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Research Article

A NUMERICAL MODEL OF FINITE DIFFERENCE (F.D) FOR DYNAMIC PILE DRIVING

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ABSTRACT
One of the serious challenges in geotechnical engineering issues is the evaluation of dynamic pile driving in soils. Effort is made in this paper to examine the process of pile driving in soils by providing a Finite Difference (FD) numerical model. Considering the axisymmetric nature of pile driving environment with a circular section in the soil and the related border conditions, a 2D numerical model is presented by FLAC 2D software. Use of a constitutive model of critical condition for soil constitutive model called BBM which was previously proposed by Alonso *et al.*, and spring-dashpot pile driving model (Smith) are among the major characteristics of the modeling conducted in this paper. At the end, the proposed numerical model is accredited by comparing the results of the model to the results of an actual pile driving project which was previously proposed by Fakharian *et al*., (2014) using M-C constitutive model.

Keywords: *Unsaturated Soils, Barcelona Basic Model (BBM), F.D Model, FLAC, Modified Cam-Clay-Dynamic Pile Driving*

INTRODUCTION
Formulation Framework for BBM Finite Difference Model
BBM model is formulated in a stress space (s, p, and q), where, q is the deviator stress, p is total net average stress and s is suction. In this paper, equal suction parameter (S_{eq}) is used instead of suction parameter (s) to propose BBM model within the framework of finite difference of FIAC software. Moreover, in all calculations of variables σ_1, σ_2 and σ_3, total stresses are net stresses. The relations of variables of the stress condition used in the BBM finite difference model are reviewed in the following.
Stress Condition Variables
Considering the subjects discussed in the previous section, the relations of p, q and S_{eq} in the BBM finite difference model are presented.

$$p_1 = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \tag{1}$$

$$q = \frac{1}{\sqrt{2}}(\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}) = \sqrt{3 j_2} \tag{2}$$

$$S_{ep} = S - S_{air} = (U_a - U_w) - S_{air} \tag{3}$$

Where,
σ_1, σ_2, σ_3 =Principal Net Total Stress
j_2 =Second Invariant of the deviator Stress Tensor
S =Matric Suction
U_a =Pore Air Pressure
U_w =Pore Water Pressure

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Strains Due to Stress in BBM Lumped Element Model

In BBM lumped element model, variables of strain changes related to p and q stresses cause volumetric strain changes (Δe). In BBM finite difference model, the relation of volumetric strain changes (Δe) is defined as follows;

$$\Delta e = \Delta e_1 + \Delta e_2 + \Delta e_3 \tag{4}$$

Where;

Δe = Volumetric Strain Increments

Δe_1, Δe, Δe_3 = Principal Strain Increments.

In this way, volumetric strain changes are calculated from the following relation based on specific volume (ν);

$$\Delta e = \frac{\Delta v}{v} \tag{5}$$

Where;

$\Delta \nu$ is specific volume changes and ν is the soil specific volume.

In BBM finite difference model and based on the relation proposed by Alonso *et al*., the relation of specific volume changes in relation to net total stress and suction parameter under initial stress path/virgin and loading-unloading conditions is provided as follows.

$$\nu = N(S_{eq}) - \lambda(S_{eq}) Ln \frac{p}{p^c} \tag{6}$$

The above relation is known as Virgin Compression Line where P^c is the reference stress parameter for $\nu = N(S_{eq})$.

(Reference Stress for v= N (S_{eq}))

In this way, Equivalent Deviator Strain such as deviator stress changes is defined from the following relation.

$$\Delta e_p = \frac{\sqrt{2}}{3}\sqrt{(\Delta e_1 - \Delta e_2)^2 + (\Delta e_2 - \Delta e_3)^2 + (\Delta e_1 - \Delta e_3)^2} \tag{7}$$

On the other hand, volumetric strain changes can be defined as the total of two elastic and plastic components:

$$\Delta e_i = \Delta e_i^e + \Delta e_i^p \quad i=1, 2,3 \tag{8}$$

where ; Δe_i^p is called plastic strain changes and Δe_i^e is called elastic strain changes.

