ...(1.4.4)$$

where g is the gravitational acceleration, β the volumetric coefficient of thermal expansion, T_ω the temperature of the wall, T_∞ the free stream temperature and χ is the distance from the wall. This number characterises the free convection flow.

(v) Modified Grashoff number - Gc

Grashoff number for mass transfer through porous medium which is defined as

$$\mathrm{Gc} = \frac{g\beta^{/}(C_S^* - C_\infty)}{V^2 U_0} \quad ...(1.4.5)$$

where $\beta^{/}$ is the concentration coefficient of volumetric expression, C_S^* the concentration at the surface, C_∞ the concentration in free stream, V_0 and U_0 are characteristic velocities.

(vi) Schmidt number - Sc

This non-dimensional parameter is encounter repeatedly in the problem of diffusion in flow system, just as Prandtl number in the problem of heat transfer in flow system. Schmidt number is defined as

$$\mathrm{Sc} = \frac{\upsilon}{D} \quad ...(1.4.6)$$

Where, D is coefficient of diffusivity.

(vii) Local Skin-friction coefficient - C_f

The dimensionless parameters defined as shearing stress on the surface of a body due to fluid motion.

Local skin-friction coefficient C_f is defined as

$$C_f = \frac{\tau_\omega}{\frac{1}{2}\rho U^2} \quad ...(1.4.7)$$

where τ_ω is the local shearing stress on the surface of the body. This dimensionless parameter has properties analogous to Euler number.

(viii) Nusselt number - Nu

Nusselt number is defined as the non-dimensional rate of heat transfer i.e. quantity of heat exchanged between the body and the fluid. Using Fourier's law of heat conduction and Newton's cooling law, the rate of heat transfer in term of Nusselt number Nu is given by

$$Nu - \frac{\alpha(\chi)L}{\kappa} = -\frac{L}{(T_\omega - T_\infty)}\left(\frac{\partial T}{\partial Y}\right)Y = 0 \quad ...(1.4.8)$$

where negative sign shows the decrease in temperature. T_ω and T_∞ are temperature of wall and free stream. κ is thermal conductivity and Y is the direction of the normal to the surface of the wall. $\alpha(x)$ is coefficient of heat transfer and L is some characteristic length in the problem.

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