Indian Journal of Fundamental and Applied Life Sciences ISSN: 2231– 6345 (Online)
An Open Access, Online International Journal Available at www.cibtech.org/sp.ed/jls/2015/01/jls.htm
2015 Vol.5 (S1), pp. 2199-2206/Seyyed and Ghanbari
Research Article

SYNCHRONIZATION OF TWO DIFFERENT FRACTIONAL CHAOTIC SYSTEMS WITH UNCERTAINTY OF FRACTION USING FUZZY SLIDING MODE CONTROLLER

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ABSTRACT
In this paper, a controller is designed to synchronize two different fractional chaotic systems using sliding mode control. Because of the presence of uncertainty and noise of the system, fuzzy sliding mode control is used to achieve an acceptable synchronization. The results have shown that the performed stability analysis is so efficient. And also the uses of the Fuzzy control as a robust nonlinear controller are shown.

Keywords: *Fractional Calculus, Fuzzy Sliding Mode Control (FSMC), LU, Volta, Synchronization, Chaos, Lyaponuv Stability Analysis*

INTRODUCTION
The concept of chaos is one of the modern concepts of science that expand our insights to the world. As its name implies, chaos is apparently a random and chaotic behavior that occurs in many real-world phenomena [5]. Differential equations that can be either in partial or ordinary form provide strong analytical framework for all natural scientists. For a long times Because of the existence of the Poincare - Bendicson theorem, it was thought that a system has an equilibrium point or a partially cycle (whether stable or unstable) [17]. However, this theorem was true only for second order systems. But popular belief was about that this theorem is also holds for higher-order systems. However, it was found that for three or higher order of the system, another phenomenon also occurs which is called chaos. After an introducing a method by Pechora and Carroll [22] in synchronization of two chaotic systems with different initial conditions, Synchronization of systems with chaotic dynamic has become a fascinating subject in different areas of research in the last two decades. The Idea is to synchronize the output of the master system to control the slave control system so that the outputs of the slave system follow the master outputs asymptotically [16]. Among the various methods of chaotic system, we can mention to the adaptive control [25], sliding mode control (variable structure) [18], the fuzzy controller [8], the active control [2], the delay feedback [21], back stepping design technique [30], etc. Stability analysis for synchronization between chaotic systems is an important issue. In recent years, a lot of studies and applications are displayed in fractional systems in various fields of science and engineering [10 & 24]. Although the fraction calculus has for about 300 year of history in mathematics, recently, its applications in various fields such as signal processing [3], image processing [19], control [13] and robotics [6] has started. These cases and many examples like these examples are completely showing the importance of fraction dynamics systems [27]. The chaotic systems with fraction degree, such as Chua circuit[28], Dufyang system [20], the jerk model [29] and Chen system [14] system etc. has attracted the researchers. There are two main reasons to attract researchers in the field of FOC. The first reason is that the Changes in the differential parameters can cause changes in the system so that it creates several different systems, which can provide various control schemes. The second reason is that in most cases, the stability analysis can be easily provided by preparation of the controllers with complex structure [26]. Sliding mode control (SMC), is based on the variable structure systems theory, which are widely used in robust nonlinear control systems. Among the attractive benefits we can name: fast response, good transient performance and no sensitive to the process parameters, external disturbance and the no dynamics model [22].
In order to improve the sliding on the sliding surface of the sliding mode controller, the fuzzy sliding mode controller is designed and its improvement is shown [23].

In contrast to the classical linear and nonlinear control theory, fuzzy logic controller hasn't made based on the mathematical model and are widely used to solve problems in uncertain and ambiguous environment with a high degree of nonlinearity [9]. The advantages of fuzzy controller to the other controller are as follows:
If we want to develop the controller, it can be done with lower cost by a fuzzy controller.
It covers wide range of operating conditions.
Are easily adjustable by the linguistic expressions.
Ability of self-regulation and nonlinear time-varying adoptability.

The Equations of Chaotic Systems

There are different systems that are chaotic behavior two systems that used in this article are the Newton-Lypnyk and Volta system. Dynamics describing of the two systems with state behavior of autonomous and phase portrait are stated here.