Specific Volume

In BBM finite difference model, specific volume parameter is calculated by dividing total volume of soil by solid particle volume. Specific volume relation is as follows.

$$V = \frac{v}{v_s} \tag{9}$$

Where; V is total volume of soil and V is solid particle volume.

It should be noted that loading-unloading paths under S=cte conditions are assumed as elastic.
$\lambda(s)$ = Material-soil stiffness parameter under unsaturated conditions.

Generalization of Hooke's law in BBM Finite Difference Model

Here, Hooke's law is developed to explain stress-strain behavior of unsaturated soils by considering the definitions of different strains (different strain conditions in BBM). Based on the Hooke's law, in the main axes we will have:

$$\Delta\sigma_2 = \alpha_1 \Delta e_2^e + \alpha_2 (\Delta e_1^e + \Delta e_3^e) \qquad (10)$$

$$\Delta\sigma_1 = \alpha_1 \Delta e_1^e + \alpha_2 (\Delta e_2^e + \Delta e_3^e) \qquad (11)$$

$$\Delta\sigma_3 = \alpha_1 \Delta e_3^e + \alpha_2 (\Delta e_1^e + \Delta e_2^e) \qquad (12)$$

Where; $\Delta\sigma_1$, $\Delta\sigma_2$ and $\Delta\sigma_3$ are the changes of principal net total stress and α_1 and α_2 are defined as follows.

$$\alpha_1 = K + \frac{4}{3}G \qquad (13)$$

$$\alpha_2 = K + \frac{2}{3}G \qquad (14)$$

Where; K and G are bulk modulus and shear modulus, respectively.

Specific volume changes under loading-unloading conditions in elastic situation and by assuming constant suction are specified from the following relation.

$$dv = -k\frac{dp}{p} \qquad (15)$$

Where; k is the elastic stiffness parameter independent from suction amount. By incorporating relations 5 and 15, the following relation is obtained.

$$-\Delta P = \frac{vp}{k} . \Delta e^e \qquad (16)$$

Yield and Potential Functions

Calculation method of elastic stresses and strains for BBM finite difference model was explained in the previous sections. Based on the model proposed by Alonso *et al.*, BBM yield function in stress space (p-q) and for constant suction (s) is calculated from the following relation.

$$f = q^2 - M^2 (p + p_s)(p_o - p) \qquad (17)$$

In the above relation, parameter M is a constant amount indicating the Slope of Critical State Line, and p_s is from the simple relation of p_s=K.S where; K is a constant parameter describing the increase in cohesion with suction, and S (suction) of soil and controls the development of yield level in an area of tensile stress.

Flow Law of BBM Finite Difference Model

As it was said earlier, BBM model is the developed version of Modified Cam-Clay Constitutive Model (MCC) and consequently, plastic strain increase vector is in line with the vector perpendicular to the yield surface. Under such conditions, it follows orthogonality law and therefore, Flow Law of BBM finite difference model is the mobile flow law. Considering these explanations, strain changes in plastic state is calculated from the following relation.

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$$\Delta e_i^p = \lambda^s . \frac{\partial f}{\partial \sigma_i} \quad i=1,2,3 \tag{18}$$

Where; $_s\lambda$ is the Stiffness Parameter for changes in Suction for Virgin States of the Soil. Considering the above relation in BBM finite difference model, each of the main stresses, namely σ_1, $\sigma 2$ and σ_1 should be derived from the yield function *f* to calculate plastic strain changes. To do this and to calculate (Δe_i^p), the function f should be first developed based on the amounts of q and p (relations 1 and 2).

