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Research Article

PARAMETER ESTIMATION OF AREA BIASED AILAMUJIA DISTRIBUTION

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ABSTRACT
Area biased Ailamujia distribution is considered. The classical maximum likelihood estimator has been obtained. Bayesian method of estimation is employed in order to estimate the scale parameter of area biased Ailamujia distribution by using quasi and gamma priors. In this paper, the Bayes estimators of the scale parameter have been obtained under squared error, precautionary and weighted loss functions.

Keywords: *Area biased Ailamujia distribution, Bayesian method, quasi and gamma priors, squared error, precautionary and weighted loss functions*

INTRODUCTION
Ailamujia distribution is proposed by Lv *et al.*, (2002). Uzma *et al.*, (2017) studied the weighted version Ailamujia distribution. Reshi *et al.*, (2019) studied the Bayesian estimation of parameter of Ailamujia distribution using Linex and entropy loss function. Ahmad *et al.*, (2017) studied the transmuted Ailamujia distribution. Area biased Ailamujia distribution is a newly proposed lifetime model formulated by Jayakumar and Elangovan (2019).The probability density function of area biased Ailamujia distribution is given by

$$f\left(x;\theta\right)=\frac{8}{3}x^3\theta^4e^{-2\theta x}\quad ; x\geq 0,\ \theta>0. \tag{1}$$

2. Classical Method of Estimation
In classical approach, mostly we use the method of maximum likelihood. The alternative approach is the Bayesian approach which was first discovered by Rev. Thomas Bayes. In this approach, parameters are treated as random variables and data is treated as fixed. Recently Bayesian approach to estimation has received great attention by most researchers.

Theorem 1. Let $x_1,x_2,..........,x_n$ be a random sample of size n having probability density function (1), then the maximum likelihood estimator of parameter θ is given by

$$\hat{\theta}=\frac{2n}{\sum_{i=1}^{n}x_i} \tag{2}$$

Proof. The joint density function or likelihood function of (1) is given by

$$f\left(\underline{x};\theta\right)=\left(\frac{8}{3}\right)^n\left(\theta\right)^{4n}\left(\prod_{i=1}^{n}x_i^3\right)e^{-2\theta\sum_{i=1}^{n}x_i} \tag{3}$$

The log likelihood function is given by

$$log\ f\left(\underline{x};\theta\right)=n\,log\frac{8}{3}+4n\,log\,\theta+log\left(\prod_{i=1}^{n}x_i^3\right)-2\theta\sum_{i=1}^{n}x_i \tag{4}$$

Differentiating (4) with respect to θ and equating to zero, we get

$$\hat{\theta} = \frac{2n}{\sum_{i=1}^{n} x_i} \tag{5}$$

3. Bayesian Method of Estimation

In Bayesian analysis the fundamental problem are that of the choice of prior distribution g (θ) and a loss function $L\left(\hat{\theta},\theta\right)$. The squared error loss function for the scale parameter θ are defined as

$$L\left(\hat{\theta},\theta\right) = \left(\hat{\theta}-\theta\right)^2 \tag{6}$$

The Bayes estimator under the above loss function, say, $\hat{\theta}_s$ is the posterior mean, i.e,

$$\hat{\theta}_S = E\left(\theta\right) \tag{7}$$

This loss function is often used because it does not lead to extensive numerical computations but several authors (Zellner, 1986; Basu and Ebrahimi, 1991) have recognized that the inappropriateness of using symmetric loss function. Norstrom (1996) introduced an alternative asymmetric precautionary loss function and also presented a general class of precautionary loss functions with quadratic loss function as a special case. A very useful and simple asymmetric precautionary loss function is given as

$$L\left(\hat{\theta},\theta\right) = \frac{\left(\hat{\theta}-\theta\right)^2}{\hat{\theta}} \quad . \tag{8}$$

The Bayes estimator under precautionary loss function is denoted by $\hat{\theta}_P$ and is obtained by solving the following equation.

$$\hat{\theta}_P = \left[E\left(\theta^2\right)\right]^{1/2} \tag{9}$$

Weighted loss function (Ahamad *et al.*, 2016) is given as

$$L\left(\hat{\theta},\theta\right) = \frac{\left(\hat{\theta}-\theta\right)^2}{\theta} \quad . \tag{10}$$

The Bayes estimator under weighted loss function is denoted by $\hat{\theta}_W$ and is obtained as

$$\hat{\theta}_W = \left[E\left(\frac{1}{\theta}\right)\right]^{-1} \tag{11}$$

Let us consider two prior distributions of θ to obtain the Bayes estimators.