2.1. Volta Chaotic System

Volta chaotic system [12] will be known with three states that the differential equation is as equation (1).

$$D^q x_1 = -x_1 - a_1 x_2 - x_3 x_2$$
$$D^q x_2 = -x_2 - b_1 x_1 - x_3 x_1 \qquad (1)$$
$$D^q x_3 = c_1 x_3 + x_1 x_2 + 1$$

The value of the system parameters as well as [11] is considered at a1 = 19, b1 = 11 and c1 = 0.73. Fuzzy portrait of the system which is derived at three Value the fraction q = 1, q = 0.98, and q = 06.9 is plotted in Figure 2-1.Another characteristic of chaotic systems is having the amazing adsorbent. Qualitatively, wonderful adsorbent are a kind of absorbent that will be absorbed to the state path and at the same time away from that. It can be seen that the adsorbents (attractors) [7] created in approximately derivatives value of 1 cannot be seen at a lower derivatives Value, so the chaotic behavior of the system decreases by reducing the Value derivative.

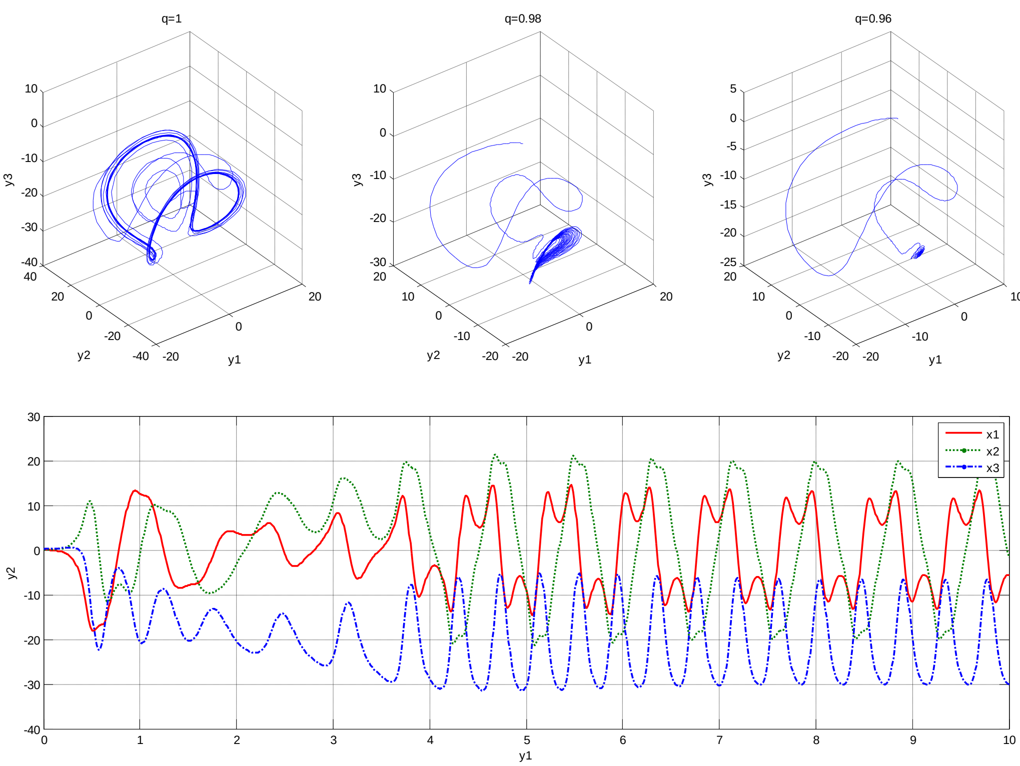

Figure 1: Portrait of a Volta fuzzy system derived at three fractional value of q = 1, q = 0.98 and q = 0.96

2.2. Unknown LU Chaotic Systems

LU chaotic system [4] will be known with three states that the differential equation is as equation (2).

$$\begin{aligned} D^q y_1 &= a_2(y_2 - y_1) + \Delta g_1(Y,t) + d_1(t) \\ D^q y_2 &= b_2 y_2 - y_1 y_3 + \Delta g_2(Y,t) + d_2(t) \\ D^q y_3 &= -c_2 y_3 + y_1 y_2 + \Delta g_3(Y,t) + d_3(t) \end{aligned} \tag{2}$$

system values parameters are such as paper [1] as a2 = 36, b2 = 20 and c2 = 3, the States of the system when derivation value is equal to q = 1 and the initil conditions are also equal to [0.2 0.2 0.1], are shown in the next figure.

