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Research Article

MHD UNSTEADY BOUNDARY LAYER FLOW OVER AN ACCELERATED PERMEABLE PLATE

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ABSTRACT
The present study examines the unsteady magnetohydrodynamic (MHD) boundary layer flow over an infinite permeable plate accelerated in its own plane, in the presence of a uniform transverse magnetic field fixed relative to the plate. The velocity of the plate is prescribed as $U_0(t) = At^{\alpha}$, where A and α are constants. A similarity transformation is employed to reduce the governing unsteady partial differential equation to a nonlinear ordinary differential equation (ODE) involving the magnetic–time parameter (M) and the permeability–time parameter (K). The reduced ODE is solved numerically using boundary-value problem techniques. The analysis demonstrates that the magnetic-time parameter M suppresses the fluid motion through Lorentz force effects, while the permeability-time parameter K introduces additional resistance due to the porous medium. The parameter α significantly influences the initial velocity gradients, and time evolution is found to thicken the boundary layer while amplifying the combined effects of M and K. Detailed numerical results supported by graphical illustrations are presented to elucidate the influence of the governing parameters on the velocity profiles and boundary layer development.

***Keywords**: MHD; Unsteady boundary layer; Accelerated permeable plate; Similarity transformation; Numerical solution.*

NOMENCLATURE

A	Constant
B_0	Constant applied magnetic field
f	Dimensionless stream function
k	Permeability of porous medium
K	Permeability-time parameter
M	Magnetic-time parameter
U_0	Accelerated velocity
t	Time
u, v	Velocity component of the fluid along the x and y directions, respectively
x, y	Cartesian coordinates along the surface and normal to it, respectively

Greek symbols

α	Acceleration parameter
ρ	Density of the fluid
Y	Kinematic viscosity
σ_e	Electrical conductivity
η	Similarity variable

Superscript

$'$	Derivative with respect to η

Subscripts:

0	Properties at the plate
∞	Free stream condition

1. INTRODUCTION

Magnetohydrodynamics (MHD) deals with the motion of electrically conducting fluids in the presence of magnetic fields, a subject first formalized by Alfven (1942), whose pioneering work laid the foundation of plasma physics, and earlier demonstrated by Hartmann (1937) in his classical study of flows under transverse magnetic fields. Since then, MHD boundary-layer flows have become an area of intense research because of their wide-ranging engineering, industrial, and geophysical applications, including MHD power generation, plasma confinement, electromagnetic pumps, nuclear fusion reactors, cooling of electronic devices, metallurgical processes, and aerospace propulsion systems Davidson (2001).

In many engineering systems, the fluid flow is inherently unsteady and frequently occurs over moving or permeable surfaces. Such configurations are encountered in a variety of practical situations, including the transient startup of MHD generators, aerodynamic boundary-layer control through suction and injection, and the motion of continuous surfaces in electrically conducting fluids. The incorporation of suction or injection through a permeable surface plays a crucial role in modifying the boundary-layer characteristics by influencing velocity, shear stress, and thermal gradients. This mechanism not only facilitates the suppression of turbulence and the delay of flow separation but also provides an effective means for regulating heat and mass transfer. Smith (2019) highlighted these effects in the context of Boundary Layer Control Using Suction and Injection, demonstrating how boundary-layer stability and flow separation can be managed through surface permeability. More recently, Liu (2024), in Effects of Permeability on Unsteady Boundary Layer Flows, extended this understanding to unsteady regimes, emphasizing that surface permeability critically alters transient flow behaviour and enhances control strategies for engineering applications. In addition, Choi (2020) investigated unsteady boundary-layer flows with variable surface velocity, demonstrating that surface acceleration or deceleration significantly modifies the temporal evolution of velocity fields and stability characteristics.

