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Research Article

SPEED CONTROL OF PERMANENT MAGNET SYNCHRONOUS MOTOR USING FEEDBACK LINEARIZATION METHOD

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ABSTRACT
In this paper, speed control of permanent magnet synchronous motor is studied using feedback linearization method. Regarding non-linear behavior of permanent magnet synchronous motor, utilizing feedback linearization method which is a non-linear control method would improve the performance of this motor. In fact, an appropriate controller is designed for such system using input-output feedback linearization method. In order to show the superiority of the designed controller, it is compared with a Fuzzy Self-adapting PID controller. The results of comparison show performance improvement of the system.

Keywords: *Permanent Magnet Synchronous Motor, Feedback Linearization Control, Using Input-Output Linearization, Fuzzy Self-adapting PID*

INTRODUCTION
Application of DC motors is restricted due to corrosion between brushes and commutators, lack of overload capability, lack of robustness, low torque and small range of speed. Therefore, AC motors are paid much attention despite expensive and complicated controllers.
In general, there are two kinds of permanent magnet AC motors: permanent magnet synchronous motor (PMSM) and brushless direct current motor (BDCM). It is cited that PMSM has better performance with respect to other AC machines. As placing permanent magnets on rotor provides a situation that there is no need to supply magnetizing current for producing constant air gap flux and the required stator current is only consumed for producing electromagnetic torque, permanent magnet motors are preferred over Dc and induction motors in recent years specially in this decade due to inherent characteristics such as high power density, low inertia, high power factor and efficiency.
Systems with linear controllers operate only around equilibrium point and as pulsations and disturbances that are imposed to the motor cause to severe current variations and consequently speed variations, it is always needed to use a controllers so that the motor is kept robust against severe and sudden state variables. Therefore, non-linear controllers are needed to control non-linear system of permanent magnet synchronous motor (Xiao, 2013; Nakai *et al.*, 2005).
In recent years, many non-linear methods are proposed to control permanent magnet synchronous motor such as adaptive method, sliding-adaptive method and also Fuzzy-adaptive method that are identical with feedback linearization method in terms of robustness and stability of state variables but they have much more complicated algorithm comparing to feedback linearization method. Therefore, feedback linearization method is one of the best controllers that is investigated in this paper.
Feedback linearization method is well used in applications like helicopter, high performance air-craft and industrial robots. This method is divided into two categories: linearization of input-state and linearization of input-output (Nakai *et al.*, 2005; Wei *et al.*, 2006; Stumberger and Hamler, 2001).
The aim of input-state linearization is stability of system which is done by obtaining a proper control rule in order to tend the state variables to zero in non-linear systems. In this method, non-linear dynamic of system is transformed into a linear system but it is possible that there would not be a non-linear transformation. In input-output linearization it is needed that non-linear output would track the desired

input and the tracking error is tend toward zero and all the state variables are limited. In this method, relative degree of system has to be calculated and in some specific systems this relative degree may not be defined. Therefore, selecting between methods depend on the model of non-linear system (Kaddouri *et al.*, 1994).

The paper is organized as follow: state space of permanent magnet synchronous motor is presented in section two. Design of feedback linearization controller with input-output method is discussed in section three and in the next section comparison is done between the designed controller and Fuzzy Self-adapting PID controller.

Modeling of Permanent Magnet Synchronous Motor

In this paper, the model for permanent magnet synchronous motor is as follow (1).

$$\begin{cases} \dot{I}_d = -\frac{R}{L_d} I_d + n_p \frac{L_q}{L_d} w_m I_q + \frac{1}{L_d} U_d \\ \dot{I}_q = -n_p \frac{L_d}{L_q} I_d w_m - \frac{R}{L_q} I_q - n_p \frac{\lambda_{PM}}{L_q} w_m + \frac{1}{L_q} U_q \\ \dot{W}_m = \frac{3n_p}{2J}\left(\lambda_{PM} I_q + (\mathrm{L}_d - \mathrm{L}_q) I_d I_q\right) - \frac{B_m}{J} w_m - \frac{T_L}{J} \end{cases} \tag{1}$$

Table 1 presents the parameters of the system.

Dynamic of the system can be illustrated as affine according to equation (2) (Xiao, 2013).

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{2}$$

where $x \in R^{3\times1}; u \in R^{3\times1}, f \in R^{3\times1}, g = \begin{bmatrix} g_1 & g_2 & g_3 \end{bmatrix}; g_i \in R^{3\times1}$. f and g are equl to (3).