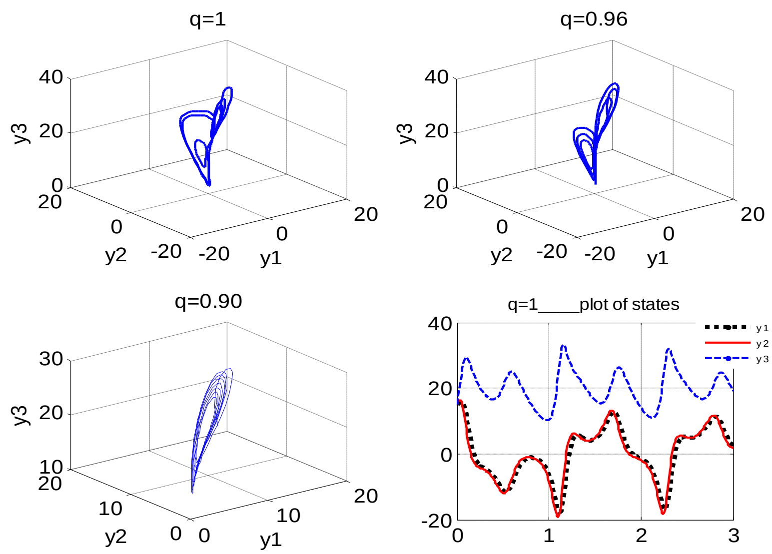

Figure 2: Portrait of a LU fuzzy system derived at three fractional value of q = 1, q = 0.96 and q = 0.9

3. Fuzzy Sliding Mode Synchronization for Fractional Systems

In synchronization problem of two systems, one of the system is known as master (drive) and another one will be slave (response). The synchronization idea means that the behavior of the system (with same or different the equations) are asymptotically equal with arbitrary initial conditions. From controlling point of view, synchronization is the one in which we use the master system equations so that we could design a controller and use for the slave system input.

Let's consider the master and the slave with differential equations of fractional degree by (3) and (4) respectively:

$$\begin{aligned} D^q x_1 &= f_1(X,t) \\ D^q x_2 &= f_2(X,t) \\ D^q x_3 &= f_3(X,t) \end{aligned} \tag{3}$$

$$\begin{aligned} D^q y_1 &= g_1(Y,t) + \Delta g_1(Y,t) + d_1(t) + u_{eq1}(t) \\ D^q y_2 &= g_2(Y,t) + \Delta g_2(Y,t) + d_2(t) + u_{eq2}(t) \\ D^q y_3 &= g_3(Y,t) + \Delta g_3(Y,t) + d_3(t) + u_{eq3}(t) \end{aligned} \tag{4}$$

While $Y = [y_1, y_2, y_3]^T, X = [x_1, x_2, x_3]^T$, the states of the system are (3) and (4) and $0 < q \le 1$.

$f_i(X,t)$ and $g_i(Y,t)$ are nonlinear function that describe the dynamic of the system. $\Delta g_i(Y,t)$ is the uncertainty of the slave system and di(t) is an active disturbance against the slave system. A fuzzy sliding mode control is proposed to design a control input of ueq(t).

3.1. Describing a Fuzzy Sliding mode Controller to Synchronize the System

Sliding mode controller design procedure is as follows:

1- Introducing a sliding surface that represents the dynamics of the system.

2- Development of the switching control lawto remain the sliding mode in all parts of the sliding surface.