Previous studies have provided foundational insights into MHD boundary-layer flows, particularly under the influence of permeable surfaces and applied magnetic fields. The pioneering works of Sakiadis (1961) on boundary-layer behaviour over continuous moving surfaces and Crane (1970) on stretching sheet flows established the fundamental framework for analyzing velocity distribution, shear stress, and flow stability in such configurations. Building on this foundation, subsequent studies examined the effects of suction and injection on flow control, stability enhancement, and heat transfer. For instance, Bansal and Dave (1987) investigated hydromagnetic flow near an accelerated plate, providing early insights into the role of magnetic fields in unsteady boundary-layer dynamics. Later, Jhankal et al. (2019) analyzed steady MHD mixed convection flow over a vertical permeable stretching sheet, demonstrating that suction enhances velocity gradients and wall shear stress. Although these works significantly advanced the understanding of steady-state phenomena, the dynamics of unsteady MHD boundary layers, particularly over accelerating surfaces, remain relatively less explored due to their nonlinear and time-dependent nature. Notable contributions in this direction include the work of Jhankal (2013), who investigated unsteady MHD boundary-layer flow and heat transfer over a stretching surface, highlighting the interplay between temporal variations and thermal transport. In another study, Jhankal (2018) examined MHD flow and heat transfer along an infinite porous hot horizontal moving plate, emphasizing the combined influence of surface permeability and heating effects. More recently, semi-analytical methods have been employed to tackle these complexities; for example, Jhankal (2023) utilized the Homotopy Perturbation Method (HPM) to effectively model unsteady MHD boundary-layer flow over a stretching plate. Complementary findings by Zhang (2022) further demonstrated the influence of magnetic fields on boundary-layer development over moving surfaces, confirming the stabilizing role of electromagnetic forces in modulating flow and thermal fields.

Despite substantial progress in the field of MHD boundary-layer flows, the problem of unsteady flow over an accelerating permeable plate under a time-dependent magnetic field has not been investigated in sufficient depth, particularly when the combined effects of acceleration, permeability, and magnetic influence are considered simultaneously. The governing equations describing such configurations are inherently nonlinear, coupled, and unsteady, which makes analytical treatment of the original partial differential equations (PDEs) extremely challenging. To overcome this difficulty, Nguyen (2021) demonstrated the effectiveness of similarity transformations in reducing unsteady boundary-layer problems into ordinary differential equations (ODEs), thereby simplifying the mathematical formulation. Numerical approaches have also played a crucial role in this area; for instance, Patel (2023) developed numerical solutions for MHD flows over stretching surfaces, while Wang (2022) carried out a detailed computational study of unsteady MHD flows, showing the significance of numerical accuracy in capturing transient flow structures. Moreover, the stabilizing influence of magnetic fields has been highlighted by Sharma (2024), who showed that magnetic forces tend to suppress instabilities and smoothen velocity profiles in unsteady boundary layers. On the other hand, surface suction and injection, which strongly affect boundary-layer thickness and wall shear stress, were examined by Khan (2023), emphasizing their critical role in modifying the flow field in the presence of magnetic effects.

Building upon this body of work, the present study focuses on unsteady MHD boundary-layer flow over an infinite permeable plate subjected to accelerated motion. The primary objective is to analyze the influence of the time-dependent parameter K and the time-dependent magnetic parameter M on the velocity distribution and overall boundary-layer development. By simultaneously considering acceleration, magnetic interaction, and surface permeability, this study provides a more comprehensive understanding of such flows, contributing not only to theoretical advancements but also to practical applications in areas such as electromagnetic flow control, plasma-assisted technologies, and high-temperature fluid systems.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The present study considers the unsteady, two-dimensional boundary-layer flow over an infinite permeable flat plate subjected to acceleration. The coordinate system is defined such that the x-axis is oriented along the direction of motion of the plate, which is assumed to start from rest and accelerate with a velocity profile $U_0 = At^{\alpha}$, where $\alpha \geq 0$ and A is a positive constant. The y-axis is taken to be normal to the plate. A uniform magnetic field of strength B_0 is applied transversely to the plate motion, i.e., along the y-axis, and is assumed to move with the plate so that it remains fixed relative to the surface. Under the boundary-layer approximations, the governing equations for the continuity and momentum of the electrically conducting fluid in the presence of this externally imposed transverse magnetic field are:

$$\frac{\partial v}{\partial y} = 0 \qquad \text{...(1)}$$

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e B_0^2}{\rho}(u - At^{\alpha}) - \frac{\nu}{k} u \qquad \text{...(2)}$$

Boundary Conditions are:

$u = 0$ for $t \leq 0$

$u = U_0 = At^{\alpha}$ for $y = 0$ and $t > 0$

$$\frac{\partial u}{\partial y} = 0, \frac{\partial^2 u}{\partial y^2} = 0 \text{ for } y \to \infty \text{ and } t > 0 \qquad \text{...(3)}$$

