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Research Article

DEVELOPMENT OF A MATHEMATICAL MODEL OF REAL-TIME PROBLEMS USING DESIGN OF EXPERIMENTS AND COMPUTER SIMULATIONS

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ABSTRACT
One of the most important steps to purchase items is the selection of suppliers. The optimal choice of suppliers in real terms is difficult, because the development of mathematical models of real-time problems is not always possible. In the circumstance, using real data and computer simulations, the mathematical model is developed and then solved. This study has modeled the problem of selecting suppliers for a real case in the steel industry as a multi-objective optimization problem and the probabilistic parameters, using design of experiments and computer simulation in which the costs, delays in delivery and quality (reduction of waste), three criteria are to be optimized. The method used is, first, it identifies the factors influencing on by design of experiments and computer simulations, calculates the prediction equations for the objective functions and uses the compromise programming method for solving the model. This method is used when due to the complexity of the real situations, mathematical modeling cannot be used practically and it should be reached by using computer simulations and the design of experiments and then optimized.

Keywords: *Suppliers' Selection, Purchasing, Experiment Design, Adaptive Planning, Multi-Objective Optimization*

INTRODUCTION
Once a market demand becoming competitive and occurring severe fluctuations of customer needs, companies are trying to provide services with maximum speed and minimum cost to maintain a competitive advantage in this market. Outsourcing all or a part of the production is one of the ways by which the companies achieve these goals. Every organization usually spends a lot of money on buying items and outsourcing. One of the most important steps in purchasing would be supplier selection, which is a strategic decision. The problem of selection of suppliers due to its importance for manufacturing companies is highly regarded. On these issues, using some criteria such as cost, preparation time, and delay in delivery, ratio of the waste and so on, we choose the suppliers. Many articles have been conducted in conjunction with selecting suppliers. In an article, Huan-Jyh Shyura and Hsu-Shih (2006) has introduced a hybrid model for supplier selection within which at the beginning, he evaluated a multi-criteria decision method to the suppliers and gave the weight to them and then would rand according to their performance. In an article, Vijay and Ravi (2007), employed several methods for selecting suppliers including the weighted objectives, goal programming and compromise programming, and compared them together then used a real numerical example in illustrating their performance. At the same year, Zhang *et al.,* (2007) have conducted the experimental studies on the future of supply management and procurement and described the results based on the change in the terms of trade, procurement strategy, structure, roles and responsibilities, skills and system development. William (2010) wrote a review of multi-criteria decision methods for the evaluation and selection of suppliers and checked the articles out based on the methods used in the six areas of AHP, ANP, CBR, DEA, GA, SMART and mathematical programming and fuzzy sets and combinations. In the same year, Liu in his article (James and Yu-Tai, 2010) used a hybrid multi-criteria model for supplier selection, first, made the objective functions to be weighted by the ANP method and then prioritized the suppliers using VIKOR. This method is useful when there are

many factors that are related. In the same year, the paper (Arijit, 2010) used a concurrent engineering approach and got the suppliers ranked with the composition of the hierarchical decision-making and QFD and by the cost factor. Liu in 2011 used a combined approach to select the suppliers. In this paper the fuzzy planning and ANP method used to select the suppliers. This model has also been applied in situations where a large number of criteria exist and communicate together. In the same year, Zidane *et al.,* 2011) in their own paper used a hybrid method to select suppliers in which both quantitative and qualitative factors are considered in selecting suppliers so that at first, using the hierarchical fuzzy decision, assessed the suppliers according to qualitative factors and then ranked them using the TOPSIS method; and at second, by the DEA method the suppliers were selected. In the same year, Chen provided a structured approach to selecting and evaluating suppliers in the supply chain (unlike previous articles all have examined the selected suppliers only in one organization, not in the supply chain). According to the paper, the criteria of the supplier selection are specified by the SWOT method then using MADA method the suppliers got ranked. In 2012 Mendoza in his article (Abraham and José, 2012) presented an analytical model to select suppliers and made inventory management engaged in the selection of suppliers. In this paper, it is presented two nonlinear integer programming models for supplier selection and the order amount aims to minimize the annual orders, stock inventory and procurement costs under the constraints of capacity of suppliers and quality. In this study, three sets of criteria for selecting suppliers are being faced, including the cost, ratio of waste, and delay in delivery ratio. Selecting suppliers is performed with the aim of minimizing the 3 objective functions such that in real case, the research (Zhen and Lixin, 2003) used a specified design of experiments and a computer simulation technique, in each experiment running, to generate response levels and the results built the regression line equation. In its follow, using an compromise programming technique, the objective functions are aggregated and the optimum obtained. In fact, in this study, using simulation results and experimental design method, the problem objective function was achieved and then using compromise programming, the objective function is optimized. Since the simulation is achieved by using a mathematical model has been presented, the results are comparable to the results of mathematical model and a model validation is done in this case. This method is applied to the problems have many complexities and there is no possible optimized mathematical modeling; and the method mentioned in the research is very useful.