$$f=\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2\right]-M^2\left[\frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)+p_s\right](p_0-\frac{\sigma_1+\sigma_2+\sigma_3}{3})$$
$$=\frac{1}{2}\left[(\sigma_1^2-2\sigma_1\sigma_2+\sigma_2^2)+(\sigma_2^2-2\sigma_2\sigma_3+\sigma_3^2)+(\sigma_1^2-2\sigma_1\sigma_3+\sigma_3^2)\right]-M^2\left[\frac{p_0}{3}(\sigma_1+\sigma_2+\sigma_3)+p_0p_s-\frac{1}{9}(\sigma_1+\sigma_2+\sigma_3)^2-\frac{p_s}{3}(\sigma_1+\sigma_2+\sigma_3)\right] \tag{19}$$

Hardening/Softening Rules Governing BBM Finite Difference Model

In BBM model, Alonso *et al.*, described the answer to identical loading under saturated (S=0) and unsaturated conditions in terms of figure 1.

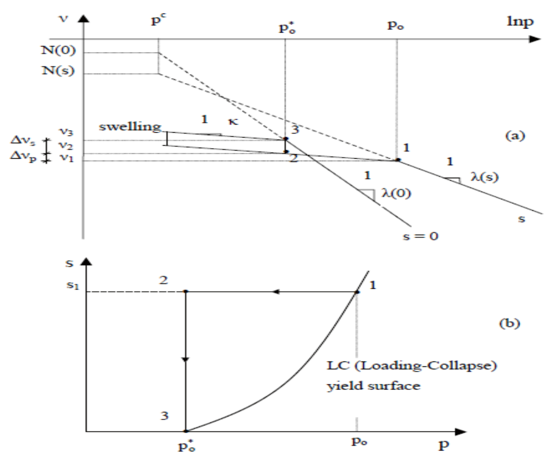

Figure 1: (a) Assumed isotropic compression lines, and (b) LC yield surface (after Alonso *et al.*, 1990)

As it is seen in figure 1, saturated sample in a stress p_o^* (point 3) and unsaturated sample in a stress larger than p_o (point 1) achieve yield. If both points 1 and 3 of yield curve are related to the similar yield curve in p and s_q space, considering the amounts of specific volume in points 1 and 3, the relation between p_o and p^* can be within a virtual path that consists an initial unloading with a constant amount (S=cte) from point 1 to point 2 followed by a decrease in suction with a constant amount (p=cte) from point 2 to point 3 as follows.

$$v_3 = v_1 + \Delta v_p + \Delta v_s \tag{40}$$

Where; v1 is the initial specific volume, Δv_p is the volume change arising from unloading, and Δv_s is the change of volume arising from suction unloading (wetting). Suction unloading (wetting) from 2 to 3 has occurred in an elastic limit which is calculated from the following relation.

$$\Delta v_s = k_s . Ln \frac{S + P}{P_{atm}} \tag{41}$$

Where; p_{atm} is the atmosphere pressure and k_s is compressibility coefficient of suction changes in the elastic limit. The amount of Δv_p with a vector from point 1 to 2 in an elastic limit is calculated from the following relation.

$$\Delta v_p = k . Ln \frac{P_o}{P_o^*} \tag{42}$$

The following relation is calculated by combining the above equations.

$$N(s) - \lambda(s) Ln \frac{p_o}{p^c} + kLn \frac{p_o}{p_o^*} + k_s Ln \frac{s + p_{atm}}{p_{atm}} = N(o) - \lambda(o) Ln \frac{p^*}{p^c} \tag{43}$$

Where; p^* is the Preconsolidation Stress for Saturated Condition, $\lambda(o)$ and $\lambda(s)$ are soil stiffness parameters in saturated (s=0) and unsaturated conditions (s), respectively, N(o) and N(s) are specific volume parameters in saturated (s=0) and unsaturated conditions (s), respectively. Therefore, considering the above explanations, specific volume changes from suction amounts of 0-s are calculated from the following relation.