(i) Quasi-prior: For the situation where the experimenter has no prior information about the parameter θ, one may use the quasi density as given by

$$g_1\left(\theta\right) = \frac{1}{\theta^d} \;\; ; \theta > 0, \; d \ge 0, \tag{12}$$

where $d = 0$ leads to a diffuse prior and $d = 1$, a non-informative prior.

(ii) Gamma prior: The most widely used prior distribution of θ is the gamma distribution with parameters α and $\beta\left(>0\right)$ with probability density function given by

$$g_2(\theta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}\ ;\ \theta>0. \tag{13}$$

3.1 Bayes Estimators under $g_1(\theta)$

The posterior density of θ under $g_1(\theta)$, on using (3), is given by

$$f(\theta/\underline{x})=\frac{\left(\frac{8}{3}\right)^n(\theta)^{4n}\left(\prod_{i=1}^{n}x_i^3\right)e^{-2\theta\sum_{i=1}^{n}x_i}\theta^{-d}}{\int_0^{\infty}\left(\frac{8}{3}\right)^n(\theta)^{4n}\left(\prod_{i=1}^{n}x_i^3\right)e^{-2\theta\sum_{i=1}^{n}x_i}\theta^{-d}d\theta}$$

$$=\frac{\theta^{4n-d}\ e^{-2\theta\sum_{i=1}^{n}x_i}}{\int_0^{\infty}\theta^{4n-d}\ e^{-2\theta\sum_{i=1}^{n}x_i}d\theta}$$

$$=\frac{\left(2\sum_{i=1}^{n}x_i\right)^{4n-d+1}}{\Gamma(4n-d+1)}\ \theta^{4n-d}\ e^{-2\theta\sum_{i=1}^{n}x_i} \tag{14}$$

Theorem 2. Assuming the squared error loss function, the Bayes estimate of the scale parameter θ, is of the form

$$\hat{\theta}_S=\frac{4n-d+1}{2\sum_{i=1}^{n}x_i} \tag{15}$$

Proof. From equation (7), on using (14),

$$\hat{\theta}_S=E(\theta)=\int\theta f(\theta/\underline{x})\,d\theta$$

$$=\frac{\left(2\sum_{i=1}^{n}x_i\right)^{4n-d+1}}{\Gamma(4n-d+1)}\int_0^{\infty}\theta^{4n-d+1}\ e^{-2\theta\sum_{i=1}^{n}x_i}d\theta$$

$$=\frac{\left(2\sum_{i=1}^{n}x_i\right)^{4n-d+1}}{\Gamma(4n-d+1)}\frac{\Gamma(4n-d+2)}{\left(2\sum_{i=1}^{n}x_i\right)^{4n-d+2}}$$

or, $$\hat{\theta}_S=\frac{(4n-d+1)}{2\sum_{i=1}^{n}x_i}.$$

Theorem 3. Assuming the precautionary loss function, the Bayes estimate of the scale parameter θ, is of the form

$$\hat{\theta}_P = \frac{\left[(4n-d+2)(4n-d+1)\right]^{\frac{1}{2}}}{2\sum_{i=1}^{n} x_i} \tag{16}$$