Where, u and v denote the velocity components in the x- and y-directions, respectively; t is the time; ρ represents the fluid density; ν is the kinematic viscosity; k denotes the permeability of the porous medium; σ_e is the electrical conductivity of the fluid; and B_0 refers to the externally imposed uniform magnetic field applied in the y-direction. The effect of the induced magnetic field is neglected under the assumption of a small magnetic Reynolds number flow. Furthermore, it is assumed that no external electric field is applied, and the contributions of the electric field arising from charge polarization as well as the Hall effect are disregarded. Here, A and α are positive constants associated with the acceleration of the plate. The momentum equation can be transformed into the corresponding ordinary nonlinear differential equation by the following transformation:

$$\eta = \frac{y}{\sqrt{\nu t}}, \; u = At^{\alpha} f(\eta) \qquad \text{…(4)}$$

Where η is the independent similarity variable. The transformed non-linear ordinary differential equations are:

$$f''(\eta) + \frac{1}{2}\eta f'(\eta) + (M + K - \alpha)f(\eta) - M = 0 \quad \ldots(5)$$

The boundary conditions are rewritten as follows:

$$f(0) = 1, f'(\infty) \to 0, f''(\infty) \to 0 \quad \ldots(6)$$

Where, $M = \frac{\sigma_e B_0^2}{\rho} t$ is the Magnetic-time parameter and $K = \frac{\nu}{k} t$ is the permeability-time parameter.

3. RESULTS AND DISCUSSION

The present investigation deals with the unsteady MHD boundary layer flow over an accelerated permeable plate under the influence of a transverse magnetic field. The transformed nonlinear governing equation (5) was solved numerically using boundary-value problem techniques subject to the boundary conditions (6), and the results are presented graphically to examine the influence of key physical parameters on the velocity field.

Figures 1 and 2 show the variation of velocity profiles with different values of the magnetic-time parameter M. It is observed that an increase in M leads to a significant reduction in the velocity of the fluid near the plate. This is attributed to the Lorentz force generated by the interaction of the magnetic field with the electrically conducting fluid, which opposes the fluid motion and thereby suppresses the velocity boundary layer thickness. For small values of M, the velocity decays more gradually away from the plate, whereas for larger M, the decay is sharper, indicating stronger damping effects. This behaviour is consistent with physical expectations, as higher magnetic fields enhance resistive forces within the flow.

Figures 3 and 4 illustrate the influence of the permeability-time parameter K on the velocity distribution. It is evident that the velocity decreases with increasing K. A higher value of K corresponds to a lower permeability of the porous medium, which introduces additional resistance to the fluid flow, thereby reducing the velocity near the plate. Conversely, when K is small (highly permeable medium), the fluid finds it easier to penetrate the porous plate, leading to a thicker boundary layer and higher velocity near the wall. This indicates that the porous medium plays a crucial role in regulating momentum transfer within the boundary layer.

The acceleration parameter α also significantly influences the velocity distribution. As seen in Figures 2 and 4, larger values of α increase the wall velocity $U_0 = At^{\alpha}$ which accelerates the fluid motion near the plate. Consequently, the velocity profiles exhibit higher magnitudes for increasing α. Physically, this means that stronger plate acceleration enhances momentum diffusion into the fluid, partially counteracting the retarding effects of both magnetic and permeability forces. Therefore, α acts as a driving parameter that competes against dissipative effects.

It is important to note that both M and K vary linearly with time, i.e., $M = \frac{\sigma_e B_0^2}{\rho} t$ and $K = \frac{\nu}{k} t$. Therefore, the temporal evolution of the velocity field is inherently coupled with their behaviour. At early times, both M and K are small, allowing the boundary layer to develop with relatively weak resistance. As time advances, the magnitudes of M and K increase, enhancing the magnetic and porous medium effects and thereby suppressing the velocity distribution. This illustrates the dual role of time: it enhances the plate velocity through $U_0 = At^{\alpha}$, while simultaneously intensifying resistive effects via larger M and K. The overall response of the velocity field depends on the balance between these competing mechanisms. For small values of α, resistive effects dominate with increasing time, whereas for larger α, plate acceleration can overcome suppression and sustain higher velocities near the plate.