$$\Delta v(p^c)\Big|_s^o = N(o) - N(s) = k_s . Ln \frac{s + p_{atm}}{p_{atm}} \tag{44}$$

Moreover, Preconsolidation Stress in unsaturated conditions and the existence of suction are calculated from the following relations.

$$P_o = P^c \left(\frac{P^*}{P^c}\right)^{\left(\frac{\lambda(o) - k}{\lambda(s) - k}\right)} \tag{45}$$

$$\lambda(s) = \lambda(o)((1 - r)e^{-\beta s} + r) \tag{46}$$

Where;
β= Parameter Controlling the rate of increase of Soil Stiffness with suction
r = Parameter defining the Maximum Soil Stiffness

Validation of BBM Finite Difference Model

In order to validate BBM model, Alonso *et al.*, compared the results obtained from the proposed relations of BBM model with those of the tests on unsaturated soil samples that had been previously conducted by Josa in 1998. The amounts of reference laboratory parameters used in this validation are summarized in table 1.

Table 1: Input parameters of FLAC software for validation – (after Alonso *et al.*, (1990))

Parameter	G	M	λ(0)	k	γ	p	β	r	k	P_{atm}	k_s	S
Value	3.3	0.82	0.14	0.15	1.915	0.045	16.4	0.26	1.24	0.1	0.01	0.01
Unit	Mpa	-	-	-	-	Mpa	Mpa^{-1}	-	-	Mpa	-	Mpa

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Important parameters of p^c or reference pressure are also considered by selecting (p=0.33) from figure 2a. The result of comparison between the relations proposed by Alonso *et al.*, in BBM model with laboratory results are shown in figure 2b.

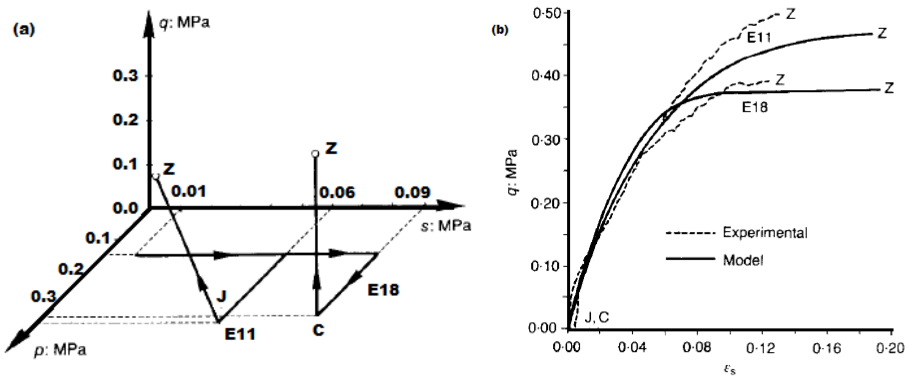

Figure 2: Shear tests on partially saturated compacted kaolin: (a) stress paths; (b) comparison of measured and predicted shear stress- strain relationships (after Alonso *et al.*, 1990)

As it is seen in the figure, the results of anticipation of Barcelona model with laboratory results in $q-\varepsilon_s$ space (deviator stress and strain) indicate the high precision of BBM for examining the collective strains in unsaturated soils. Considering the procedure explained in this paper for presenting BBM finite difference model and in order to compare BBM finite elements model with laboratory amounts and relations of initial BBM model with regard to the amounts of previous table and addition of $\gamma = 2\text{MPa}$ amounts and a relative large initial amount for K to an amount of K=8000MPa and its placing in the framework of BBM finite difference proposed in this paper, deviator stress curve (q) and shear strain ε_s of BBM finite model are obtained as the following chart.

