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A RELIABLE METHOD FOR BOUNDARY LAYER FLOW PAST A POROUS STRETCHING PLANE WITH HEAT TRANSFER IN PRESENCE OF A TRANSVERSE MAGNETIC FIELD

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ABSTRACT
In this paper, we applied the He's Homotopy Perturbation Method (HPM) to discuss the porous stretching plane in the boundary layer with heat transfer in the presence of a transverse magnetic field. The governing nonlinear partial differential equations were transformed using a suitable similarity transformation, and the resulting ordinary differential equations were solved by He's homotopy perturbation method. The main advantage of HPM is that it does not require the small parameters in the equations, and hence the limitations of traditional perturbation can be eliminated. The results reveal that the proposed method is very effective and simple and can be applied to other nonlinear problems. The influence of various relevant physical characteristics is presented and discussed.

Keywords: *Homotopy perturbation method (HPM), boundary layer flow, stretching plane, heat transform.*

NOMENCLATURE

B_0	Constant applied magnetic field, $[\mathrm{Wb\ m^{-2}}]$
f	Dimensionless stream function, [−]
k_0	Permeability of porous medium, [Darcy]
K	Permeability parameter $(= h^2/k_0)$, [−]
Pr	Prandtl number $(= \upsilon/\alpha)$, [−]
m	Velocity exponent parameter, [−]
M	Magnetic parameter $(= \sigma B_0^2 h^2/\rho\nu)$, [−]
T	Temperature of the fluid, [K]
u, v	Velocity component of the fluid along the x and y directions, respectively, $[\mathrm{m\ s^{-1}}]$
x, y	Cartesian coordinates along the surface and normal to it, respectively, [m]

Greek symbols

ρ	Density of the fluid, $[\mathrm{Kg\ m^{-3}}]$
υ	Kinematic viscosity, $[\mathrm{m^2\ s^{-1}}]$
σ	Electrical conductivity, $[\mathrm{m^2 s^{-1}}]$
θ	Dimensionless temperature, $[= \frac{T-T_\infty}{T_0-T_\infty}]$

Superscript

$'$	Derivative with respect to y

Subscripts:

0	Properties at the plane
∞	Free stream condition

1. INTRODUCTION

The study of boundary layer flow and heat transfer over a stretching plane in a viscous fluid is of considerable interest because of its ever increasing industrial applications and important bearings on several technological processes. Metal and polymer sheets can be produced as sheeting material in a variety of industrial manufacturing processes. The speed of heat transmission at the stretching surface affects the final product's quality. In today's metallurgical and metal-working operations, the study of magneto-hydrodynamic (MHD) movement of an electrically conducting fluid is of great interest. the process of cooling the first wall inside a nuclear reactor containment vessel, where the hot plasma is, and the process of fusing metals in an electrical furnace by employing a magnetic field. In controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching sheet, applied magnetic field may play an important role.

Blasius (1908) proposed the basic issue when he thought about the boundary layer flow on a stationary flat plate. Sakiadis (1961) was the first to study the boundary layer flow over a stretched sheet, in contrast to Blasius (1908). Crane (1970) later expanded on this concept for the two-dimensional situation where the velocity is inversely proportional to the distance from the plate. Numerous authors have thought about different parts of this issue and come up with comparable answers. Since then, other authors have researched numerous facets of this issue. For instance, Keller and Magyari (1999, 2000). In a porous media across a stretched sheet, Sriramulu et al. (2001) investigated the steady flow and heat transfer of a viscous incompressible fluid. Partha et al. (2005) investigated a related issue by taking an exponentially extending surface into consideration. Grubka and Bobba (1985) investigated the temperature distribution for the uniform surface heat flux condition, while Dutta et al. (1985) reported the temperature field in the flow across a linearly expanding surface susceptible to a varied surface temperature. Elbashbehy (1998) as well as Lin and Chen (1998) examined the surface heat flow and power-law velocity conditions on a stretched horizontal surface. In their studies of the impact of magnetic fields using stretching surfaces, Jhankal and Kumar (2013, 2015) studied the effect of magnetic field by, by considering stretching surface. According to Kumaran et al. (2009), a magnetic field causes streamlines to become steeper, which causes the boundary layer to become thinner.