So, to find sliding surface and reach the control law u(t), it is necessary to understand and control the dynamics of the system. And any state outside of the sliding surface can be stimulated to reach the sliding surface in a limited time.

$$e_i = x_i - y_i \tag{5}$$

Wide sliding surfaces can be defined for this article which the most appropriate definition foe simple and useful design is as equation (6).

$$s(t) = c_1e_1 + c_2e_2 + c_3e_3 \tag{6}$$

The values of C1, C2, and C3 are the control variables that dynamics behavior will be defined by correct selection of them. This selection includes the zero setting of slip sliding surface in the finite time and not changing the sliding surface after reaching to the states. Once you reach the sliding surface, then it is said that sliding mode has occurred.

Similarly, the design of sliding mode control can be defined in two phases [23]:

1- reaching phase when $S(t) \neq 0$

2- sliding Phase by S (t) =0

A sufficient condition for error, which moves from the first phase to the second phase is shown in into Equation (7):

$$S(t)\dot{S}(t) \leq 0 \tag{7}$$

This condition is called as sliding condition. In the absence of uncertainty and external disturbances, The corresponding balance control force, ueq (t), can be obtained by $\dot{S}(t) = 0$.

The following equation will perform the classic derivative of a fractioning.

$$\dot{S}(t) = D^{1-q}(D^q(S(t))) = 0 \rightarrow D^q(S(t)) = 0 \tag{8}$$

The control signal in the following equation will stimulate the dynamics to reach to the sliding surface.

$$D^q(S(t)) = C_1D^qe_1 + C_2D^qe_2 + C_3D^qe_3 =$$

$$C_1D^q(x_1 - y_1) + C_2D^q(x_2 - y_2) + C_3D^q(x_3 - y_3)$$

$$= \; C_1(f_1(X,t) - g_1(Y,\mathrm{t}) - \mathrm{u}_{eq1}(\mathrm{t}))$$

$$+ \; C_2(f_2(X,t) - g_2(Y,\mathrm{t}) - \mathrm{u}_{eq2}(\mathrm{t}))$$

$$+ \; C_3(f_3(X,t) - g_3(Y,\mathrm{t}) - \mathrm{u}_{eq3}(\mathrm{t}))$$

So the stability control law is in the form of the equation (9).

$$u_{eqi}(t) = f_i(X,t) - g_i(Y,\mathrm{t}) \tag{9}$$

To improve the robustness of the system against external disturbance and uncertain state which will remain on the sliding surface, switching control functions can be combined as equation (10):

$$u_i(t) = u_{eq_i}(t) + KFSMC(S) \tag{10}$$

That the FSMC (S) represents the output of the fuzzy system which its input is s. Input and output membership functions of the fuzzy system is shown in figure 3.

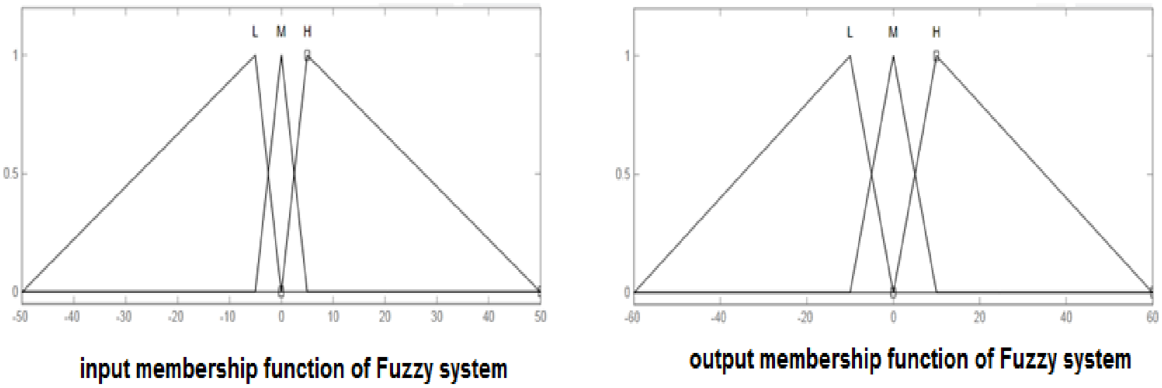

Figure 3: Input and output Membership functions of fuzzy system

For each input and output, three Membership functions are determined as negative (l), zero (m) and positive (h) which have the above values.
Three Fuzzy rules are defined as follows:
If the input is L, the output is also L
Centroid is used for the defuzzification.

