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Research Article

FUZZY LINEAR PROGRAMMING AND A MODEL FOR LOCATING NEW FACILITIES AMONG THE EXISTING FACILITIES

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ABSTRACT
In many real life situations, the decision maker may not be in a position to specify the objective and/ or constraint functions precisely but rather can specify them in a "fuzzy sense". In such situations, it is desirable to use some fuzzy linear programming type of modeling. The technique of fuzzy linear programming enlarges the range of applications of the linear programming method. It enables us to consider tolerances for values of decision model parameters in a more natural and direct way. This paper is devoted to fuzzy linear programming and a model for locating some new facilities among the existing facilities in set and gives convex solution, then formulates the model for fuzzy data by an integer linear programming and then solves the model with an example in order to minimize the total cost related to the displacement by assuming step distance among them.

Keywords: *Facility, Linear Fuzzy Programming, Integer Programming*

INTRODUCTION
The problems related to the configuration and locating facilities, are among the problems that has been discussed and analyzed for centuries. Although configuration and locating problems has been considered for many years, but by the advent of operation research, it should be considered more than before.
The configuration and locating facilities means the way of configuring facilities and determining their main position configuration. In other words, locating means the selection of the most desirable place for establishing the facilities optimally on the basis of the identified quantitative and qualitative factors.
For locating, the location models such as mathematical models and qualitative models are used. Generally, the mathematical models are divided into descriptive and prescriptive models. In descriptive model, the model is used for describing the system's behavior, such as the queuing models that are among the descriptive models.
But the application of the prescriptive models is to recommend doing an action that is in some cases optimal. The mathematical programming models are known as prescriptive models.
In mathematical programming models, selecting a suitable criterion and standard is considered. So, the criterion in which the best solution is chosen among the possible solutions, the best criterion is minimizing passed distance. Selecting this solution is in a way that if we minimize passed distances, the transportation costs become minimized. The amount of focus on these criteria was studied by Wolman and Bufa. We can classify the facilities location problems on the basis of the number of new facilities. Also, we can consider the location of new facilities regional and point. In some of the facilities locating problems, the number of new facilities is known as decision variables.
Also, the location of the new facilities may be dependent or independent of the remaining new facilities. If we consider the locations for the new and existing facilities regional, the facilities locating problem is classified as a facilities configuration problem that, in this case, the size and form of the facilities location is known as the decision variables.
The location of facilities may be static or dynamic, or it can be definite. Also, depending on the size of facilities, they consider their location in the form of point or area.
When there is a quantitative and qualitative relation, we should identify the facilities that have this relation. In some cases, the degree of this relation from the places of the facilities is independent.
The experience has proved that, in the final decision about the facilities locating and configuration, we should take into account the qualitative factors with the quantitative results.

In some cases, among the solutions that obtained through solving mathematical model, impractical solutions appear. Such solutions should be explained as criteria that the accepted practical solutions will be measured with them. So, we should see the given models like other models as design tools.
In this research quantitative results are emphasized meanwhile qualification values of facilities locating and configuration problems are not neglected.
So, by formulating the multiple locating mathematical models of facilities for fuzzy data, we could increase the decision power for locating on the basis of different components.
In brief, we can say that the multiple optimized locating of the facilities, gives the most desirable and optimized possible position for the configuration of the new facilities in the set.
In the paper, we offer a model for locating the multiple optimized facilities in the convex set using the fuzzy data assuming the step distance among them.
The remaining of the paper is organized as follows. This introduction is followed by a brief review of the concept of Fuzzy set theory in Section 2. Section 3 sets out a Literature review of Facility Location and Layout.
In section 4, we will have general approach toward a mathematical model for multiple optimized locating of the facilities, and in this approach, we assumed the step distance among facilities. We have defined a mathematical model for the optimized multiple locating of the facilities in the convex set in section 5. In section 6, we extend the mathematical model of multiple optimized locating of facilities in the convex set for the fuzzy data.

