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Research Article

DUALITY IN MULTIPLE CRITERIA AND MULTIPLE CONSTRAINT LEVELS LINEAR PROGRAMMING WITH FUZZY PARAMETERS

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ABSTRACT
In this paper, multiple criteria and multiple constraint levels linear programming with fuzzy parameters (FMC^2LP) is introduced based on fuzzy relations. The dual of FMC^2LP is constructed on the basis that MC^2LP can be decreased to a multiple criteria linear programming ($MCLP$). The concepts of α - feasible solution and (α,β) - maximal and minimal solutions are defined. Finally, the weak and strong duality theorems are obtained.

Keywords: *Multiple Criteria and Multiple Constraint Levels Linear Programming, Duality, Fuzzy Numbers, α - Feasible Solution, (α,β) - Maximal and Minimal Solutions*

INTRODUCTION

In this paper, multiple criteria and multiple constraint levels linear programming (MC^2LP) that is one of the important fields of multiple criteria decision making (MCDM) is considered to study. In the linear programming (LP) and multiple criteria linear programming ($MCLP$) models a single person is considered in charge of decision making, for example the production manager. This may not be true assumption in actual practice. Therefore, we consider a group of decision makers (DMs), including the president, chief financial officer and controller share the decision making, responsibility. The preference values of each involved manager on resource availability can be treated as an individual level of the availability. Then, the obtained model has multiple discrete levels of resource availability. A problem with such structure is called MC^2LP. The solution of this problem is called a potential solution (Shi, 2001). Each potential solution optimizes the MC^2LP problem under a certain range of decision parameters that are the criteria and constraint level weight vectors.

Shi in his book (Shi, 2001) studied MC^2LP problem, integer MC^2LP problem and transportation MC^2LP problem. Shi (Shi, 2001) studied MC^2LP problems in which objective functions are considered as fuzzy decisions. Instead of finding a set of potential solutions for MC^2LP problem, they presented the decision makers goal-seeking and compromise behavior to attain a set of satisficing solutions between an upper and a lower aspiration level.

In the classic MC^2LP, all data are exactly known. However, crisp data may not be available, because data in many real applications cannot be precisely measured. On the other hand, duality concept provides the useful information about the primal problem.

Furthermore, solving the dual problem is often easier than its primal. Therefore, this paper studies MC^2LP with fuzzy data (FMC^2LP) and extends duality concept to FMC^2LP. For this, FMC^2LP is firstly introduced and is converted to an $MCLP$ with fuzzy parameters ($FMCLP$) and then this problem is transform to an linear programming with fuzzy parameters (FLP). In this situation, the dual of FMC^2LP can be acquired by applying the concept of possibility and necessity relations on the obtained FLP. We consider the possibility relation for primal problem and the necessity relation for dual problem.

The reminder of this paper is structured as follows. Section 2 is about fuzzy preliminaries. Section 3 recalls the formulation of MC^2LP and its properties. In the fourth section, we consider fuzzy version of MC^2LP. Section 5 is about maximizing objective function and the next section, we consider fuzzy version of FMC^2LP and extend the duality concept for this problem. Conclusions are given in the last section.

Preliminaries

Fuzzy set theory which was firstly introduced by Zadeh (Zadeh, 1965; Zadeh, 1999) has been extensively applied in much area of science. The basic definitions of fuzzy set theory are as follows:

Let X denotes a universal set. Then a fuzzy subset $\tilde{A}$ of X is defined by its membership function as

$$\mu_{\tilde{A}} : X \rightarrow [0,1]$$

which assigns to each element $x \in X$ a real number $\mu_{\tilde{A}}(x)$ in the interval [0,1]. The support of a fuzzy set $\tilde{A}$ on X, denoted by supp(A), is the set of points in X in which $\mu_{\tilde{A}}(x)$ is positive, i.e.

$$supp(A) = \{x \in X \mid \mu_{\tilde{A}}(x) > 0\}$$

The α - level set of a fuzzy set $\tilde{A}$ is defined as an ordinary set A_α for which the degree of its membership function exceeds the level α:

$$[A]_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\}, \quad \alpha \in [0,1].$$

We denote the set of all fuzzy subsets of X by $F(X)$.