Nonlinear issues predominate in the boundary layer flow region. These nonlinear equations should be solved using other suitable methods because most of them lack an accurate solution, with the exception of a small number of them that have a precise analytical solution. The majority of scientists think that combining numerical and semi-exact analytical techniques can provide useful findings. This research introduces and uses a semi-exact approach termed HPM to study boundary layer flow over a stretched vertical surface with heat flux. The initial work in HPM was studied by J. H. He (1998, 2000, 2001, 2005, 2009) and after that these investigations inspired a lot of researchers Ganji and Rajabi (2006), Ganji and Sadighi (2006), Ariel et al. (2006), Zhang and He (2006), Ganji and Ganjin (2008), Beléndez et al. (2008), Ma et al. (2008), Siddiqui et al. (2008), Zhang et al. (2008), Jhankal (2014, 2015) to solve nonlinear equations with this method.

The problem of boundary layer flow via a porous stretching plane with heat transfer in the presence of a transverse magnetic field is studied in the present study utilizing the homotopy perturbation method (HPM). The impact of several relevant characteristics is shown and discussed.

1.1 Basic idea of homotopy perturbation method (HPM):

To illustrate the basic ideas of the HPM, we consider the following nonlinear differential equation.

$$A(u) - f(r) = 0, \; r \in \Omega \qquad \dots(1)$$

Subject to the boundary conditions

$$B\left(u, \frac{\partial u}{\partial \eta}\right) = 0, \; r \in \Gamma \qquad \dots(2)$$

Where A is a general differential operator, B is a boundary operator, f(r) is a known analytical function and Γ is the boundary of the domain Ω. A can be divided into two parts which are L and N, where L is linear and N is nonlinear. Therefore equation (1) can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0, \; r \in \Omega \qquad \dots(3)$$

By the homotopy perturbation technique, we construct a homotopy

$v(r, p): \Omega \times [0,1] \to R$, which satisfies:

$$H(v, p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \; p \in [0,1], r \in \Omega \qquad \dots(4)$$

Where $p \in [0,1]$ is an embedding parameter and u_0 is an initial approximation that satisfies the boundary condition. Obviously, from these definitions we will have:

$$H(v, 0) = L(v) - L(u_0) = 0$$

$$H(v,1) = A(v) - f(r) = 0$$

The changing process of p from zero to one is just that of $v(r,p)$ from $u_0(r)$ to $u(r)$. In topology, this is called deformation and $L(v) - L(u_0)$ and $A(v) - f(r)$ are called homotopy. According to the HPM, we can first use the embedding parameter p as a "small parameter" and assuming that the solution of (4) can be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 \ldots \qquad \ldots(5)$$

Setting $p = 1$, results in the approximate solution of (1):

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \cdots \qquad \ldots(6)$$

The convergence and stability of this method was shown in Hosein Nia et al. (2008).

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a steady, two dimensional boundary layer flow over a porous stretching plane of a viscous incompressible electrically conducting fluid, where the x-axis is along the stretching plane and y-axis perpendicular to it, the applied magnetic field B_0 is transversely to x-axis. Under the boundary layer approximation, the governing equations of continuity, momentum and energy under the influence of externally imposed transverse magnetic field are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots(7)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u - \frac{\upsilon}{k_0}u \qquad \ldots(8)$$

$$v\frac{\partial T}{\partial y} = \frac{\upsilon}{Pr}\frac{\partial^2 T}{\partial y^2} \qquad \ldots(9)$$

Along with the boundary conditions for the problem are given by:

$$y = 0: u = mx, v = 0, T = T_0 \ (m > 0)$$
$$y \to \infty: u = 0, T = T_\infty \qquad \ldots(10)$$

Here since temperature filed varies with regard to y only, so $\frac{\partial T}{\partial x} = 0$. Also, we introduce the following non-dimensional quantities:

$\bar{x} = \frac{x}{h}, \bar{y} = \frac{y}{h}, \bar{u} = \frac{uh}{\nu}, \bar{v} = \frac{vh}{\nu}, \theta(y) = \frac{T - T_\infty}{T_0 - T_\infty}$ (where T_0 is the temperature at plane and T_∞ is the temperature of surrounding). …(11)