Fuzzy Set Theory

Fuzzy set theory has been studied extensively over the past 30 years. Most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive processes (see for example Zadeh, 1965). Fuzzy set theory is now applied to problems in engineering, business, medical and related health sciences, and the natural sciences.
In an effort to gain a better understanding of the use of fuzzy set theory and to provide a basis for future research, a literature review of fuzzy set theory has been conducted.
Over the years there have been successful applications and implementations of fuzzy set theory. Fuzzy set theory is being recognized as an important problem modeling and solution technique. A summary of the findings of fuzzy set theory may benefit researchers.
Kaufmann and Gupta (1988) report that over 7,000 research papers, reports, monographs, and books on fuzzy set theory and applications have been published since 1965. Table 1 provides a summary of selected bibliographies on fuzzy set theory and applications. The objective of Table 1 is not to identify every bibliography and extended review of fuzzy set theory, rather it is intended to provide the reader with a starting point for investigating the literature on fuzzy set theory.
The bibliographies encompass journals, books, edited volumes, conference proceedings, monographs, and theses from 1965 to 1994. The bibliographies compiled by Gaines and Kohout (1977), Kandel and Yager (1979), Kandel (1986), and Kaufmann and Gupta (1988) address fuzzy set theory and applications in general. The bibliographies by Zimmerman (1983) and Lai and Hwang (1994) review the literature on fuzzy sets in operations research and fuzzy multiple objective decision making respectively. Maiers and Sherif (1985) review the literature on fuzzy industrial controllers and provide an index of applications of fuzzy set theory to twelve subject areas including decision making, economics, engineering and operations research.
As evidenced by the large number of citations found in Table 1, fuzzy set theory is an established and growing research discipline. The use of fuzzy set theory as a methodology for modeling and analyzing decision systems is of particular interest to researchers due to fuzzy set theory's ability to quantitatively and qualitatively model problems which involve vagueness and imprecision. Karwowski and Evans (1986) identify the potential applications of fuzzy set theory to the following areas: new product development, facilities location and layout, production scheduling and control, inventory management, quality and cost benefit analysis. Karwowski and Evans identify three key reasons why fuzzy set theory is relevant to production management research.

Table 1: Selected Bibliographies of Fuzzy Set Theo

Reference Author(s)	Number of reference citations
Gaines and Kohout (1977)	763 (with 401 additional on topics closely related to fuzzy systems theory)
Kandel and Yager (1979)	1799
Zimmerman (1983)	54 (emphasis on fuzzy sets in operations research)
Maiers and Sherif (1985)	450 (emphasis on fuzzy sets and industrial controllers)
Kandel (1986)	952
Kaufmann and Gupta (1988)	220
Lai and Hwang (1994)	695 (emphasis on fuzzy multiple objective decision making)

First, imprecision and vagueness are inherent to the decision maker's mental model of the problem under study. Thus, the decision maker's experience and judgment may be used to complement established theories to foster a better understanding of the problem. Second, in the production management environment, the information required to formulate a model's objective, decision variables, constraints and parameters may be vague or not precisely measurable.

Third, imprecision and vagueness as a result of personal bias and subjective opinion may further dampen the quality and quantity of available information. Hence, fuzzy set theory can be used to bridge modeling gaps in descriptive and prescriptive decision models in production management research

Evans *et al.*, (1987) introduce a fuzzy set theory based construction heuristic for solving the block layout design problem. Qualitative layout design inputs of 'closeness' and 'importance' are modeled using linguistic variables. The solution algorithm selects the order of department placement which is manual. The algorithm is demonstrated by determining a layout for a six department metal fabrication shop. The authors identify the need for future research toward the development of a heuristic that address both the order and placement of departments, the selection of values for the linguistic variables, and the determination of membership functions.

Raoot and Rakshit (1991) present a fuzzy layout construction algorithm to solve the facility layout problem. Linguistic variables are used in the heuristic to describe qualitative and quantitative factors that affect the layout decision. Linguistic variables capture information collected from experts for the following factors: flow relationships, control relationships, process and service relationships, organizational and personnel relationships, and environmental relationships. Distance is also modeled as a fuzzy variable and is used by the heuristic as the basis for placement of departments. Three test problems are used to compare the fuzzy heuristic with ALDEP and CORELAP. The authors note that the differences achieved by each of the three methods are a function of the different levels of reality that they use.

Raoot and Rakshit (1993) formulate the problem of evaluating alternative facility layouts as a multiple criteria decision model (MCDM) employing fuzzy set theory. The formulation addresses the layout problem in which qualitative and quantitative factors are equally important. Linguistic variables are used to capture experts' opinions regarding the primary relationships between departments. Membership functions are selected based on consultation with layout experts. The multiple objectives and constraints of the formulation are expressed as linguistic patterns. The fuzzy MCDM layout algorithm is demonstrated for the layout of an eight department facility.