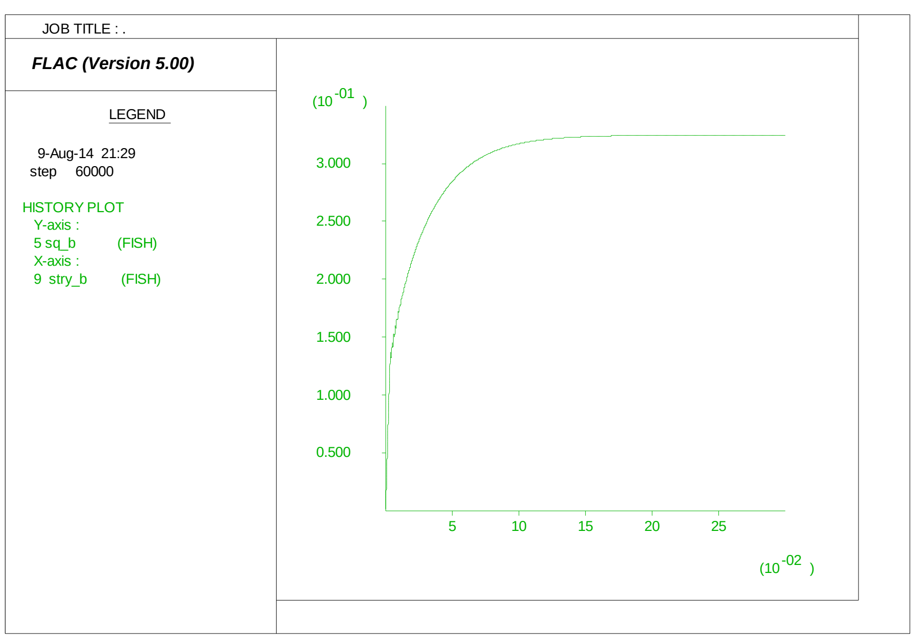

Figure 3: F.D FLAC Code shear stress-strain relationships results (Amir *et al.*, 2014)

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Dynamic Pile Driving Model in Unsaturated Soil

After ensuring the validity of BBM unsaturated constitutive model in previous sections, in this section a pile in a layer soil that had been previously presented by Fakharian *et al.*, (2014) is presented again as the default constitutive model by using the capabilities of FLAC software and BBM constitutive model. All the defaults considered in this section are based on the paper of Fakharian *et al.*, (2014) with a difference that despite the paper of Fakharian *et al.*, (2014) in which FLAC default constitutive model; i.e., Mohr-Coulomb had been used, in the new finite difference model, BBM unsaturated constitutive model is used which was referred in previous sections of this paper. Soil profile and pile driving finite difference model are shown in figures 4 and 5, respectively.

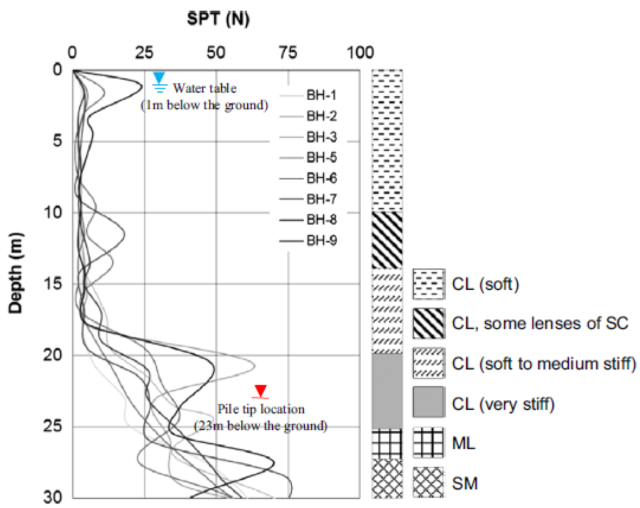

Figure 4: SPT and soil profile (Fakharian, 2003)