A fuzzy subset $\tilde{P}$ of $F(X) \times F(X)$ is called a fuzzy relation on X, i.e $\tilde{P} \in F(F(X) \times F(X))$.

A fuzzy relation $\tilde{Q}$ on X is called a fuzzy extension of relation P, if for each $x, y \in X$

$$\mu_{\tilde{Q}}(x, y) = \mu_P(x, y)$$

Let $\tilde{d}$ is a fuzzy quantity. Following notation are used as:

$$\tilde{d}^L(\beta) = inf\{t \mid t \in [\tilde{d}]_\beta\}, \quad \tilde{d}^R(\beta) = sup\{t \mid t \in [\tilde{d}]_\beta\}$$

Let A and B are fuzzy sets with the membership functions $\mu_A : R \rightarrow [0,1]$ and $\mu_B : R \rightarrow [0,1]$, respectively. We shall consider

$$\mu_{Pos}(A, B) = sup\{min(\mu_A(x), \mu_B(y)) \mid x \leq y, \quad x, y \in R\} \qquad (a)$$

$$\mu_{Nec}(A, B) = inf\{max(1 - \mu_A(x), 1 - \mu_B(y)) \mid x > y, \quad x, y \in R\} \qquad (b)$$

where equations (a) and (b) are respectively called possibility and necessity relations. Possibility and necessity relations (a) and (b) were originally introduced as possibility and necessity indices in (Dubois and Prade, 1983). This relations can be alternatively written as:

$$\mu_{Pos}(A, B) = (A \leq^{Pos} B), \quad \mu_{Nec}(A, B) = (A <^{Nec} B)$$

where μ_{Pos} and μ_{Nec} are the membership functions of the fuzzy relation on R. By $A \geq^{Pos} B$ and $A >^{Nec} B$ we mean $B \leq^{Pos} A$ or $B <^{Nec} A$, respectively.

Multiple Criteria and Multiple Constraint Linear Programming

In general, $MCLP$ can be written as:

$$\max \quad Cx$$

$$
\begin{aligned}
& s.t. \\
& \quad Ax \le d \\
& \quad x \ge 0
\end{aligned}
\tag{1}
$$

The objective function is to maximize q different criteria written by the $(q \times n)$ matrix C (criterion matrix), while the $(m \times n)$ matrix A is the unit consumption of resources and the $(m \times 1)$ vector d is the constraint (resource availability) level.

To construction MC^2LP, we replace d by an $(m \times r)$ matrix D and the problem (1) is converted to:

$$
\begin{aligned}
& \max \quad Cx \\
& s.t. \\
& \quad Ax \le D \\
& \quad x \ge 0
\end{aligned}
\tag{2}
$$

The MC^2LP implies that **x** is feasible if Ax lies in a convex set generated by the columns of D. For solving the problem (2), Shi (Shi,) introduced two weight parameters, the constraint parameter $\gamma^t = (\gamma_1, \ldots, \gamma_r) > 0$ and the criteria parameter $\lambda^t = (\lambda_1, \ldots, \lambda_q) > 0$ into the formulation. Thus, the MC^2LP can be transformed to:

$$
\begin{aligned}
& \max \quad \lambda^t Cx \\
& s.t. \\
& \quad Ax \le D\gamma \\
& \quad x \ge 0
\end{aligned}
\tag{3}
$$

A solution of MC^2LP called potential solution is formally defined as an extension of non-dominated solution for the multiple criteria program.