Raoot and Rakshit (1994) present a fuzzy set theory-based heuristic for the multiple goal quadratic assignment problem (QAP). The objective function in this formulation utilizes 'the mean truth value', which indicates the level of satisfaction of a layout arrangement to the requirements of the layout as dictated by a quantitative or qualitative goal. The basic inputs to the model are expert's opinions on the qualitative and quantitative relationships between pairs of facilities. The qualitative and quantitative relationships are captured by linguistic variables, membership 15 functions are chosen arbitrarily. Three linguistic patterns (one quantitative; two qualitative) are employed by the heuristic to locate facilities. The performance of the heuristic is tested against a set of test problems taken from the open literature. The results of the comparison indicate that the proposed fuzzy heuristic performs well in terms of the quality of the solution.

Dweiri and Meier (1996) define a fuzzy decision making system (FDMS) consisting of four principal components: (i) fuzzification of input and output variables; (ii) the experts' knowledge base; (iii) fuzzy decision making; and (iv) defuzzification of fuzzy output into crisp values. The analytical hierarchy process is used to weight factors affecting closeness ratings between departments. A computer program based on FDMS then generates activity relationship charts which, in turn, are developed into layouts by FZYCRLP (Fuzzy Computer Relationship Layout Planning - a modified version of CORELAP). Simulation is used to compare layouts generated under FZYCRLP and CORELAP for a set of twelve test problems involving layouts ranging in size from seven to seventeen departments. Layouts generated under FZYCRLP performed well under fuzzy and non-fuzzy evaluation metrics.

Many of the factors affecting facility layout and location problems are difficult to precisely measure and therefore require considerable human judgment. Closeness measures are a key input in nearly all facility layout models and are often determined in the form of closeness ratings that are described by degree of importance in linguistics terms such as 'absolutely necessary', 'very important', and 'undesirable'. Subjective weights are often used in conjunction with closeness measures when utilizing a scoring criterion to determine the layout of departments in a facility. Fuzzy set theory effectively models the linguistic aspects of specifying closeness measures and the subjectivities involved when specifying closeness weights. Facility location models may also require the determination of subjective factor weights to measure the relative importance of various factors influencing the location decision. Single and multiple criteria optimization procedures are frequently used in modeling facility location problems. Fuzzy set theory allows subjectivity in the parameters of these models to be incorporated into the model formulation and solution.

Facility Location and Layout Literature

The problems of facility location and layout have been studied extensively in the engineering literature.

Narasimhan (1979) presents an application of fuzzy set theory to the problem of locating gas stations. Fuzzy ratings are used to describe the relative importance of eleven attributes for a set of three location alternatives. A Delphi-based procedure was applied, and the input of decision makers was used to construct membership functions for three importance weights for judging attributes. Computations are summarized for the selection decision. The author concludes that the procedure presented is congruent to the way people make decisions. The procedure provides a structure for organizing information, and a systematic approach to the evaluation of imprecise and unreliable information.

Darzentas (1987) formulates the facility location problem as a fuzzy set partitioning model using integer programming. This model is applicable when the potential facility points are not crisp and can best be described by fuzzy sets. Linear membership functions are employed in the objective function and constraints of the model. The model is illustrated with an example based on three location points and four covers.

Mital *et al.,* (1987) and Mital and Karwowski (1989) apply fuzzy set theory in quantifying eight subjective factors in a case study involving the location of a manufacturing plant. Linguistic descriptors are used to describe qualitative factors in the location decision, such as community attitude, quality of schools, climate, union attitude, nearness to market, police protection, fire protection, and closeness to port.

Bhattacharya *et al.,* (1992) present a fuzzy goal programming model for locating a single facility within a given convex region subject to the simultaneous consideration of three criteria: (i) maximizing the minimum distances from the facility to the demand points; (ii) minimizing the maximum distances from the facilities to the demand points; and (iii) minimizing the sum of all transportation costs. Rectilinear distances are used under the assumption that an urban scenario is under investigation. A numerical example consisting with three demand points is given to illustrate the solution procedure.