3.2. Stability Analysis

By choosing Lyapunov function as $V_{smc} = \frac{s^2(t)}{2}$, reaching condition can be guaranteed as follows:
While $\left|D^{1-q}(\Delta g_i(Y,t))\right|$ and $\left|D^{1-q}(d_i(t))\right|$ are limited (for example $\left|D^{1-q}(\Delta g_i(Y,t))\right| \le \alpha \quad and \quad \left|D^{1-q}(d_i(t))\right| \le \beta$) and as a result:

$$\dot{V}_{smc}(t) = S(t)\dot{S}(t)$$

$$= S.\sum_{j=1}^{3} D^{1-q}(C_1(f_1(X,t) - g_1(Y,\mathrm{t}) - \mathrm{u}_{eq1}(\mathrm{t})) + C_2(f_2(X,t) - g_2(Y,\mathrm{t}) - \mathrm{u}_{eq2}(\mathrm{t}))$$

$$+C_3(f_3(X,t) - g_3 Y,\mathrm{t}) - \mathrm{u}_{eq3}(\mathrm{t}))) \qquad (11)$$

$$= S.\sum_{j=1}^{3} D^{1-q}(-\Delta g_j(Y,t) - d(t) - K_j FSMC(S)).c_j$$

$$\le S\sum_{j=1}^{3}\left\{-K_j FSMC(S) - \alpha_j - \beta_j\right\} \le 0$$

Because we have:

$$\begin{aligned} &if \quad S<0 \quad then \quad FSMC(S)<0 \quad then \quad -K_j FSMC(S) - \alpha_j - \beta_j > 0 \\ &if \quad S>0 \quad then \quad FSMC(S)>0 \quad then \quad -K_j FSMC(S) - \alpha_j - \beta_j < 0 \end{aligned} \qquad (12)$$

Equation (8.3) admits the existence of sliding mode dynamics. Thus, the system is Asymptotically stable.According to equations (6-3) and (7-3), the final control effect of Fuzzy sliding mode controller for fraction degree of the chaotic systems can be introduced by the equation (13):

$$u(t) = f_i(X,t) - g_i(Y,\mathrm{t}) + K_i FSMC(S) \qquad (13)$$

Conclusion

4.1. Describing the Sliding Mode Controller to Synchronize the System

The designed controller for two different chaotic systems Lu and Volta in equation (14) will be as:

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$$u_1(t) = -x_1 - a_1 x_2 - x_3 x_2 - (a_2(y_2 - y_1)) + K_1 FSMC(S)$$
$$u_2(t) = -x_2 - b_1 x_1 - x_3 x_1 - (b_2 y_2 - y_1 y_3) + K_2 FSMC(S) \quad (14)$$
$$u_3(t) = c_1 x_3 + x_1 x_2 + 1 - (-c_2 y_3 + y_1 y_2) + K_3 FSMC(S)$$

4.2. Numerical Results

The simulation results are shown in Figures 4, 5, 6 and 7, while the parameters are selected as $C_1 = C_2 = C_3 = 1,\ K_1 = 5, K_2 = 6, K_3 = 7$.Control signal, sliding surface and synchronize of the X and Y for q = 0.99 is shown in Figure 3.Uncertainty of the system is also considered as $\Delta g_i(Y, t) = 0.5(\sqrt{y_1^2 + y_2^2 + y_3^2})\sin(t)$ and $d_i(t) = 0.5\sin(t)$.

Note that the control is enabled at t = 5s. the importance of the sliding mode control are shown in the simulation results and a quick synchronization between master and slave is obtained. This also proves the strength of the designed controller. It can be seen by figures of 4, 5 and 6 that after starting the commands, the synchronization between two systems are quickly done. This is obvious to see in the errors between two systems that the fuzzy controller was able to operate without any uncertainty and disturbance so that it shows the strengths of the system.