In following, the problem is addressed. The real case study is introduced. Then the experimental design method was used to find the factors influencing on; finally, the problem optimization and conclusion are prepared. In this paper, real cases discussed in the paper (Zhen and Lixin, 2003) are used. The paper (Zhen *et al.,* 2003) has investigated a model for multi-objective linear programming (MOLP) for the purchase of raw materials for the steel plant and it is discussed on the selection of suppliers, product selection and the order amount as a key factor in optimizing ordering policy is discussed. In this paper, the integrating compromise programming has been used in solving the problem. In this paper a real steel plant is investigated. The plant introduced requires to more than 100 kinds of raw material and 30 million tons of raw materials in order to produce energy for manufacturing iron and steel used. The raw materials are included in 4 categories: basic raw materials, oil, iron, waste of steel and ferroalloy. An important issue here is how to make decision on purchase to achieve lower cost raw materials for timely to meeting the demand, quality, quantity, and the maturity date. To achieve these goals are often in conflict with each other. These objectives can be divided into 3 categories, including: decisions regarding the selection of the product components which ought to be bought, decide on the quantity of product procurements and on the supplier selection. Selection of the components from different raw materials involves different costs and the selection that what type of components to choose has a significant impact on production costs. Different components or various combinations of components may produce the same product but by different costs. To determine the amount of the purchasing raw materials, it should be considered the components' portion of the product and purchased based on. As well, selecting suppliers has a direct impact on the product components; this is because each supplier provides a specific component with quality and cost different from other suppliers, therefore the change means a change in selected components, quality and cost. This is the problem about selecting suppliers. In this article we focus on the problem of selecting suppliers. The purpose of modeling is to decide on the matter that what components of the raw materials must be purchased and how many will each be ordered from suppliers. To select suppliers, 3 types of objective functions are more important: quality, price and delivery date. To demonstrate the quality and the date of delivery, the standard rate of waste in materials and the ratio of the delay in the delivery of supplies were used, respectively. Hence, there are 3 objective functions in the problem.

- Minimize the cost.
- Minimize rates of waste.
- Minimize delay in delivery.

All 3 objective functions are of minimizing requirements. This type of multi-objective modeling gives many advantages over one-objective modeling some is as follows:

- Various parameters can be evaluated in common units and it is not needed to change units into a single unit.
- Multi-objective technique enables decision-maker to enter his or her own experience and perspective into the model.
- Multi-objective technique enables decision-maker to review systematically effects of various decisions on an objective function and in the respective objective function space.

Formulated problem is as follows (Zhen and Lixin, 2003):

Min $Z_1=\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij}$

Min $Z_2=\sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij} X_{ij}$

Min $Z_3=\sum_{i=1}^{m}\sum_{j=1}^{n} s_{ij} X_{ij}$

X_{ij} = the number of order of j^{th} component of the product from i^{th} supplier.
C_{ij} = purchasing cost per unit of j^{th} component of product from i^{th} supplier.
r_{ij} = ratio of delay in delivery of j^{th} component of product from i^{th} supplier.
S_{ij} = ratio of the waste of j^{th} component of product from i^{th} supplier.
m = the number of suppliers.
n = the number of components of the product.
These limitations include the following:

- Budget constraints.
- Limitation of the number of production according to the demand.
- Limitation of the relevant technology to the portion of components' combination of the product.

In this model, the objective function values r_{ij} and S_{ij} and C_{ij} are probabilistic elements and has a triangular distribution.