Chung and Tcha (1992) address the location of public supply-demand distribution systems such as a water supply facility or a waste disposal facility. Typically, the location decision in these environments is made subject to the conflicting goals minimization of expenditures and the preference at each demand site to maximizing the amount supplied. A fuzzy mixed 0-1 mathematical programming model is formulated to study both uncapacitated and capacitated modeling scenarios. The objective function includes the cost of transportation and the fixed cost for satisfying demand at each site. Each cost is represented by a linear membership function. Computational results for twelve sample problems are demonstrated for a solution heuristic based on Erlenkotter's dual-based procedure for the uncapacitated facility location problem. Extension to the capacitated case is limited by issues of computational complexity and computational results are not presented.

Bhattacharya *et al.,* (1993) formulate a fuzzy goal programming model for locating a single facility within a given convex region subject to the simultaneous consideration of two criteria: (i) minimize the sum of all transportation costs; and (ii) minimize the maximum distances from the facilities to the demand points. Details and assumptions of the model are similar to Bhattacharya *et al.,* (1992). A numerical example consisting of two facilities and three demand points is presented and solved using LINDO.

Grobelny (1987a, 1987b) incorporates the use of 'linguistic patterns' in solving the facility layout problem. Linguistic patterns are statements, based on the fuzzy aggregated opinions of experts, which can be used as recommendations when solving a layout problem and as criteria for evaluating an existing algorithm. For example, if the flow of materials between departments is high, then the departments should be located close to each other. The linking between the departments and the distance between the departments represent linguistic (fuzzy) variables; the 'high' and 'close' qualifications represent values of the linguistic variables. The evaluation of a layout is measured as the grade of satisfaction as measured by the mean truth value, of each linguistic pattern by the final placement of departments. Both the 1987a and 1987b models are construction type algorithms based on a modification of Hillier and Conner's HC-66 layout algorithm.

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Mathematical Model of the Multiple Optimized Locating of the Facilities

Let the points $P_1, P_2, \ldots, P_m$, be the places of m facilities in the workshop, so the coordinate of these points will be:

$P_1 = (a_1, b_1)$, $P_2 = (a_2, b_2), \ldots, P_m = (a_m, b_m)$

In other words,

$P_i = (a_i, b_i)$ ؛ $i = 1, 2, \ldots, m$

If we want to establish n new facilities in the workshop by the purpose of minimizing the total transportation cost between all of the workshop facilities in the points of $Z_1, \ldots, Z_n$, so we can obtain the best coordinate of the point with the mathematical model of multiple optimized locating of the facilities.

Introduction of the parameters:

i : the existing facilities index

j, k : new facilities index

m: the number of the existing facilities

n: the number of the new facilities

$P_i = (a_i, b_i)$: the coordinate of the location of new facilities

$Z_j = (x_j, y_j)$ = the coordinate of the location of j new facility

$Z_k = (x_k, y_k)$ = the coordinate of the k new facility

w_{ij} = annual cost of transportation per unit traveled distance between i^{th} existing facility and j^{th} new facility.

V_{jk} = annual transportation cost per unit traveled distance between j^{th} existing facility and k^{th} new facility

We should explain that between the existing facilities with new facilities and among the new facilities, the flow exchange exists that creates cost. So, in this locating, $V_{jk} \neq 0$ and $j \neq k$.

$\mathrm{f}(Z_1,\ldots,Z_n)$: the total annual cost of transportation according to the location of new facilities $Z_n,\ldots,Z_1$.

The model definition: $\mathrm{f}(Z_1,\ldots,Z_n) = \sum_{1\le j<k\le n} V_{jk} d(Z_j, Z_k) + \sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij} d(Z_j, P_i)$

In this model, the total annual cost of transportation is the sum of annual cost of transportation among new facilities and the annual transportation cost between new facilities with the existing facilities.