Concepts of Potential Solutions (Seiford and Zhu, 1979)

A basis J is called a potential basis a potential solution for the MC^2LP if and only if there exists $\gamma^0 > 0$ and $\lambda^0 > 0$ such that J is an optimal basis for the following problem:

$$
\begin{aligned}
& \max \quad (\lambda^0)^t Cx \\
& s.t. \\
& \quad Ax \le D\gamma^0 \\
& \quad x \ge 0
\end{aligned}
$$

Given a basis J for an MC^2LP

(i) $\Gamma(J) = \{\gamma > 0 \mid B^{-1}D\gamma \ge 0\}$ is called primal parameter set and

(ii) $\Lambda(J) = \{\lambda > 0 \mid \lambda^t(C_B B^{-1}R - C_R) \ge 0\}$ is called dual parameter set.

The matrices R and B are sub-matrices of A which R consists of non-basic variables and B is a non-singular matrix. Given a basis J for an MC^2LP

(i) The basis solution $x(J,\gamma) = B^{-1}D\gamma$ is feasible if and only if $\gamma \in \Gamma(J)$.

(ii) The solution $x(J,\gamma)$ is optimal if and only if $\gamma \in \Gamma(J)$ and $\lambda \in \Lambda(J)$.

(iii) The objective function value of the problem is denoted by $V(J,\lambda,\gamma)=\lambda^t C_B B^{-1} D\gamma$, which is a function of (γ,λ). Generally, (ii) of definition 3 is called the optimality condition.

(i) J is a primal potential basis if and only if $\Gamma(J)\neq\varnothing$.

(ii) J is a dual potential solution if and only if $\Lambda(J)\neq\varnothing$.

(iii) J is a potential basis if and only if the cartesian product $\Gamma(J)\times\Lambda(J)\neq\varnothing$.

Special Potential Solution

If the basic variable $x(J,\gamma)=B^{-1}D\gamma$ is considered as a parameter form with respect the parameter γ, then the condition $\gamma_1+\gamma_2+\ldots+\gamma_r=1$ can set as a constraint, MC^2LP becomes a LP as

$$\begin{aligned} \max \quad & \Sigma_{k=1}^{q}\lambda_k C^k x \\ s.t. \quad & \\ & Ax\leq \Sigma_{g=1}^{r}\gamma_g D_g \\ & \Sigma_{g=1}^{r}\gamma_g=1 \\ & x\geq 0,\gamma_g\geq 0, g=1,\ldots,r \end{aligned} \tag{4}$$

Note that in (4), C^k is the k th row of the $(q\times n)$ criterion matrix. A non-dominated solution of (4) with $\gamma>0$ is called a potential solution, and a non-dominated solution of (4) with $\gamma\geq 0$ is called a weak potential solution.

$x^*\in X=\{x\,|\,Ax\leq d, x\geq 0 \quad for \quad some \quad d\in H(D)\}$ is a weak potential solution if and only if there is a $\lambda^t=(\lambda_1,\lambda_2,\ldots,\lambda_q)^t$ with $0\leq\lambda_k\leq 1, k\in\{1,2,\ldots,q\}$, such that x^* solves (4). For each given $(\gamma^0)^t=(\gamma_1^0,\gamma_2^0,\ldots,\gamma_r^0)^t>0$, with $\Sigma_{g=1}^{r}\gamma_g^0=1$, x^* is a potential solution of (3) if and only if x^* is a non-dominated solution of (5):

$$\begin{aligned} \max \quad & (C^1x, C^2x,\ldots,C^qx) \\ s.t. \quad & \\ & Ax\leq D\gamma^0 \\ & x\geq 0 \end{aligned} \tag{5}$$

$x^*\in X$ is a potential solution of (4) if and only if there is an $\varepsilon>0$ and $\lambda^t=(\lambda_1,\lambda_2,\ldots,\lambda_q)^t$ with $0\leq\lambda_k\leq 1, k\in\{1,2,\ldots,q\}$, such that x^* solves (6):

$$\begin{aligned} \max \quad & \Sigma_{k=1}^{q}\lambda_k C^k x \\ s.t. \quad & \\ & Ax\leq \Sigma_{g=1}^{r}\gamma_g D_g \\ & \Sigma_{g=1}^{r}\gamma_g=1 \\ & x\geq 0,\gamma_g\geq\varepsilon>0, g=1,\ldots,r \end{aligned} \tag{6}$$