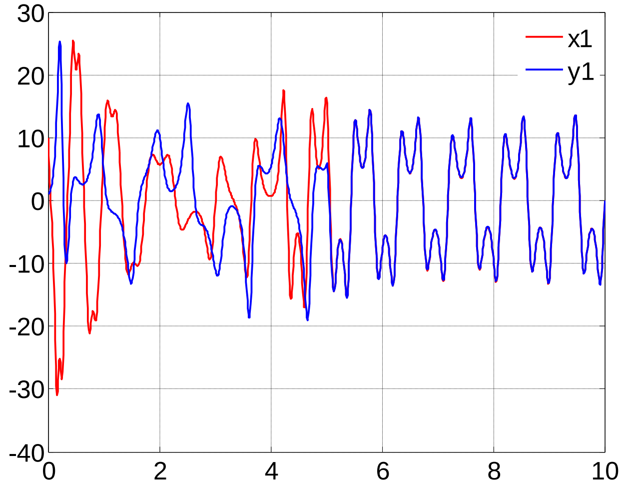

Figure 4: First state diagram of two synchronize systems

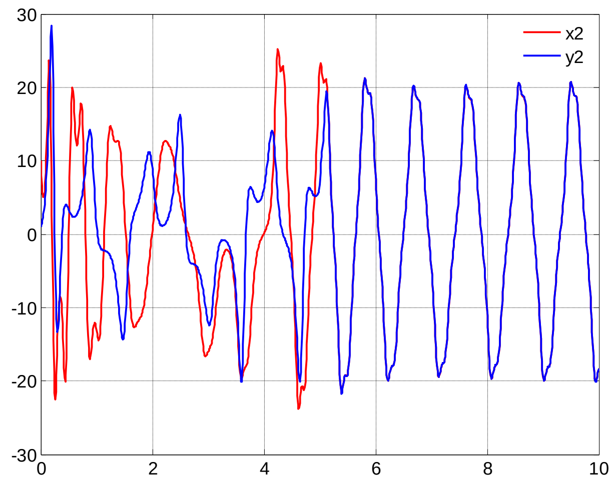

Figure 5: Second state diagram of two synchronize systems

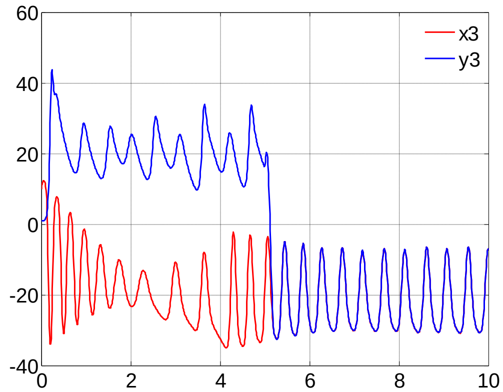

Figure 6: Third state diagram of two synchronize systems

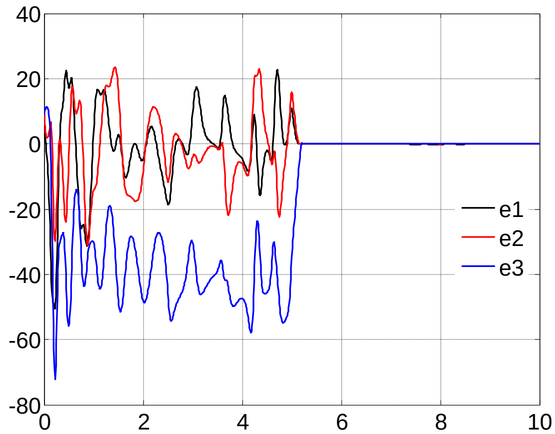

Figure 7: Error diagram of two synchronize systems

CONCLUSION

In this paper, a method for synchronization of two indefinite Volta and Le chaotic systems has been determined. This method is designed based on the degree fraction of fuzzy sliding mode controller. The main advantage of the proposed method is to provide the robustness and stability analysis of the system. Robustness has also been demonstrated by various simulations.

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