Since we assume that the binary distance among each of facilities follows a step pattern, so the model would be as follows:

$$\mathrm{f}(Z_1,\ldots,Z_n) = \sum_{1\le j<k\le n} V_{jk}\left\{|x_j - x_k| + |y_j - y_k|\right\} + \sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij}\left\{|x_j - a_i| + |y_j - b_i|\right\}$$

The purpose of this problem is minimizing the total transportation cost between workshop facilities. So we have:

$$\min \mathrm{f}(Z_1,\ldots,Z_n) = \min \sum_{1\le j<k\le n} V_{jk}\left\{|x_j - x_k| + |y_j - y_k|\right\} + \min \sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij}\left\{|x_j - a_i| + |y_j - b_i|\right\}$$

In this function, since the variables x and y are independent, so we can divide the total annual transportation cost into two independent elements:

$$\min \mathrm{f}(Z_1,\ldots,Z_n) = \min \mathrm{f}_1(x_1,\ldots x_n) + \min \mathrm{f}_2(y_1,\ldots y_n)$$

In this relation, $\mathrm{f}_1(x_1,\ldots,x_n)$ and $\mathrm{f}_2(y_1,\ldots,y_n)$ are the first and second elementsof the material flow exchange, respectively.

$$\min \mathrm{f}_1(x_1,\ldots,x_n) = \min \sum_{1\le j<k\le n} V_{jk}|x_j - x_k| + \min \sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij}|x_j - a_i|$$

,

$$\min \mathrm{f}_2(y_1,\ldots,y_n) = \min \sum_{1\le j<k\le n} V_{jk}|y_j - y_k| + \min \sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij}|y_j - b_i|$$

Solving the Model:

The defined target function is a nonlinear target function. So, for solving it, it is necessary to convert it into linear. For converting nonlinear model into linear model, it is necessary to know this principle:

By assuming the variables q, p, b, a, and if $a - b - p + q = 0$ and $p \geq 0$ and $q \geq 0$ and pq=0, so $|a - b| = p + q$

Minimizing f_1 is equivalent to the following linear planning problem:

$$\min \mathrm{f}_1(x_1,\ldots,x_n) = \sum_{1\le j<k\le n} V_{jk}(p_{jk} + q_{jk}) + \sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij}(r_{ij} + s_{ij})$$

$$S.t.$$
$$x_j - x_k - p_{jk} + q_{jk} = 0$$
$$x_j - r_{ij} + s_{ij} = a_i$$
$$x_j, p_{jk}, q_{jk}, r_{ij}, s_{ij} \ge 0$$
$$\begin{cases} i = 1,\dots,m \\ j = 1,\dots,n \\ 1 \le j < k \le n \end{cases}$$

Minimizing f_2 is equivalent to the following linear planning problem:

$$\min f_2(y_1,\dots,y_n) = \sum_{1\le j<k\le n} V_{jk}(p'_{jk} + q'_{jk}) + \sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij}(r'_{ij} + s'_{ij})$$
$$S.t.$$
$$y_j - y_k - p'_{jk} + q'_{jk} = 0$$
$$y_j - r'_{ij} + s'_{ij} = b_i$$
$$y_j, p'_{jk}, q'_{jk}, r'_{ij}, s'_{ij} \ge 0$$
$$\begin{cases} i = 1,\dots,m \\ j = 1,\dots,n \\ 1 \le j < k \le n \end{cases}$$

So, we can solve the obtained independent linear planning problem by the simplex algorithm method and obtain the optimal solution which is the optimal coordinate of the location of new facilities. The use of linear planning for solving the problem of optimal locating for multi facilities assures that the optimal obtained solutions of the problem, minimizes the target function.

Mathematical Model of Multiple Optimal Locating of Facilities in the Convex Set Using the Fuzzy Data

Here, we extend the mathematical model obtained in the previous section for fuzzy data:

Assume that we have a set of L facility in the workshop that all of them occupy the same space. The coordination of the place of these existing facilities in the workshop is presented as follows:

$$(\tilde{x}_t, \tilde{y}_t) \quad ، \quad t = 1,\dots,L$$

Also, assume that we divide the remaining workshop space into M convex point (it should be noted that we assumed the convex regions to be like rectangle).

We display these convex regions with O_t and define it as follows:

$$O_t = \left\{(x,y)\,\middle|\, \tilde{a}_t \precsim x \precsim \tilde{b}_t \ ,\ \tilde{c}_t \precsim y \precsim \tilde{d}_t\right\} \qquad for \quad t = 1,\dots,M$$

Here, the numbers $\tilde{d}_t, \tilde{c}_t, \tilde{b}_t, \tilde{a}_t$, and (x, y) are fuzzy numbers.