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MC^2LP With Fuzzy Parameters

The extended form of MC^2LP (2) can be written as:

$$\begin{aligned} \max \quad & z_k = \Sigma_{j=1}^{n} c_{kj} x_j \quad k = 1 \cdots, q \\ s.t. \quad & \\ & \Sigma_{j=1}^{n} a_{ij} x_j \quad P \quad (d_{i1}, \cdots, d_{ig}), \quad g = 1, \cdots, r \\ & x_j \geq 0, \quad j = 1, \ldots, n \end{aligned} \tag{7}$$

where $P = (\leq or = or \geq)$. Considering data as fuzzy numbers, an FMC^2LP is as:

$$\begin{aligned} \max \quad & \tilde{z}_k = \Sigma_{j=1}^{n} \tilde{c}_{kj} x_j \quad k = 1 \cdots, q \\ s.t. \quad & \\ & \Sigma_{j=1}^{n} \tilde{a}_{ij} x_j \quad \tilde{P} \quad (\tilde{d}_{i1}, \cdots, \tilde{d}_{ig}), \quad g = 1, \cdots, r \\ & x_j \geq 0, \quad j = 1, \ldots, n \end{aligned} \tag{8}$$

By defining the constraint parameter $\gamma^t = (\gamma_1, \ldots, \gamma_r) > 0$ and the criteria parameter $\lambda^t = (\lambda_1, \ldots, \lambda_q) > 0$ into the formulation, the problem can be transformed to:

$$\begin{aligned} \max \quad & \Sigma_{j=1}^{n} (\Sigma_{k=1}^{q} \lambda_k \tilde{c}_{kj}) x_j \\ s.t. \quad & \\ & \Sigma_{j=1}^{n} \tilde{a}_{ij} x_j \quad \tilde{P} \quad \Sigma_{g=1}^{r} \tilde{d}_{ig} \gamma_g, \ g = 1, \cdots, r \\ & \Sigma_{g=1}^{r} \gamma_g = 1 \\ & \gamma_g \geq \varepsilon > 0, g = 1, \ldots, r \\ & x_j \geq 0, \quad j = 1, \ldots, n \end{aligned} \tag{9}$$

which is equivalent to

$$\begin{aligned} \max \quad & \Sigma_{j=1}^{n} (\Sigma_{k=1}^{q} \lambda_k \tilde{c}_{kj}) x_j \\ s.t. \quad & \\ & \Sigma_{j=1}^{n} \tilde{a}_{ij} x_j - \Sigma_{g=1}^{r} \tilde{d}_{ig} \gamma_g \quad \tilde{P} \quad 0, \ g = 1, \cdots, r \\ & \Sigma_{g=1}^{r} \gamma_g = 1 \\ & \gamma_g \geq \varepsilon > 0, g = 1, \ldots, r \\ & x_j \geq 0, \quad j = 1, \ldots, n \end{aligned} \tag{10}$$

Now,we consider matrix $\tilde{A}$ as follows:

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} & \approx d_{11} & \approx d_{12} & \cdots & \approx d_{1r} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} & \approx d_{21} & \approx d_{22} & \cdots & \approx d_{2r} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} & \approx d_{m1} & \approx d_{m2} & \cdots & \approx d_{mr} \\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -1 \end{bmatrix}$$