Table 1: Description of the motor parameters (Kaddouri *et al.*, 1994)

Parameter	**Definition**	**Unit**
R	Stator resistance	Ω
L_d	d-axis stator inductance	mH
L_q	q-axis stator inductance	mH
J	Moment of inertia	$kg.m^2$
n_p	Number of rotor pole-pairs	
λ_{PM}	magnetic flux of Permanent magnet	Wb
B_m	Friction factor	$Nm.s/rad$
i_d	d-axis stator current	A
i_q	q-axis stator current	A
u_d	d-axis stator voltage	V
u_q	q-axis stator voltage	V
T_L	Load torque	Nm
Wm	Mechanical angular speed of rotor	rpm

$$f=\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix};\ \begin{cases} f_1=-\dfrac{R}{L_d}x_1+n_p\dfrac{L_q}{L_d}x_2x_3 \\ f_2=-n_p\dfrac{L_d}{L_q}x_1x_3-\dfrac{R}{L_q}x_2-n_p\dfrac{\lambda_{PM}}{L_q}x_3 \\ f_3=\dfrac{3n_p}{2J}\left(\lambda_{PM}x_2+(\mathrm{L}_d-\mathrm{L}_q)\mathrm{x}_1x_2\right)-\dfrac{B_m}{J}x_3 \end{cases};$$

$$g_1=\begin{bmatrix} \dfrac{1}{L_d} \\ 0 \\ 0 \end{bmatrix};\ g_2=\begin{bmatrix} 0 \\ \dfrac{1}{L_q} \\ 0 \end{bmatrix};\ g_3=\begin{bmatrix} 0 \\ 0 \\ \dfrac{-1}{J} \end{bmatrix} \tag{3}$$

Input and outputs of the system are

$$y=h(x)=x=\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}=\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} i_d \\ i_q \\ w_m \end{bmatrix};$$

$$u=\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}=\begin{bmatrix} u_d \\ u_q \\ T_L \end{bmatrix} \tag{4}$$

As shown in abovementioned equations, permanent magnet synchronous motor is a multiinput-multioutput system.

Design of Feedback Linearization Controller using Input-output Method

Feedback linearization controller is designed through the following three steps (Kaddouri *et al.*, 1994; Slotine and Li, 1991).

1. Obtaining Relative Degree

First, relative degree of each outputs of the system has to be obtained. For this purpose, outputs are differentiates to a certain rank so that an explicit relation is achieved between input and output.

$$y_1=x_1\rightarrow \dot{y}_1=\dot{x}_1=f_1+\frac{1}{L_d}u_d\rightarrow r_1=1;$$

$$y_2=x_2\rightarrow \dot{y}_2=\dot{x}_2=f_2+\frac{1}{L_q}u_q\rightarrow r_2=1;$$

$$y_3=x_3\rightarrow \dot{y}_3=\dot{x}_3=f_3-\frac{1}{J}T_L\rightarrow r_3=1;$$

It is shown that the system inputs are obtained after one time differentiation. Therefore, relative degree of outputs is equal to one.

2. Design of Linear Control Signal

If the relative degree of the system is equal to one in a single input-single output, signal v is selected according to equation (5) and if the relative degree is equal to two, this signal is selected according to equation (6). Therefore, higher order differentiations of the desired output and error are used in design of signal v with increase in relative degree of system.

$$v=\dot{y}_d-k_0e \tag{5}$$

$$v=\ddot{y}_d-k_0e-k_1\dot{e} \tag{6}$$

Therefore, signal v is selected in order to stabilize the dynamic of the system as follow:

$$\dot{y} = v \rightarrow \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \dot{y}_{1d} - k_1 e_1 \\ \dot{y}_{2d} - k_2 e_2 \\ \dot{y}_{3d} - k_3 e_3 \end{bmatrix}; \quad (7)$$

$$\begin{cases} e_1 = y_1 - y_{1d} \\ e_2 = y_2 - y_{2d} \\ e_3 = y_3 - y_{3d} \end{cases}$$