Now, we want to find points as the place of optimal new facilities in M convex region for placing n new facility, so that the target function becomes minimized.

To this end, we define the following nonlinear mathematical model:

$$\min d = \sum_{1\le j<k\le n}\left\{\left|\tilde{x}_j + \tilde{x}_k\right| + \left|\tilde{y}_j + \tilde{y}_k\right|\right\} + \sum_{t=1}^{L}\sum_{j=1}^{n}\left\{\left|\tilde{x}_j - \tilde{x}_t\right| + \left|\tilde{y}_j - \tilde{y}_t\right|\right\}$$

$S.t.$

$$\begin{cases} \tilde{a}_1 \tilde{\leq} \tilde{x}_j \tilde{\leq} \tilde{b}_1 & , \quad \tilde{c}_1 \tilde{\leq} \tilde{y}_j \tilde{\leq} \tilde{d}_1 \\ \tilde{a}_1 \tilde{\leq} \tilde{x}_k \tilde{\leq} \tilde{b}_1 & , \quad \tilde{c}_1 \tilde{\leq} \tilde{y}_k \tilde{\leq} \tilde{d}_1 \end{cases}$$

or

$$\begin{cases} \tilde{a}_2 \tilde{\leq} \tilde{x}_j \tilde{\leq} \tilde{b}_2 & , \quad \tilde{c}_2 \tilde{\leq} \tilde{y}_j \tilde{\leq} \tilde{d}_2 \\ \tilde{a}_2 \tilde{\leq} \tilde{x}_k \tilde{\leq} \tilde{b}_2 & , \quad \tilde{c}_2 \tilde{\leq} \tilde{y}_k \tilde{\leq} \tilde{d}_2 \end{cases}$$

or

$\vdots$

$$\begin{cases} \tilde{a}_M \tilde{\leq} \tilde{x}_j \tilde{\leq} \tilde{b}_M & , \quad \tilde{c}_M \tilde{\leq} \tilde{y}_j \tilde{\leq} \tilde{d}_M \\ \tilde{a}_M \tilde{\leq} \tilde{x}_k \tilde{\leq} \tilde{b}_M & , \quad \tilde{c}_M \tilde{\leq} \tilde{y}_k \tilde{\leq} \tilde{d}_M \end{cases}$$

Now, like the reason given to the previous section, we change the current mathematical model to the following form:

$$\min d = \sum_{1 \leq j < k \leq n} \left\{ (\tilde{p}_{jk} + \tilde{q}_{jk}) + (\tilde{p}'_{jk} + \tilde{q}'_{jk}) \right\} + \sum_{t=1}^{L} \sum_{j=1}^{n} \left\{ (\tilde{r}_{tj} + \tilde{s}_{tj}) + (\tilde{r}'_{tj} + \tilde{s}'_{tj}) \right\}$$

$S.t.$

$$\begin{array}{ll} \tilde{x}_j \tilde{\leq} \tilde{b}_u + \tilde{M}\alpha_u \quad , & \tilde{x}_k \tilde{\leq} \tilde{b}_u + \tilde{M}\alpha_u \\ \tilde{x}_j \tilde{\geq} \tilde{a}_u - \tilde{M}\alpha_u \quad , & \tilde{x}_k \tilde{\geq} \tilde{a}_u - \tilde{M}\alpha_u \\ \tilde{y}_j \tilde{\leq} \tilde{d}_u + \tilde{M}\alpha_u \quad , & \tilde{y}_k \tilde{\leq} \tilde{d}_u + \tilde{M}\alpha_u \\ \tilde{y}_j \tilde{\geq} \tilde{c}_u - \tilde{M}\alpha_u \quad , & \tilde{y}_k \tilde{\geq} \tilde{c}_u - \tilde{M}\alpha_u \end{array}$$

$$\begin{cases} \tilde{x}_j - \tilde{x}_k = \tilde{p}_{jk} - \tilde{q}_{jk} & , \quad \tilde{y}_j - \tilde{y}_k = \tilde{p}'_{jk} - \tilde{q}'_{jk} \\ \tilde{x}_j - \tilde{x}_t = \tilde{r}_{tj} - \tilde{s}_{tj} & , \quad \tilde{y}_j - \tilde{y}_t = \tilde{r}'_{tj} - \tilde{s}'_{tj} \end{cases}$$