Substituting in (11) in governing equations, equations are reduced to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots(12)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{\partial^2 u}{\partial y^2} - Mu - Ku \qquad \ldots(13)$$

$$\frac{\partial^2 \theta}{\partial y^2} + f(y)Pr\frac{\partial \theta}{\partial y} = 0 \qquad \ldots(14)$$

Where bar has been dropped for our convenience
Along with the boundary conditions for the problem are given by:

$$y = 0: u = mx, v = 0, \theta = 1$$
$$y \to \infty: u = 0, \theta = 0 \qquad \ldots(15)$$

we set the similarity solution of the form

$$u = mxf'(y) \qquad \ldots(16)$$

Also using the continuity equation (12) with equation (16), we have

$$v = -m[f(y) - f(0)] \qquad \ldots(17)$$

Using equations (16) and (17), equations (13) and (14) becomes

$$m\left[f'^2(y) - f(y)f''(y)\right] = f'''(y) - Mxf'(y) - Kxf'(y) \qquad \ldots(18)$$

$$\theta''(y) + mPrf(y)\theta'(y) = 0 \qquad \ldots(19)$$

Along with boundary conditions:

$$y = 0: f' = 1, \theta = 1$$
$$y \to \infty: f' = 0, \theta = 0 \qquad \ldots(20)$$

Where we take f(0) = 0, without loss of generality.

SOLUTION WITH HOMOTOPY PERTURBATION METHOD

According to the HPM, the homotopy form of equation (18) and (19) are constructed as follows:

$$(1-p)(f''' - Mxf' - Kxf') + p\left(f''' - mf'^2 + ff'' - Mxf' - Kxf'\right) = 0 \quad \ldots(21)$$

$$(1-p)(\theta'') + p(\theta'' + mPrf\theta') = 0 \quad \ldots(22)$$

We consider f and θ as the following:

$$f = f_0 + pf_1 + p^2 f_2 \ldots$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 \ldots \quad \ldots(23)$$

By substituting equation (23) into (21) and (22), and then

(I) Terms independent of p give

$$f_0''' - (M+K)xf_0' = 0 \quad \ldots(24)$$

$$\theta_0''=0 \quad \ldots(25)$$

The boundary conditions are

$$f_0'(0) = 1,\ f_0'(\infty) = 0,\ \theta_0(0) = 1,\ \theta_0(\infty) = 0. \quad \ldots(26)$$

(II) Terms containing only p give

$$f_1''' - mf_0'^2 + f_0 f_0'' - (M+K)xf_1' = 0 \quad \ldots(27)$$

$$\theta_1'' + mPrf_0\theta_0' = 0 \quad \ldots(28)$$

The boundary conditions are

$$f_1'(0) = 0,\ f_1'(\infty) = 0,\ \theta_0(0) = 0,\ \theta_1(\infty) = 0. \quad \ldots(29)$$

(III) Terms containing only p^2 give

$$f_2''' - 2mf_0'f_1' + f_0 f_1'' + f_1 f_0'' - (M+K)xf_2' = 0 \quad \ldots(30)$$

$$\theta_2'' + mPr(f_0\theta_1' + f_1\theta_0') = 0 \quad \ldots(31)$$

The boundary conditions are

$$f_2'(0) = 0,\ f_2'(\infty) = 0,\ \theta_0(0) = 0,\ \theta_2(\infty) = 0. \quad \ldots(32)$$

The equations (24)-(25), (27)-(28) and (30)-(31) are solved with boundary conditions (26), (29) and (32) respectively, the boundary condition η=∞ were replaced by those at η=4 in accordance with standard practice in the boundary layer analysis. If p→1, we can find the approximate solution of equations (18) and (19).

RESULTS AND DISCUSSION

For the purpose of discussing the result, the numerical calculations are presented in the form of non-dimensional velocity and temperature profiles. Numerical computations have been carried out for different values of the magnetic field parameter (M), permeability parameter (K), and Prandtl number (Pr).