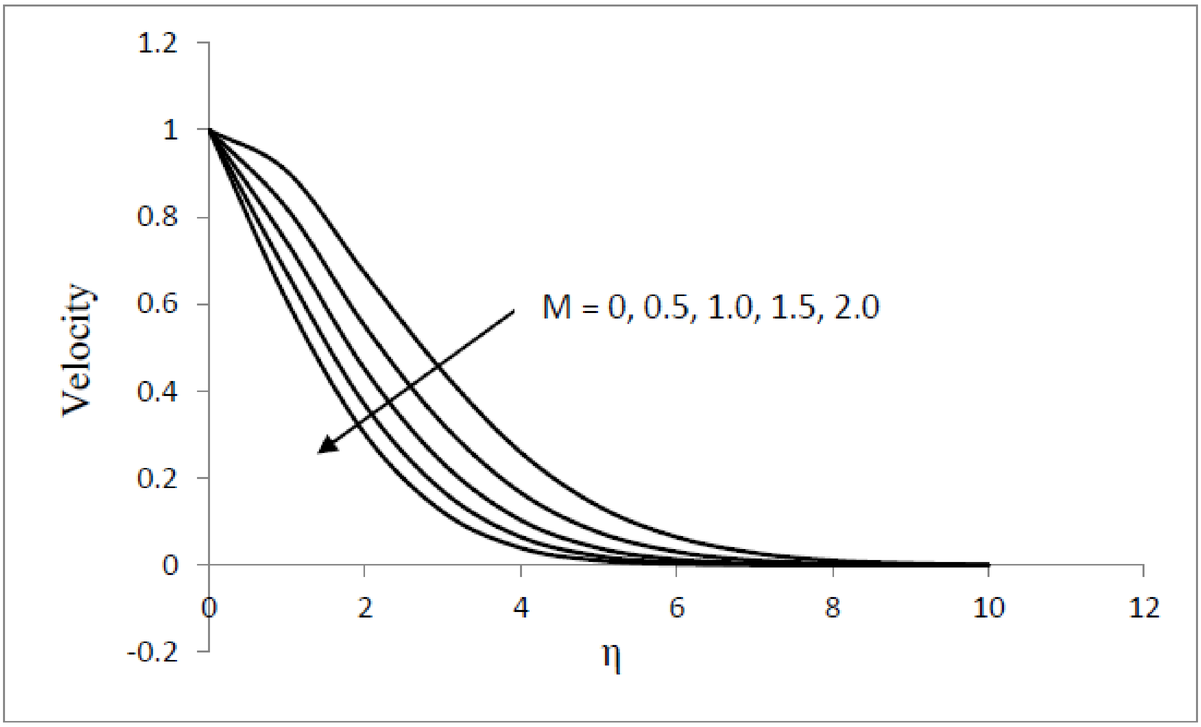

Figure 1: Velocity profile for various values of M when K = 0 and α = 0.