MATERIALS AND METHODS

This is a case study on a steel plant with 30 suppliers and 100 products. Data were collected for 7 months and 7 suppliers and 13 products were selected for investigation. Limitations of this case are as follows (to a complete explanation of the study and the limitations, it is referred to (Zhen and Lixin, 2003):

$0.112X_{11}+0.127X_{31}+0.122X_{41}+0.115X_{51}+0.119X_{71}+0.0654X_{12}+0.0621X_{22}+0.0586X_{32}+0.0602X_{62}+0.195X_{33}+0.185X_{53}+0.09521X_{14}+0.0975X_{34}\leq 16.373,$

$1.2X_{11}+0.9X_{31}+X_{41}+1.1X_{51}+.95X_{71}\geq 60,$

$1.2X_{12}+0.95X_{22}+1.15X_{32}+1.05X_{62}\geq 30,$
$1.3X_{33}+1.1X_{53}\geq 10,$

$1.12X_{14}+1.24X_{34}\geq 70,$

$2X_{12}+2X_{22}+2X_{32}+2X_{62}+3X_{33}+3X_{53}=X_{11}+X_{31}+X_{41}+X_{51}+X_{71}+X_{14}+X_{34},\ X_{ij}\geq 0$, i=1,2,…,7 ; j=1,2,3,4.

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In this paper, the objective function was regarded to adapt further with the reality and do computer simulation with probabilistic parameters have a triangular distribution.

Design of Experiments

Because of the uncertainty of the factors' levels of this problem, this research will have to use heuristic technique to get them. According to the modeling presented there are 4 limits and the last one is of equality type, so it is failed to find out a combination of factors that satisfies all constraints especially the last, using meta-heuristic innovative methods. The method used in this study is that the problem model along with the aggregated objective function introduced by Zhen and Lixin (2003) has been written in Lingo software with the real-time constraints. The probabilistic parameters' values of the objective function have been altered in the range of slightly more than the maximum and slightly less than the least and the optimum solution has been found. This was run 19 times, and 19 sets of answers were obtained to the decision variables that satisfy the constraints of the problem. In fact, for each variable, it is gained 19 values and the maximum and minimum values were intended as in boundary. Using this method, 19 values, that is, 19 different combinations of the factors that satisfy all constraints were obtained, as a result, for each factor, 19 different values have been found. Among the values, the maximum was considered as an upper boundary and the minimum as a lower. The values obtained are shown in Table 1 (X_{ij} = the number of i^{th} item purchased from j^{th} seller). Computer simulation has been used in making replication in each experiment design running.

Process of making simulation is in Appendix 1. In each the simulation running, the values are set at levels introduced in the pilot project and made run the model. It is shown, on the software outputs, the values of 3 response variables. For each combination of variables, it is done 3 times and we entered the results of experiments in the table.

Table 1: Variables' levels in experiment design

Low Level	High Level	Factor
50	0	$\mathbf{X_{11}}$
19.50171	0	$\mathbf{X_{31}}$
10.36517	0	$\mathbf{X_{41}}$
54.54545	0	$\mathbf{X_{51}}$
37.18061	0	$\mathbf{X_{71}}$
43.58931	0	$\mathbf{X_{12}}$
43.98983	0	$\mathbf{X_{22}}$
44.42275	0	$\mathbf{X_{32}}$
44.2285	0	$\mathbf{X_{62}}$
0.002537	0	$\mathbf{X_{33}}$
19.61918	9.08791	$\mathbf{X_{53}}$
62.5	0	$\mathbf{X_{14}}$
56.45161	0	$\mathbf{X_{34}}$

Because it is a difficult and time-consuming simulation, there are only 16 possible experiments to the problem; this for reason, this problem used the sectorial design of experiment of 2^{13-9}. Note that due to many factors (13 factors) CCD design is not allowed for this problem because it engages in limitation on the number of factors (9 factors). Pilot project was implemented in Minitab software.

Results of the Design of Experiment

With the arrival of the sectorial design of experiment of 2^{13-9} in Minitab software and observing the output, it estimated regression line equation. On Figures 1,2 and 3, it can be visible sub-graph of distribution of errors. It can be determined that it is acceptable assuming a normal distribution of errors.