Proof. From equation (9), on using (14),

$$\left(\hat{\theta}_P\right)^2 = E\left(\theta^2\right) = \int \theta^2 f\left(\theta / \underline{x}\right) d\theta$$

$$= \frac{\left(2\sum_{i=1}^{n} x_i\right)^{4n-d+1}}{\Gamma(4n-d+1)} \int_0^{\infty} \theta^{4n-d+2}\, e^{-2\theta \sum_{i=1}^{n} x_i}\, d\theta$$

$$= \frac{\left(2\sum_{i=1}^{n} x_i\right)^{4n-d+1}}{\Gamma(4n-d+1)} \frac{\Gamma(4n-d+3)}{\left(2\sum_{i=1}^{n} x_i\right)^{4n-d+3}}$$

$$= \frac{(4n-d+2)(4n-d+1)}{\left(2\sum_{i=1}^{n} x_i\right)^2}$$

$$\Rightarrow \qquad \hat{\theta}_P = \frac{\left[(4n-d+2)(4n-d+1)\right]^{\frac{1}{2}}}{2\sum_{i=1}^{n} x_i}$$

Theorem 4. Assuming the weighted loss function, the Bayes estimate of the scale parameter θ, is of the form

$$\hat{\theta}_W = \frac{4n-d}{2\sum_{i=1}^{n} x_i} \tag{17}$$

Proof. From equation (11), on using (14),

$$\hat{\theta}_W = \left[E\left(\frac{1}{\theta}\right)\right]^{-1} = \left[\int \frac{1}{\theta} f\left(\theta / \underline{x}\right) d\theta\right]^{-1}$$

$$= \left[\frac{\left(2\sum_{i=1}^{n} x_i\right)^{4n-d+1}}{\Gamma(4n-d+1)} \int_0^{\infty} \theta^{4n-d-1}\, e^{-2\theta \sum_{i=1}^{n} x_i}\, d\theta\right]^{-1}$$

$$=\left[\frac{\left(2\sum_{i=1}^{n}x_i\right)^{4n-d+1}}{\Gamma(4n-d+1)}\frac{\Gamma(4n-d)}{\left(2\sum_{i=1}^{n}x_i\right)^{4n-d}}\right]^{-1}$$

$$=\left[\frac{2\sum_{i=1}^{n}x_i}{4n-d}\right]^{-1}$$

or, $\hat{\theta}_W=\dfrac{4n-d}{2\sum_{i=1}^{n}x_i}$.

3.2 Bayes Estimators under $g_2(\theta)$

Under $g_2(\theta)$, the posterior density of θ, using equation (3), is obtained as

$$f(\theta/\underline{x})=\frac{\left(\frac{8}{3}\right)^n(\theta)^{4n}\left(\prod_{i=1}^{n}x_i^3\right)e^{-2\theta\sum_{i=1}^{n}x_i}\frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}}{\int_0^\infty\left(\frac{8}{3}\right)^n(\theta)^{4n}\left(\prod_{i=1}^{n}x_i^3\right)e^{-2\theta\sum_{i=1}^{n}x_i}\frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}d\theta}$$

$$=\frac{\theta^{4n+\alpha-1}e^{-\left(\beta+2\sum_{i=1}^{n}x_i\right)\theta}}{\int_0^\infty\theta^{4n+\alpha-1}e^{-\left(\beta+2\sum_{i=1}^{n}x_i\right)\theta}d\theta}$$

$$=\frac{\theta^{4n+\alpha-1}e^{-\left(\beta+2\sum_{i=1}^{n}x_i\right)\theta}}{\Gamma(4n+\alpha)\Big/\left(\beta+2\sum_{i=1}^{n}x_i\right)^{4n+\alpha}}$$

$$=\frac{\left(\beta+2\sum_{i=1}^{n}x_i\right)^{4n+\alpha}}{\Gamma(4n+\alpha)}\theta^{4n+\alpha-1}e^{-\left(\beta+2\sum_{i=1}^{n}x_i\right)\theta} \tag{18}$$

Theorem 5. Assuming the squared error loss function, the Bayes estimate of the scale parameter θ, is of the form

$$\hat{\theta}_S=\frac{4n+\alpha}{\beta+2\sum_{i=1}^{n}x_i} \tag{19}$$

Proof. From equation (7), on using (18),

$$\hat{\theta}_S = E(\theta) = \int \theta f(\theta/\underline{x})\, d\theta$$

$$= \frac{\left(\beta + 2\sum_{i=1}^{n} x_i\right)^{4n+\alpha}}{\Gamma(4n+\alpha)} \int_0^{\infty} \theta^{4n+\alpha} e^{-\left(\beta+2\sum_{i=1}^{n} x_i\right)\theta} d\theta$$