Also, suppose $\tilde{c} = (\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_n, \overbrace{0,0,\ldots,0}^{r})$ such that $\tilde{c}_j = \Sigma_{k=1}^{q} \lambda_k \tilde{c}_{kj}$ $(j = 1,\ldots,n)$, $X = (x_1, x_2, \ldots, x_n, \gamma_1, \gamma_2, \ldots, \gamma_r)^t$ and $\tilde{B} = (\overbrace{0,0,\ldots,0}^{m}, 1, \overbrace{-\varepsilon, -\varepsilon, \ldots, -\varepsilon}^{r})^T$. Hence, model (10) is became as follows:

$$\begin{aligned} &\max \quad \Sigma_{j=1}^{n+r} \tilde{c}_j \; X_j \\ &s.t. \\ &\quad \Sigma_{j=1}^{n+r} \tilde{A}_{ij} X_j \;\; \tilde{P} \;\; \tilde{B}_i, \quad i = 1, \cdots, m+r+1 \\ &\quad X_j \geq 0, \quad j = 1, \ldots, n \end{aligned} \tag{11}$$

A fuzzy set $\tilde{X}$, whose membership function $\mu_{\tilde{X}}$ is defined for all $x \in R$ by

$$\mu_{\tilde{X}}(x) = \begin{cases} \min\{\mu_{\tilde{P}}(\tilde{a}_{11}x_1 + \ldots + \tilde{a}_{1n}x_n - \tilde{d}_{11}\gamma_1 - \ldots - \tilde{d}_{1r}\gamma_r, 0), \ldots, \mu_{\tilde{P}}(\tilde{a}_{m1}x_1 + \ldots \\ + \tilde{a}_{mn}x_n - \tilde{d}_{m1}\gamma_1 - \ldots - \tilde{d}_{mr}\gamma_r, 0), \mu_{\tilde{P}}(\gamma_1 + \ldots + \gamma_r, 1), \\ \mu_{\tilde{P}}(-\gamma_1, -\varepsilon), \ldots, \mu_{\tilde{P}}(-\gamma_r, -\varepsilon)\} \\ 0, \qquad otherwise \end{cases}$$

is called the fuzzy set of feasible region of the FMC^2LP problem. For $\beta \in (0,1]$, a vector $x \in [\tilde{X}]_\beta$ is called the β-feasible solution of the FLP (11). Notice that the feasible region $\tilde{X}$ is a fuzzy set. On the other hand, β-feasible solution is a vector belonging to the β-cut of the feasible region $\tilde{X}$.

Maximizing the Objective Function

Now we look for the "best" fuzzy quantities $\tilde{z}_k$ with respect to the given fuzzy constraints, or, in other words, with respect to the fuzzy set of feasible region of (8). Knowing the weights $\lambda_k, k = 1,..q$ of the objectives we shall deal with the associated problem (11), particularly, with the single objective function $\tilde{z} = \tilde{c}_1 x_1 \tilde{+} \ldots \tilde{+} \tilde{c}_{n+r} x_{n+r}$. Special relations are defined by Ramik (Ramik, 2006b). Let $\tilde{\prec}$ be a fuzzy relation on R, let $\tilde{a}$, $\tilde{b}$ be fuzzy sets of R and let $\alpha \in (0,1]$. It is said that $\tilde{a}$ is α-less than $\tilde{b}$ with respect to $\tilde{\prec}$ and write $\tilde{a} \tilde{\prec}_\alpha \tilde{b}$ if $\mu_{\tilde{\prec}}(\tilde{a}, \tilde{b}) \geq \alpha$ and $\mu_{\tilde{\prec}}(\tilde{b}, \tilde{a}) < \alpha$. It is called $\tilde{\prec}_\alpha$ the α-relation on R with respect to $\tilde{\prec}$. Notice that $\tilde{\prec}_\alpha$ is a binary relation on the set of fuzzy sets $F(R)$ being constructed from a fuzzy relation $\tilde{\prec}$ on the level of $\alpha \in (0,1]$. If $\tilde{a}$ and $\tilde{b}$ are crisp numbers corresponding to real numbers a and b, respectively, and is a fuzzy extension of relation $\tilde{\prec}$ then $\tilde{a} \tilde{\prec}_\alpha \tilde{b}$ if and only if $a \leq b$. Now, modifying the well known concept of potential solution in MC^2LP optimization or nondominated solution optimization in $MCLP$, we define maximization (or equivalently minimization) of the objective function of FLP problem (11). We shall consider a fuzzy relation $\tilde{\prec}$ on R being a fuzzy extension of the usual binary relation $\leq$ on R, see also Ramik (Ramik, 2006a). Here, we allow for independent, i.e. different satisfaction levels: $\alpha \neq \beta$, where is considered for the objective functions α and β for the constraints (Ramik, 2006a). Let $\tilde{c}_j, \tilde{a}_{ij}$ and $\tilde{b}_i$,