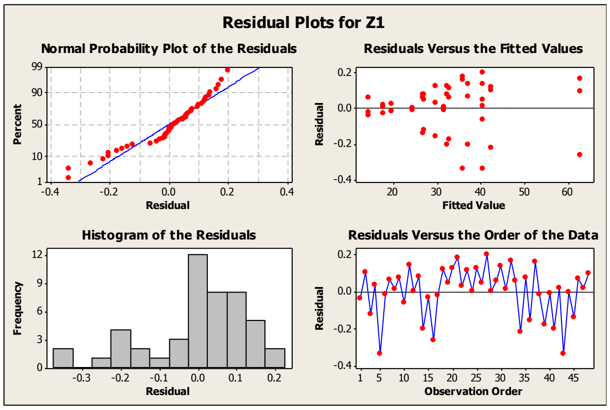

Figure 1: Distribution of errors for the objective function Z1

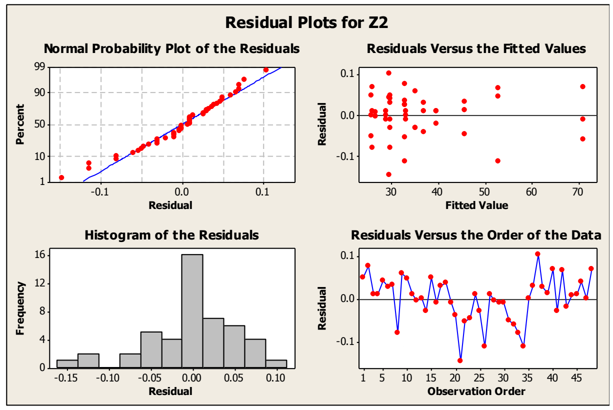

Figure 2: Distribution of errors for the objective function Z2

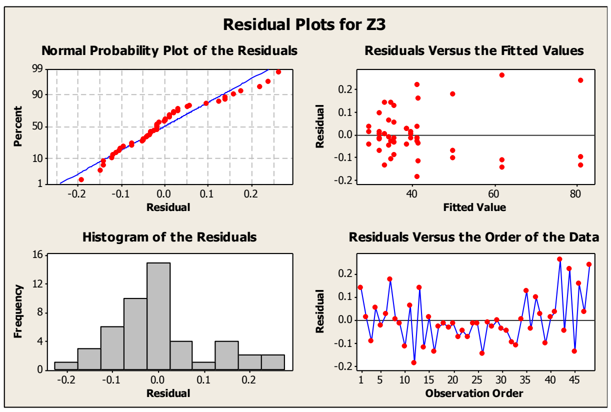

Figure 3: Distribution of errors for the objective function Z3

Based on the results of design of experiments and outputs from the software, prediction equations will be estimated as follows:

$Z1= 32.2269+ 3.7827X_{11}+ 1.1015X_{31}+ 0.7990X_{41}+ 3.0456X_{51}+ 6.2094X_{71}+ 1.3052X_{12}+ 6.9860X_{22}+ 1.0123X_{32}+ 1.1598X_{62}+0.9540X_{53}+ 2.1415X_{14}+ 2.0852X_{34}$

$Z2= 36.0573+ 2.5806X_{11}+ 1.5265X_{31}+ 0.6948X_{41}+ 3.2035X_{51}+ 3.2281X_{71}+ 2.2485X_{12}+ 4.0535X_{22}+ 3.0794X_{32}+ 6.7119X_{62}+0.6206X_{53}+ 3.2302X_{14}+ 3.8406X_{34}$

$Z3= 41.4354+ 5.0725X_{11}+ 1.0029X_{31}+ 0.8517X_{41}+ 4.0996X_{51}+ 2.3529X_{71}+ 3.7192X_{12}+ 3.0179X_{22}+ 3.0783X_{32}+ 4.8562X_{62}+ 0.9837X_{53}+ 5.8317X_{14}+ 4.8767X_{34}$

Decision variable X33 can be seen to be omitted in the output of the model because it doesn't any significant impact on the software outputs.