By selecting the above rule, dynamic of the system error would be equal to

$$\begin{cases} \dot{y}_1 = \dot{y}_{1d} - k_1 e_1 \\ \dot{y}_2 = \dot{y}_{2d} - k_2 e_2 \\ \dot{y}_3 = \dot{y}_{3d} - k_3 e_3 \end{cases} \rightarrow \begin{cases} \dot{y}_1 - \dot{y}_{1d} + k_1 e_1 = 0 \\ \dot{y}_2 - \dot{y}_{2d} + k_2 e_2 = 0 \rightarrow \\ \dot{y}_3 - \dot{y}_{3d} + k_3 e_3 = 0 \end{cases} \quad (8)$$

$$\begin{cases} \dot{e}_1 + k_1 e_1 = 0 \\ \dot{e}_2 + k_2 e_2 = 0 \\ \dot{e}_3 + k_3 e_3 = 0 \end{cases}$$

With selecting positive values for k_1, k_2, k_3, the above dynamic is stable and error tends toward zero. It has to be noted that in this compensation, error tends toward zero with rank one dynamic i.e. without overshoot.

3. Design of Non-Linear Control Signal

Control equation for linearization of system dynamic in input-output method is as follow

$$v = \alpha(x) + \beta(x)u \quad (9)$$

$$\alpha(x) = \begin{bmatrix} L_f^{r_1}(h_1) \\ L_f^{r_2}(h_2) \\ L_f^{r_3}(h_3) \end{bmatrix};$$

$$\beta(x) = \begin{bmatrix} L_{g_1}(\mathrm{L}_f^{r_1-1}\mathrm{h}_1) & L_{g_2}(\mathrm{L}_f^{r_1-1}\mathrm{h}_1) & L_{g_3}(\mathrm{L}_f^{r_1-1}\mathrm{h}_1) \\ L_{g_1}(\mathrm{L}_f^{r_2-1}\mathrm{h}_2) & L_{g_2}(\mathrm{L}_f^{r_2-1}\mathrm{h}_2) & L_{g_3}(\mathrm{L}_f^{r_2-1}\mathrm{h}_2) \\ L_{g_1}(\mathrm{L}_f^{r_3-1}\mathrm{h}_3) & L_{g_2}(\mathrm{L}_f^{r_3-1}\mathrm{h}_3) & L_{g_3}(\mathrm{L}_f^{r_3-1}\mathrm{h}_3) \end{bmatrix};$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

In order to calculate the matrix, r=1 and differentiation of each array are obtained.

The obtained values are expressed in equation (10).

$$\alpha = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix}; \beta = \begin{bmatrix} \frac{1}{L_d} & 0 & 0 \\ 0 & \frac{1}{L_q} & 0 \\ 0 & 0 & -\frac{1}{J} \end{bmatrix} \quad (10)$$

Therefore, the control signal imposed to the process is equal to

$$u = \beta^{-1}(v - \alpha) \quad (11)$$

$$\begin{cases} u_1 = L_d\left(v_1 - f_1(x)\right) \\ u_2 = L_q\left(v_2 - f_2(x)\right) \rightarrow \\ u_3 = -J\left(v_3 - f_3(x)\right) \end{cases} \tag{12}$$

$$\begin{cases} u_1 = L_d\left(\dot{y}_{1d} - k_1 e_1 - f_1(x)\right) \\ u_2 = L_q\left(\dot{y}_{2d} - k_2 e_2 - f_2(x)\right) \\ u_3 = -J\left(\dot{y}_{3d} - k_3 e_3 - f_3(x)\right) \end{cases}$$

Control signal v and u are designed by carrying out the above steps and the process for speed control is formed by having state space model of motor according to block-diagram of Figure 1. Matlab-simulink can simulate the mentioned control process (Kaddouri *et al.*, 1994; Slotine and Li, 1991).

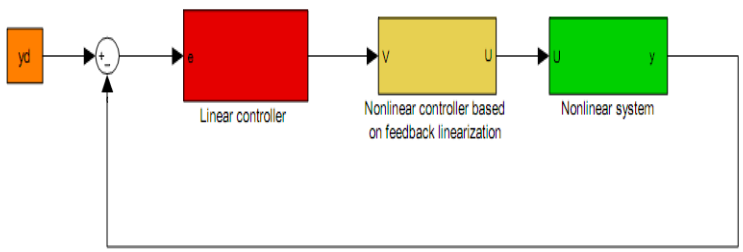

Figure 1: Block-diagram of input-output method

Simulation and Comparison Results

In this section, firstly permanent magnet synchronous motor is simulated using feedback linearization controller and the speed of the system is investigated and then the results are compared withFuzzy Self-adapting PID controller of ref (8).