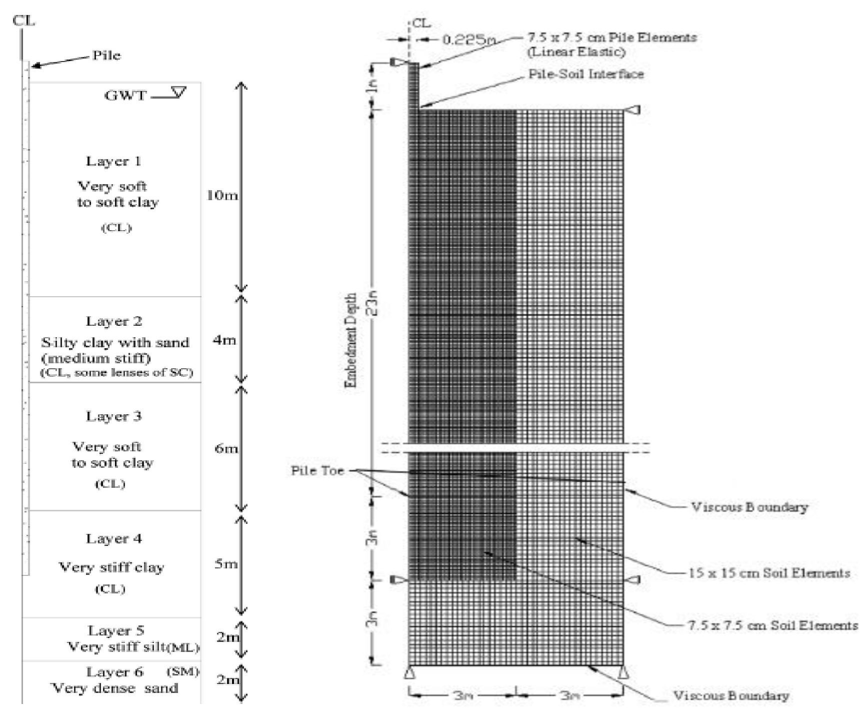

Figure 5: (a) The axisymmetric pile–soil model for pile driving simulation. (b) Geometry, mesh and boundaries of the axisymmetric pile–soil numerical models- the Finite Difference Method (FDM) (Fakharian *et al.*, 2014)

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In this section, by using the existing relations and the information available in the paper of Fakharian *et al.*, (2014), input parameters of BBM constitutive model are calculated to rewrite pile driving in unsaturated soil and the summary of which is provided in table 2.

Table 2: Converted input parameters of FLAC software for dynamic pile driving model

Layer	el	Cc	Cs	landa	kappa	M	layer tickness	Pc
1	0/84	0/2856	0/03864	0/124	0/017	1/03	10	1/84E+05
2	1/005	0/3417	0/04623	0/148	0/020	1/20	4	2/45E+05
3	0/84	0/2856	0/03864	0/124	0/017	1/03	6	3/80E+05
4	1/005	0/3417	0/04623	0/148	0/020	1/37	5	4/13E+05
5	1/005	0/3417	0/04623	0/148	0/020	1/37	2	4/49E+05
6	0/462069	0/157103	0/021255	0/068	0/009	1/64	2	4/51E+05
block	0/7	0/238	0/0322	0/103	0/014	1/37	3	4/28E+05

Validation of Dynamic Pile Driving Model with BBM Unsaturated Model

In this section, the results obtained from pile driving model in an unsaturated environment are compared to those of the paper of Fakharian *et al.*, (2014) and are then validated by using the above information and by applying the loading similar to the aforesaid paper. In figure 6, dynamic loading of the new model is shown. After applying the above loading on the new pile driving finite difference model which is coded based on BBM constitutive model, the information related to the response of the aforesaid pile and its dynamic load carrying capacity is presented as compared to the results of pile driving finite difference model based on M-C constitutive model and indicates high precision of pile driving finite difference model based on BBM constitutive model.