Optimization of the Model

To optimize this Model, the compromise programming method was used. This method to optimize used the compatible functions. It is used the distance metric in L-P methods in order to measure how close the existing solution is to the ideal solution. The measurement of the deviation as a compatible function will be as follows (AsgharPur, 2011):

$L\text{-}P=[\sum_{j=1}^{k}\gamma_j[f_j(x^{*j})-f_j(x)]^p]^{1/p}$

Here the objective is to making maximization. The x_j* represents the ideal solution in j^{th} objective optimization. The x represents a given solution and γ_j reflects the importance (weight) for the j^{th} objective when $\gamma_j > 0$. L-P compatible function in order to minimize the deviation from the ideal should be minimal.

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To solve the problem includes the 3 integrated objective functions of minimizing type, we use the formula presented above as follows (Andre and Romer, 2008):

$$L-P=\left[\sum_{i=1}^{k}\gamma_j\left[\frac{f_j(x)-f_j(x*j)}{f_j(x^{-j})-f_j(x*j)}\right]^p\right]^{1/p}$$

The value of the parameter in the formula according to (Zhen and Lixin, 2003) as follows:
γ_1=0.5, γ_2=0.2 , γ_3=0.3
Also here let p = 1.
Due to the distance metric introduced, the aggregated objective function is rewritten as follows:

F_a= (0.5^1)*((((32.2269+ 3.7827*x_{11}+ 1.1015*x_{31}+ 0.7990*x_{41}+ 3.0456*x_{51}+ 6.2094*x_{71}+ 1.3052*x_{12}+ 6.9860*x_{22}+ 1.0123*x_{32}+ 1.1598*x_{62}+0.9540*x_{53}+ 2.1415*x_{14}+ 2.0852*x_{34})-(302.1262))/(749.6259-302.1262))^1)+ (0.2^1)*((((36.0573+ 2.5806*x_{11}+ 1.5265*x_{31}+ 0.6948*x_{41}+ 3.2035*x_{51}+ 3.2281*x_{71}+ 2.2485*x_{12}+ 4.0535*x_{22}+ 3.0794*x_{32}+ 6.7119*x_{62}+0.6206*x_{53}+ 3.2302*x_{14}+ 3.8406*x_{34})-(439.4523))/(736.4576-439.4523))^1)+ (0.3^1)*((((41.4354+ 5.0725*x_{11}+ 1.0029*x_{31}+ 0.8517*x_{41}+ 4.0996*x_{51}+ 2.3529*x_{71}+ 3.7192*x_{12}+ 3.0179*x_{22}+ 3.0783*x_{32}+ 4.8562*x_{62}+ 0.9837*x_{53}+ 5.8317*x_{14}+ 4.8767*x_{34})-(583.3673))/(913.34-583.3673))^1)

The results of solving the problem by Lingo software using the objective function is shown in Table 2.

Table 2: Results of the solving the mathematical model

X11	50	**X62**	0
X31	0	**X33**	0
X41	0	**X53**	9.090909
X51	0	**X14**	66.41454
X71	0	**X34**	0
X12	0	**X32**	**0**
X22	44.57091		

Table 3: The objective functions' values

Aggregated Objective Function Value	0.7318		
Objective Num.1	683.6337	**Opt**	302.12
Objective Num.2	565.9295	**Opt**	439.45
Objective Num.3	825.8233	**Opt**	583.36

The integrated optimal objective function's value of the original paper (Zhen and Lixin, 2003) is by 6.20 (considering all the parameters in this article to be certain) that compared with the results obtained by the method and of course, considering price, waste rate and delay in the delivery to be possible, the value is obtained by 21.74 representing a very good approximation and the solution close to the optimal in this way and it is of many applications when mathematical modeling is difficult.

RESULTS AND DISCUSSION

In this study, the supplier selection process has been investigated under three criteria: cost, procurement schedule and waste rate so that according to data collected by the simulation and with help of designing the experiments, factors have been identified, influencing the problem, and afterwards regression equations have been formed for the 3 criteria said. Finally, by adaptive planning, these functions become integrated and a solution near optimal has been obtained. By comparing the results obtained with the solutions of the original paper, it can be seen there are not many changes in the solutions (despite of the change in original problem and of considering the factors to be possible) and this implies the correctness

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of the activities carried out in this research. Topic for future research will be to consider this problem in a supply chain.