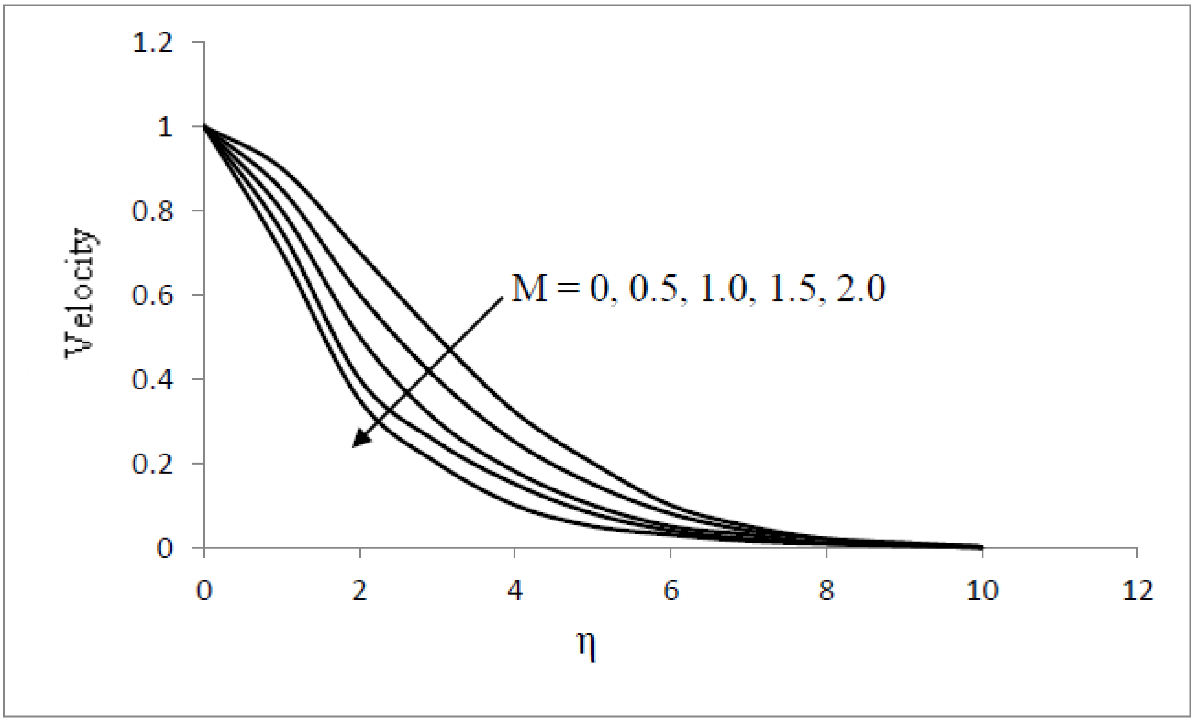

Figure 2: Velocity profile for various values of M when K = 1.0 and α = 1.0.

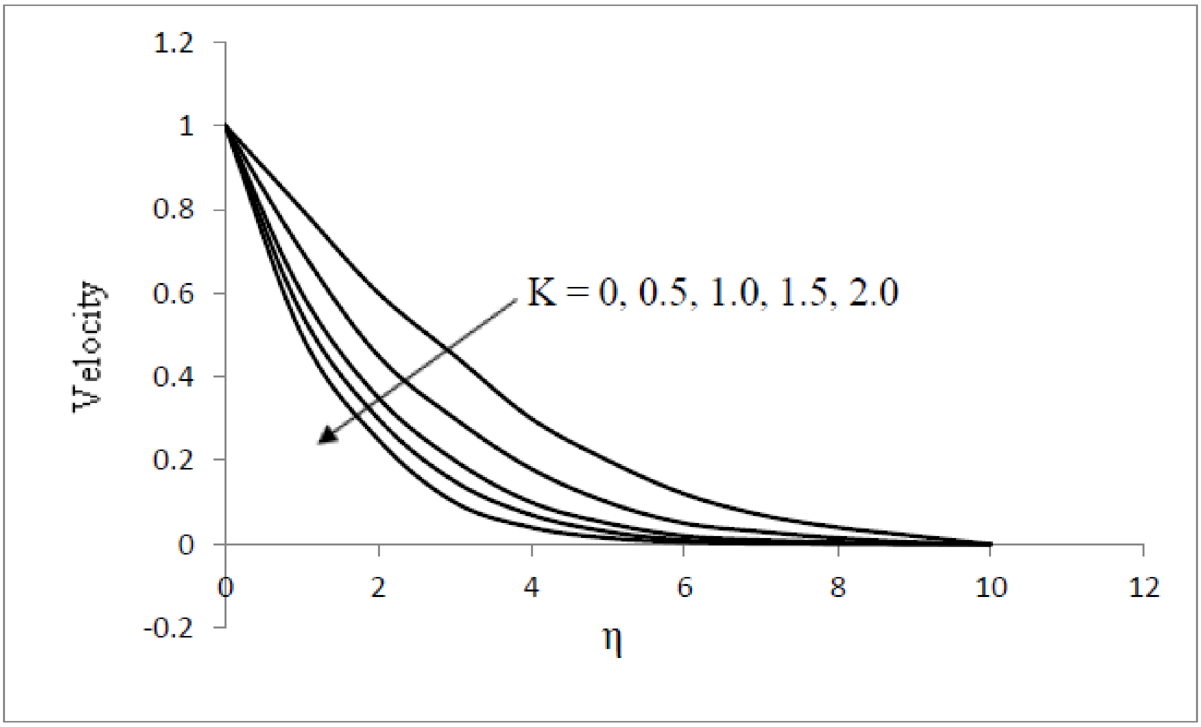

Figure 3: Velocity profile for various values of K when M = 1.0 and α = 1.0.