$i = 1,\ldots,m+r+1,\ j = 1,\ldots n+r$, be fuzzy quantities on R. Let $\tilde{\prec}$ be a fuzzy relation on R, being also a fuzzy extension of the usual binary relation $\le$ on R. A β- feasible solution of (11), $x \in [\tilde{X}]_\beta$ is called the (α,β)-maximal solution of (11) with respect to $\tilde{\prec}$ if there is no $x' \in [\tilde{X}]_\beta$ such that $\tilde{c}^T x \tilde{\prec} \tilde{c}^T x'$.

Dual Problems and Duality Theorem

Model (11) is a linear programming and its dual is as follows:

$$\min \quad u = u_{m+1} - \varepsilon \Sigma_{g=2}^{r+1} u_{m+g}$$
$$s.t.$$
$$\Sigma_{i=1}^{m+r+1} u_i \tilde{A}_{ij} \quad \tilde{Q} \quad \tilde{c}_j, \quad j = 1,\ldots,n+r$$
$$u_i \ge 0, \quad i = 1,\ldots,m+r+1 \qquad (12)$$

Here, we consider either $\tilde{P} = \le^{Pos}$, $\tilde{Q} = <^{Nec}$ or $\tilde{P} = <^{Nec}$, $\tilde{Q} = \le^{Pos}$. In problem (11), maximization is considered with respect to fuzzy relation $\tilde{P}$. On the other hand, minimization is considered with respect to fuzzy relation $\tilde{Q}$, which can be formulated analogically.

In the following duality theorems we present two versions: (I) for fuzzy relation $\le^{Pos}$ in problem (11) and fuzzy relation $<^{Nec}$ in problem (12), and (II), for fuzzy relation $<^{Nec}$ in problem (11) and fuzzy relation $\le^{Pos}$ in problem (12). In order to prove duality results we assume $\alpha = \beta$. Otherwise, the duality theorems in our formulation do not hold, for more details see (Ramik, 2006a). Moreover, we assume that each objective function is associated with a weight $\lambda_k > 0, \quad k = 1,\ldots,q$, such that $\Sigma_{k=1}^{q} \lambda_k = 1$ where λ_k may be interpreted as a relative importance of the k th objective function. The corresponding proofs can be found in the study of Ramik (Ramik, 2006a).

Weak Duality Theorem. Suppose $\tilde{c}_{kj}$, $\tilde{A}_{ij}$ and $\tilde{B}_i$ are fuzzy numbers, $k = 1,\ldots,q, i = 1,\ldots,m+r+1, j = 1,\ldots,n+r$.

(I) Let $\tilde{X}$ and $\tilde{Y}$ are respectively the feasible regions of problems (11) and (12) in which $\tilde{P} = \le^{Pos}$ and $\tilde{Q} = <^{Nec}$. If $X = (x_1, x_2,\ldots,x_n,\gamma_1,\gamma_2,\ldots,\gamma_r) \in [\tilde{X}]_\alpha$ and $u = (u_1,u_2,\ldots,u_{m+r+1}) \in [\tilde{Y}]_{1-\alpha}$, then

$$\Sigma_{j=1}^{n}\Sigma_{k=1}^{q} \lambda_k \tilde{c}_{kj}^R(\alpha) x_j = \Sigma_{j=1}^{n+r} \tilde{c}_j^R(\alpha) X_j \le \Sigma_{i=1}^{m+r+1} \tilde{B}_i^R(\alpha) u_i$$