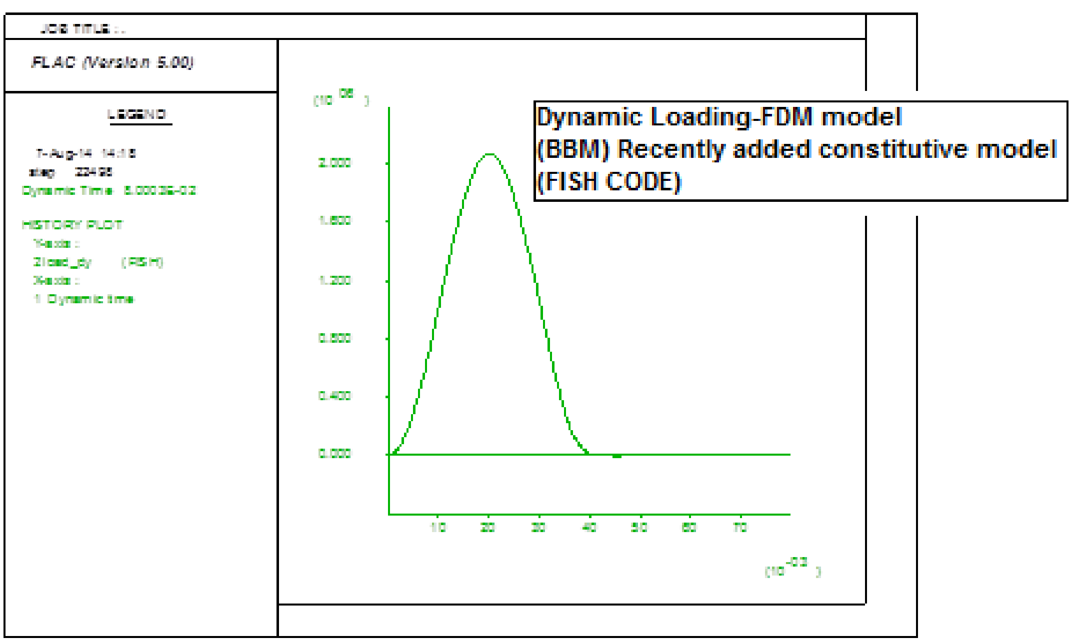

Figure 6: Dynamic Loading of Plie

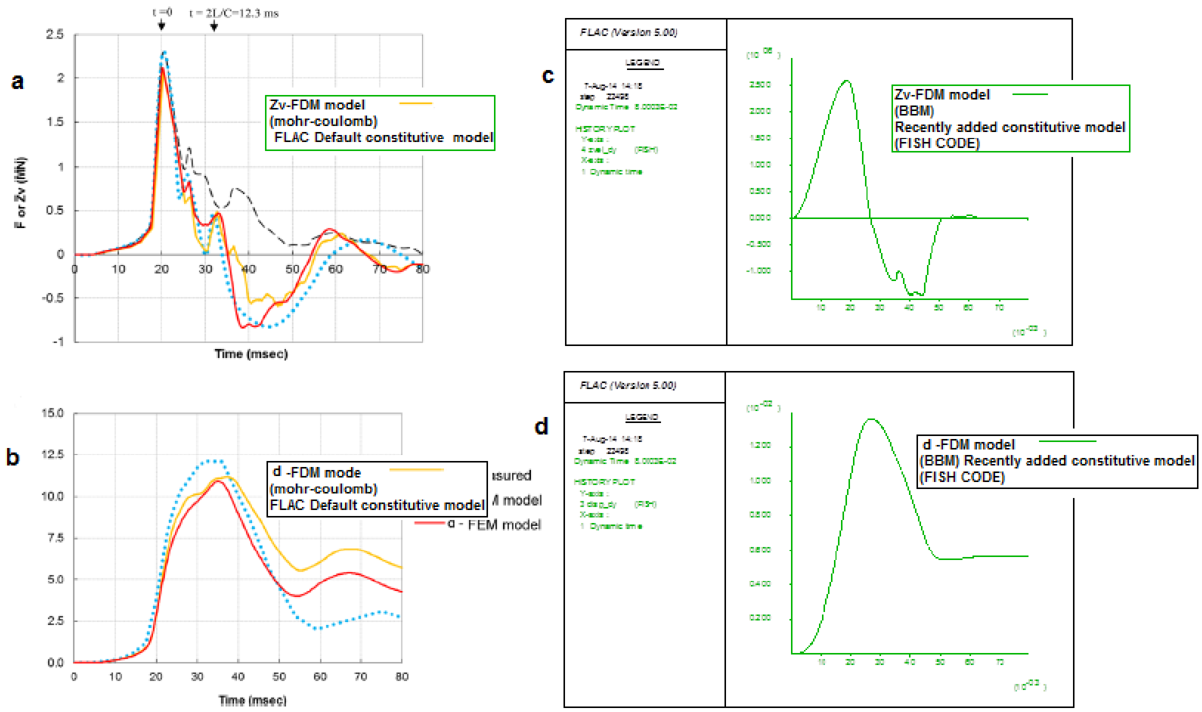

Figure 7: Comparing the results of the response of pile driving finite difference model based on BBM constitutive model with pile driving finite difference model based on M-C constitutive model

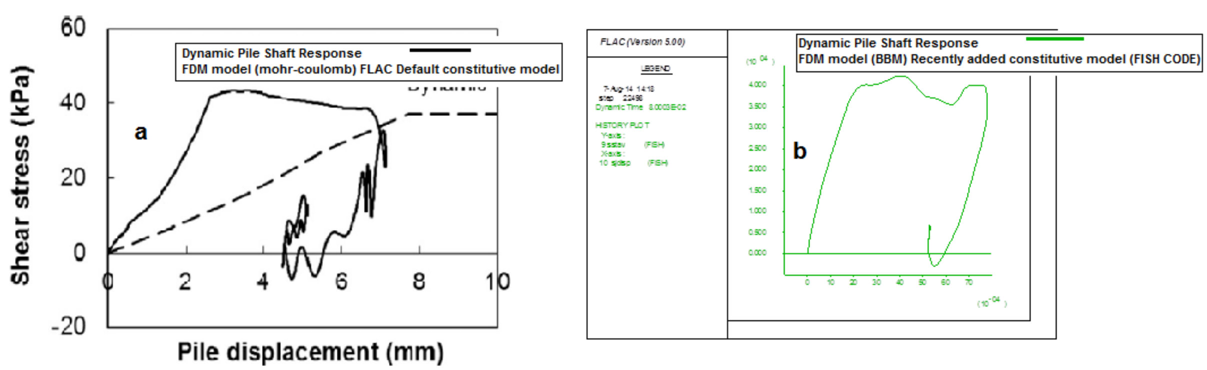

Figure 8: Comparing the results of dynamic load carrying capacity of pile driving finite difference model based on BBM constitutive model with pile driving finite differenced model based on M-C constitutive model

Conclusion

As it is specified in figure 2b related to the paper of Alonso *et al.*, (1990), maximum point of strain chart related to the numerical model proposed by Alonso *et al.*, (1990) with E18 laboratory information input shows 0.015 and 0.37 MPa amounts for stress and strain parameters, respectively. Similarly, by inputting the information related to the paper of Alonso *et al.*, (1990) (table 1) in the finite difference code written in FLAC software (figure 3), 0.017 and 0.37 MPa amounts are obtained for stress and strain parameters, respectively. This shows a precision with 10% difference in stress parameter and 8% in strain parameter. Moreover, by comparing the results of pile driving finite difference model based on BBM constitutive model with the same model based on M-C constitutive model which indicates the high precision of the proposed model, the effect of unsaturated parameters of soil such as S and S_r on dynamic pile load carrying capacity can be easily examined.

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