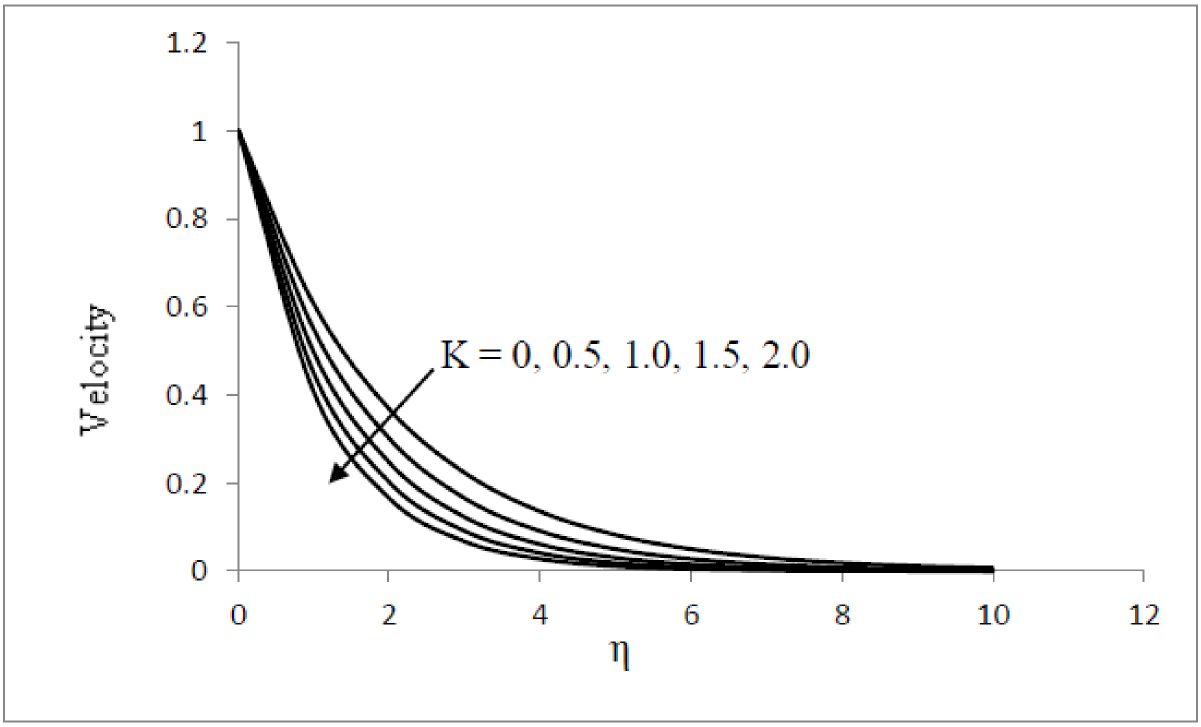

Figure 4: Velocity profile for various values of K when M = 2.0 and α = 2.0.

4. CONCLUSIONS

A mathematical model has been formulated to investigate the unsteady boundary layer flow over an accelerated permeable plate in the presence of a transverse magnetic field. The governing equation was reduced to a system of nonlinear ordinary differential equations and solved numerically using boundary-value problem techniques for different values of the pertinent physical parameters. Based on the results obtained, the following conclusions are drawn:

- The magnetic-time parameter M acts predominantly as a damping mechanism, leading to a suppression of the velocity field and a reduction in boundary layer thickness.
- The permeability-time parameter K introduces additional flow resistance, thereby diminishing the fluid velocity as the medium becomes less permeable.
- The acceleration parameter α enhances the velocity in the vicinity of the plate, partially counteracting the dissipative effects induced by magnetic and porous resistances.
- Time exerts a dual influence: it augments the wall velocity through the term t^{α}, while simultaneously intensifying the resistive contributions from the magnetic and porous media.

Overall, the flow behaviour reflects a subtle balance between acceleration-induced momentum enhancement and resistive suppression associated with magnetic and porous effects. These findings are consistent with the expected physical characteristics of unsteady MHD boundary layer flows over accelerated porous surface.

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