(II) Let $\tilde{X}$ and $\tilde{Y}$ are respectively the feasible regions of problems (11) and (12) in which $\tilde{P} = <^{Nec}$ and $\tilde{Q} = \le^{Pos}$. If $X = (x_1, x_2,\ldots,x_n,\gamma_1,\gamma_2,\ldots,\gamma_r) \in [\tilde{X}]_{1-\alpha}$ and $u = (u_1,u_2,\ldots,u_{m+r+1}) \in [\tilde{Y}]_\alpha$, then

$$\Sigma_{j=1}^{n}\Sigma_{k=1}^{q} \lambda_k \tilde{c}_{kj}^L(\alpha) x_j = \Sigma_{j=1}^{n+r} \tilde{c}_j^L(\alpha) X_j \le \Sigma_{i=1}^{m+r+1} \tilde{B}_i^L(\alpha) u_i$$

Strong Duality Theorem. Consider $\tilde{c}_{kj}$, $\tilde{A}_{ij}$ and $\tilde{B}_i$ are fuzzy numbers, $k = 1,\ldots,q, i = 1,\ldots,m+r+1, j = 1,\ldots,n+r$.

(I) Suppose $[\tilde{X}]$ is the feasible region of problem (11) with $\tilde{P} = \le^{Pos}$, $[\tilde{Y}]$ is the feasible region of problem (12) with $\tilde{Q} = <^{Nec}$. If for some $\alpha \in (0,1)$, $[\tilde{X}]_\alpha$ and $[\tilde{Y}]_{1-\alpha}$ are nonempty, then there exists $X = (x_1, x_2, ..., x_n, \gamma_1, \gamma_2, ..., \gamma_r) \in [\tilde{X}]_\alpha$ which is (α, α)- maximal solution of (11) with respect to $\leq^{Pos}$, and $u = (u_1, u_2, ..., u_{m+r+1}) \in [\tilde{Y}]_{1-\alpha}$ which is $(1-\alpha, 1-\alpha)$- minimal solution of (12) with respect to $<^{Nec}$, such that

$$\Sigma_{j=1}^{n}\Sigma_{k=1}^{q}\lambda_k \tilde{c}_{kj}^{R}(\alpha)x_j = \Sigma_{j=1}^{n+r}\tilde{c}_j^{R}(\alpha)X_j = \Sigma_{i=1}^{m+r+1}\tilde{B}_i^{R}(\alpha)u_i$$

(II) Let $[\tilde{X}]$ be the feasible region of problem (11) in which $\tilde{P} =<^{Nec}$ and $[\tilde{Y}]$ be the feasible region of problem (12) in which $\tilde{Q} =<^{Pos}$. If for some $\alpha \in (0,1)$, $[\tilde{X}]_{1-\alpha}$ and $[\tilde{Y}]_\alpha$ are nonempty, then there exists $X = (x_1, x_2, ..., x_n, \gamma_1, \gamma_2, ..., \gamma_r) \in [\tilde{X}]_{1-\alpha}$ which is $(1-\alpha, 1-\alpha)$-maximal solution of (11) with respect to $<^{Nec}$ and $u = (u_1, u_2, ..., u_{m+r+1}) \in [\tilde{Y}]_\alpha$ which is (α, α)- minimal solution of (12) with respect to $\leq^{Pos}$, such that

$$\Sigma_{j=1}^{n}\Sigma_{k=1}^{q}\lambda_k \tilde{c}_{kj}^{L}(\alpha)x_j = \Sigma_{j=1}^{n+r}\tilde{c}_j^{L}(\alpha)X_j = \Sigma_{i=1}^{m+r+1}\tilde{B}_i^{L}(\alpha)u_i$$

CONCLUSION

In this paper, the dual of MC^2LP when data are fuzzy parameters was presented, while the concept of the possibility and necessity relations was used. In continuation, weak and strong duality theorems corresponding to these models were stated. As we know, the possibility and necessity relations do not have self-duality property. In the future, we will apply credibility theory on FMC^2LP to obtained its dual in which has the self-duality property.

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