$$= \frac{\left(\beta + 2\sum_{i=1}^{n} x_i\right)^{4n+\alpha}}{\Gamma(4n+\alpha)} \frac{\Gamma(4n+\alpha+1)}{\left(\beta + 2\sum_{i=1}^{n} x_i\right)^{4n+\alpha+1}}$$

or, $$\hat{\theta}_S = \frac{4n+\alpha}{\beta + 2\sum_{i=1}^{n} x_i}.$$

Theorem 6. Assuming the precautionary loss function, the Bayes estimate of the scale parameter θ, is of the form

$$\hat{\theta}_P = \frac{\left[(4n+\alpha+1)(4n+\alpha)\right]^{\frac{1}{2}}}{\beta + 2\sum_{i=1}^{n} x_i} \tag{20}$$

Proof. From equation (9), on using (18),

$$\left(\hat{\theta}_P\right)^2 = E\left(\theta^2\right) = \int \theta^2 f(\theta/\underline{x})\, d\theta$$

$$= \frac{\left(\beta + 2\sum_{i=1}^{n} x_i\right)^{4n+\alpha}}{\Gamma(4n+\alpha)} \int_0^{\infty} \theta^{4n+\alpha+1} e^{-\left(\beta+2\sum_{i=1}^{n} x_i\right)\theta} d\theta$$

$$= \frac{\left(\beta + 2\sum_{i=1}^{n} x_i\right)^{4n+\alpha}}{\Gamma(4n+\alpha)} \frac{\Gamma(4n+\alpha+2)}{\left(\beta + 2\sum_{i=1}^{n} x_i\right)^{4n+\alpha+2}}$$

$$= \frac{(4n+\alpha+1)(4n+\alpha)}{\left(\beta + 2\sum_{i=1}^{n} x_i\right)^2}$$

or, $$\hat{\theta}_P = \frac{\left[(4n+\alpha+1)(4n+\alpha)\right]^{\frac{1}{2}}}{\beta + 2\sum_{i=1}^{n} x_i}.$$

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Theorem 7. Assuming the weighted loss function, the Bayes estimate of the scale parameter θ, is of the form

$$\hat{\theta}_W = \frac{4n+\alpha-1}{\beta+2\sum_{i=1}^{n} x_i} \tag{21}$$

Proof. From equation (11), on using (18),

$$\hat{\theta}_W = \left[E\left(\frac{1}{\theta}\right)\right]^{-1} = \left[\int \frac{1}{\theta} f\left(\theta/\underline{x}\right) d\theta\right]^{-1}$$

$$= \left[\frac{\left(\beta+2\sum_{i=1}^{n} x_i\right)^{4n+\alpha}}{\Gamma(4n+\alpha)} \int_0^\infty \theta^{4n+\alpha-2}\, e^{-\left(\beta+2\sum_{i=1}^{n} x_i\right)\theta} d\theta\right]^{-1}$$

$$= \left[\frac{\left(\beta+2\sum_{i=1}^{n} x_i\right)^{4n+\alpha}}{\Gamma(4n+\alpha)} \frac{\Gamma(4n+\alpha-1)}{\left(\beta+2\sum_{i=1}^{n} x_i\right)^{4n+\alpha-1}}\right]^{-1}$$

$$= \left[\frac{\beta+2\sum_{i=1}^{n} x_i}{4n+\alpha-1}\right]^{-1}$$

or, $$\hat{\theta}_W = \frac{4n+\alpha-1}{\beta+2\sum_{i=1}^{n} x_i}.$$

Conclusion

In this paper, we have obtained a number of estimators of parameter of area biased Ailamujia distribution. In equation (2) we have obtained the maximum likelihood estimator of the parameter. In equation (15), (16) and (17) we have obtained the Bayes estimators under squared error, precautionary and weighted loss functions using quasi prior. In equation (19), (20) and (21) we have obtained the Bayes estimators under squared error, precautionary and weighted loss functions using gamma prior. In the above equation, it is clear that the Bayes estimators depend upon the parameters of the prior distribution.

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