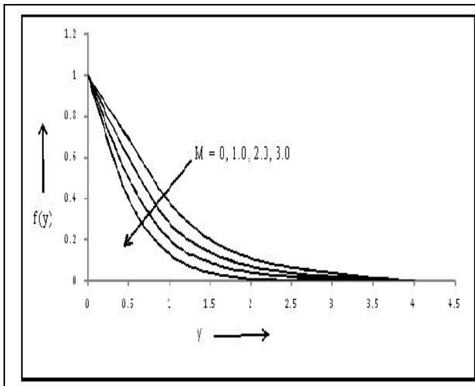

Figure 1: Effect of M on the velocity profile for K=1, and Pr=1.

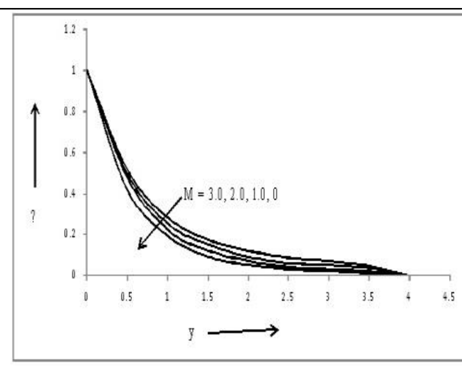

Figure 2: Effect of M on the temperature profiles for M=1, and Pr=1.

The effect of magnetic parameter (M) on the velocity profile for K=1.0, and Pr = 1.0 is presented in figure 1, it can be observed that velocity profiles are decreases as the M increase.
The effect of magnetic parameter (M) on the temperature profile for K=1.0 and Pr = 1.0 is presented in figure 2, it can be concluded that the temperature profiles are increases as M increase.
The effects of permeability parameter (K) on the velocity profile for M=1.0 and Pr = 1.0 is presented in figure 3, it can be found that velocity profiles are decreases as the K increase.
The effects of Prandtl number (Pr) on the temperature profile for M=1.0 and K=1.0 is presented in figure 4, it can be observed that temperature profiles decreases as Pr increase.

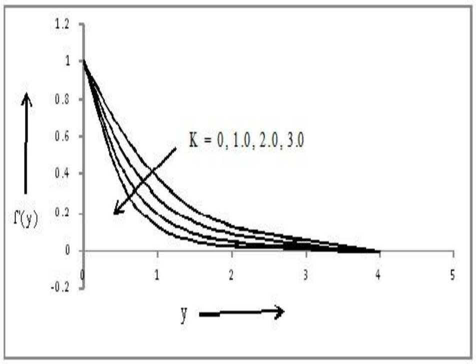

Figure 3: Effect of K on the velocity profiles for M=1, and Pr=1.

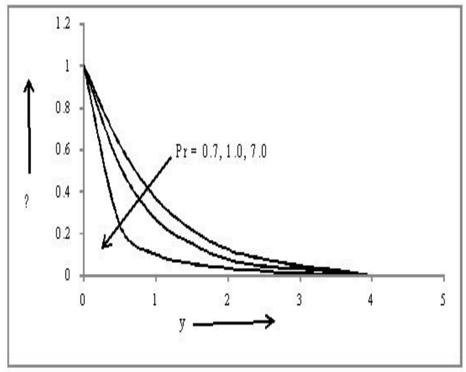

Figure 4: Effect of Pr on the temperature profiles for M=1, and K=1

CONCLUSIONS

A mathematical model has been presented for the problem of boundary layer flow via a porous stretching plane with heat transfer in presence of transverse magnetic field. The governing equations are approximated to a system of non-linear ordinary differential equations. HPM has been carries out for various values of the dimensionless parameters of the problem. The conclusions of present study are given below:

- The momentum boundary layer thickness reduces as magnetic parameter is increased. This is because the variation of M leads to the variation of the Lorentz force due to the magnetic field, and the Lorentz force produces more resistance to the transport phenomena.
- The thermal boundary thickness and heat transfer rate increases with increases in magnetic parameter (M) also it is observed that effect of M on temperature is very small when Pr is small.
- Boundary layer thickness is decreases by increasing the permeability parameter but quit slowly.
- The increase of Prantle number (Pr) results in the decrease of temperature distribution. The reason is that smaller values of Pr are equivalent to increasing thermal conductivity and therefore heat is able to diffuse from the heated surface more rapidly than for higher values of Pr.

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