Parameters of permanent magnet synchronous motor used in simulation are presented in Table 2 according to ref (8). Besides, reference input is considered a step signal with amplitude of 1500 in design of feedback linearization controller and friction is neglected. To reach the desired damping, constant parameters of k_1, k_2, k_3 are determined to 120 that are expressed in equation (8). The simulation results are shown in Figure 2.

Table 2: Parameters of the simulated motor (Wang *et al.*, 2007)

Parameter	Value
R	**4.495**
L_d	**0.027**
L_q	**0.067**
J	**0.00179**
n_p	**2**
λ_{PM}	**0.12**
B_m	**0**

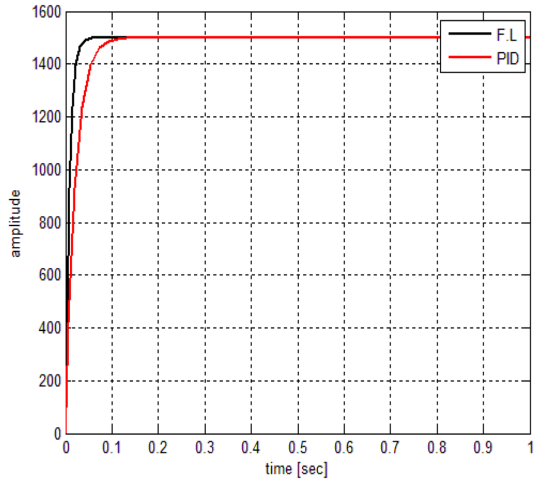

Figure 2: Diagram of rotor angular speed with feedback linearization controller and Fuzzy Self-adapting PID controller

It is shown by comparing the two waveforms that feedback linearization controller can well track the purposed output and has a lower rise time and reaches to the desired output quicker comparing to Fuzzy Self-adapting PID controller.

CONCLUSION

In this paper, speed of permanent magnet synchronous motor is controlled by feedback linearization method. The simulations results of the proposed method show that the stability of the permanent magnet synchronous motor system is guaranteed. Besides, the proposed controller is compared with Fuzzy Self-adapting PID controller in order to verify the performance of the proposed controller. It is concluded that feedback linearization controller has better performance.

REFERENCES

Bentaallah Abderrahim Massoum, Ahmed Benhamida Farid and Meroufel Abdelkader (2012). Adaptive feedback linearization control for asynchronous machine with nonlinear for natural dynamic complete observer. *Journal of Electrical Engineering* 88-94.

Bobaşu E, Popescu D and Ivanov S (No Date). Robust Control Law Design for a Synchronous Motor Using Feedback Linearization Method. *Automatic Control Department, University of Craiova, Romania.*

Kaddouri Azeddine, Akhrlf Ouassima, Le-huy Hoang and Ghribl Mohsen (1994). Nonlinear feedback control of permanent magnet synchronous motors 77– 80.

Low KS, Rahman MF and Lim KW (1995). The dq transformation and feedback linearization of a permanent magnet synchronous motor. *IEEE Catalogue* 292-296.

Nakai H, Ohtani H, Satoh E and Inaguma Y (2005). Development and testing of the torque control for the permanent-magnet synchronous motor. *IEEE Transactions on Industrial Electronics* **52**(3) 800-806.

Slotine JJE and Li W (No Date). *Applied Nonlinear Control*. Englewoodcliffs (NJ: prentice-Hall).

Stumberger B and Hamler A (2001). Analysis of Interior Permanent Magnet SynchronousMotor Designed for Flux Weakening Operation. *IEEE Transaction on Magnetic* **37**(5) 3644-3647.

Sue Shinn-Ming and Pan Ching-Tsai (2008). Voltage Constraint Tracking Based Field Weakening Control of IPM Synchronous Motor Drives. *IEEE Transactions on Industrial Electronics* **55**(1) 340-347.

Wang Limie, Tian Mingxiu and Gao Yanping (2007). Fuzzy Self-adapting PID Control of PMSM Servo System. *Electric Machines & Drives Conference*, IEMDC '07, *IEEE International* **1**.

Wei DQ, Luo XS, Fang JQ and Wang BJ (2006). Controlling chaos in permanent magnet synchronous motor based on the differential geometry method. *Acta Physica Sinica* **55**(1) 54-59.

Xiao-Jing Song (2013). Design and Simulation of PMSM Feedback Linearization Control System. *Telkomnika* **11**(3) 1245-1250.