$$\tilde{p}_{jk}, \tilde{q}_{jk}, \tilde{p}'_{jk}, \tilde{q}'_{jk}, \tilde{r}_{tj}, \tilde{s}_{tj}, \tilde{r}'_{tj}, \tilde{s}'_{tj} \tilde{\geq} 0$$

$$\alpha_1 + \alpha_2 + \cdots + \alpha_M = M - 1$$
$$\alpha_u \in \{0,1\} \ ; \ u = 1, \cdots, M$$

$$\begin{cases} t = 1, \cdots, L \\ j = 1, \cdots, n \\ 1 \leq j < k \leq n \end{cases}$$

The current linear mathematical model can be converted into the following standard form:

$$\min d = \sum_{1 \leq j < k \leq n} \left\{ (\tilde{p}_{jk} + \tilde{q}_{jk}) + (\tilde{p}'_{jk} + \tilde{q}'_{jk}) \right\} + \sum_{t=1}^{L} \sum_{j=1}^{n} \left\{ (\tilde{r}_{tj} + \tilde{s}_{tj}) + (\tilde{r}'_{tj} + \tilde{s}'_{tj}) \right\}$$

$$S.t.$$

$$\tilde{x}_j + \tilde{e}_u - \tilde{M}\alpha_u = \tilde{b}_u \quad , \quad \tilde{x}_k + \tilde{e}'_u - \tilde{M}\alpha_u = \tilde{b}_u$$

$$\tilde{x}_j - \tilde{f}_u + \tilde{M}\alpha_u = \tilde{a}_u \quad , \quad \tilde{x}_k - \tilde{f}'_u + \tilde{M}\alpha_u = \tilde{a}_u$$

$$\tilde{y}_j + \tilde{g}_u - \tilde{M}\alpha_u = \tilde{d}_u \quad , \quad \tilde{y}_k + \tilde{g}'_u - \tilde{M}\alpha_u = \tilde{d}_u$$

$$\tilde{y}_j - \tilde{h}_u + \tilde{M}\alpha_u = \tilde{c}_u \quad , \quad \tilde{y}_K - \tilde{h}'_u + \tilde{M}\alpha_u = \tilde{c}_u$$

$$\begin{cases} \tilde{x}_j - \tilde{p}_{jk} + \tilde{q}_{jk} = \tilde{x}_k \quad , \quad \tilde{y}_j - \tilde{p}'_{jk} + \tilde{q}'_{jk} = \tilde{y}_k \\ \tilde{x}_j - \tilde{r}_{tj} + \tilde{s}_{tj} = \tilde{x}_t \quad , \quad \tilde{y}_j - \tilde{r}'_{tj} + \tilde{s}'_{tj} = \tilde{y}_t \end{cases}$$

$$\tilde{p}_{jk} , \tilde{q}_{jk} , \tilde{p}'_{jk} , \tilde{q}'_{jk} , \tilde{r}_{tj} , \tilde{s}_{tj} , \tilde{r}'_{tj} , \tilde{s}'_{\tilde{t}j} \geq 0$$

$$\tilde{e}_u , \tilde{e}'_u , \tilde{f}_u , \tilde{f}'_u , \tilde{g}_u , \tilde{g}'_u , \tilde{h}_u , \tilde{h}'_u \gtrsim 0$$

$$\alpha_1 + \alpha_2 + \cdots + \alpha_M = M - 1$$

$$\alpha_u \in \{0,1\} \text{ ؛ } u = 1, \cdots, M$$

$$\begin{cases} t = 1, \cdots, L \\ j = 1, \cdots, n \\ 1 \leq j < k \leq n \end{cases}$$

CONCLUSION

In this paper, we have formulated the multiple optimal locating mathematical models of the facilities using linear integer programming. Then, we extended it for the fuzzy data and finally, we defined the fuzzy multiple optimal locating mathematical model in a convex set.

It seems that it is important to design this mathematical model for optimal locating the new facilities among existing facilities layout. Because in the real world, just the decision component in establishing new facilities cannot be distance between facilities.

So, solving the fuzzy multiple optimal locating mathematical model in the convex model, gives us acceptable and desirable limit in order to decide for the location of new facilities, that undoubtedly, the fuzzy thought helps us in more actualizing common multiple locating mathematical model of the facilities.

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