Appendix 1 (Simulation Software Problem Arena)

Once it became clear the factors' levels and the experimental design was made, in each of factors' levels coming in the experiment design rows, there must exist 3 values of 3 replications for each of 3 objective functions. It was used simulation in making replication performed by Arena software. This simulation made the objective function modeled, and it was run 3 times and the results entered into the pilot project once the factors' levels entering into every pilot project row of the simulation. Briefly, the simulation model is constructed as follows.

For each of the possible elements a variable was defined whose value increases when entering a module each time. Once Entering, the possible amount of element ij multiplied by the number of that commodity X_{ij} and it shows sum of the possible elements for each category (C_{ij}, r_{ij} and S_{ij}) and the objective function value (Z_1 or Z_2 or Z_3) on that phase.

At the end 'n' items enter in the model; that is, each objective function would be calculated 'n' times and the entire operation can be performed in 20 replications, we consider their average of 20 replications as the objective function value. This describes that entities arrive at the model exponentially distributed. Then to calculate the first objective function with respect to the specified values of the X_{ij}'s, the possible C_{ij}'s would be generated by a specified triangular distribution and multiplied by the relevant X_{ij}. This is done for all C_{ij} X_{ij} that the result is Z_1. Producing n entities, the value of Z_1 are calculated by n times, and the average value will be equal to the objective function Z_1 of this phase. Each entity listed in the following of the model produced the possible values of r_{ij} and S_{ij} are as Z_1, Z_2 and Z_3 too. These operations are repeated up to 20 times until the standard deviation of the process is reduced and the values of Z_i's generated have more credibility. In this way, one obtains the values of the response variable. Computer simulation model is in form of Figure 4.

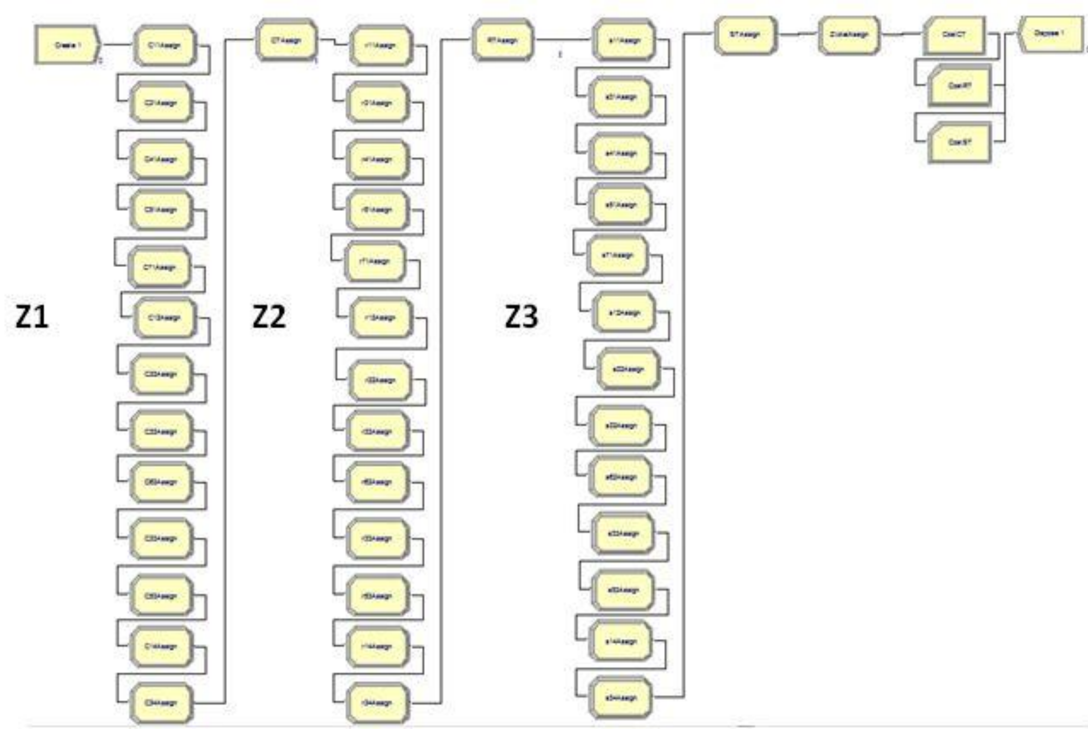

Figure 4: Simulated Model in Arena Software

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