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DESIGNING AND SIMULATING AN OPTIMAL LINEAR CONTROLLER FOR VEHICLE ACTIVE SUSPENSION SYSTEM CONSIDERING THE NONLINEAR DYNAMICS OF HYDRAULIC ACTUATOR

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ABSTRACT
In this article an optimal linear quadratic regulator (LQR) controller is designed by considering theNonlinear Dynamics of Hydraulic Actuator for controlling vehicle active suspension system; and the aim of this design is constraining the vibrations on vehicle body. Here the concepts of active suspension systems are surveyed and improving the passenger's comfort and ride quality are the most important aims of this article. From different points of view, suspension systems have different categories and different types of quarter car models are explained. Also, history and researches conducted in the field of vehicle suspension systems are stated and driving features and performance of suspension systems are surveyed. Ultimately, the optimal linear control method is explained and after conducting simulations, the results of simulation and controller design are surveyed. A quarter car suspension system model and a nonlinear hydraulic actuator model are used for simulating the control system. It could be concluded that the main aim of designing the vehicle (car) active suspension systems is making a peace between ride quality steering capability suspension displacement and energy consumption. Results show that by optimal controlling of an active suspension system used by a hydraulic actuator, the vibrations of vehicle body significantly decrease and as a result the passenger's comfort and ride quality are significantly improved.

Keywords: Active Suspension System, Non-linear Control, Ride Comfort, Actuator Dynamics, Linear Quadratic Regulator (LQR)

INTRODUCTION
A road as flat and smooth as possible, is not a good place for moving one or a few tons of metal with high speed. Thus there is a need for a system that has the ability to decrease bumps, shakes and vibrations caused by the road condition. Additionally, a vehicle must be flexible toward the change of applied load and change of point of gravity and it must have the ability to deal with them; in case of lack of a system for changing the balance, the car is deviated at the first turn or overturned. Development of control methods for vehicle active/inactive suspension systems is one of the main issues of car industries. A good suspension system must be able to improve the ride quality, steering capability and also passengers' comfort, at the same time.
For increasing the passenger's comfort, the vertical acceleration of the vehicle caused by road vibrations must be limited which means that the suspension system must absorb the road vibrations and prevent it from transferring to the vehicle body and passengers. In other words, the contact of tire with road surface must be decreased as much as possible. On the other hand, for increasing the vehicle steering capability, tire must have the maximum contact with road surface. Primary suspension systems were consisted of a few springs and dampers and since no active element was used in their structure, they were called inactive suspension systems. The flexibility of this system was very weak and only the hardness of spring and damper was changeable in the suspension structure. Thus, regarding the limitations existing in inactive systems, researchers turned toward active and inactive suspension systems; systems that in addition to inactive elements, had a force applying actuator. During the recent years different control methods have been used for controlling the active suspension system; and the main method has been the optimal control method. In most conducted works, the model considered for the vehicle has been a linear model and in

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that model the optimal linear control methods such as LQ, LQR and LQG are used; whereas the actual model of vehicle is nonlinear.

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Creation of suspension system dates back to more than five hundred years ago before the Sumerians; and it began when the wheel was invented; and since then it had a continuous development. In Persepolis a part of this invention dating back to 2500 years ago is observable. Iranians were the first people who optimized wheel and added tread to it. The first shock absorbers were installed on carriages with leaf spring in 1904; and it resulted in softness and speed of this vehicle. Before that this vehicle was not very comfortable.

The optimal control methods of vehicle suspension system caught the attention of global car industries from 1970's onwards (Camino *et al*., 1999). As it is observable in most studies, the performance of vehicle suspension system significantly improves with an optimal controller. LQR optimal linear control method is one of the most important control methods in vehicle suspension system (Thompson, 1976). This control method minimizes the cost functional.

In a research conducted by Rizzoni & Engelman (1993), it has been observed that LQG controllers designed by them do not result in desirable performance of the system; because the hydraulic driving force must be very low for lack of system instability. Recently in a research conducted in University of Michigan (1996), LQG controllers are used on a quarter car test model and it has been observed that the test model is only stable in small control gains and in these gains the desirable performance of system is unachieved.

Linear stimulating dynamics are used in both conducted researches and the LQG controller designed in them is based on the system dynamics and linear stimulating dynamics. A method called LTR was suggested by Bird Well in 1990 in order to improve the system stability area. The error of this method was that it disturbed the system performance.

Surveying the Active Suspension System

A soft spring (smaller k_s) is required for having desirable vibration isolation in a wide frequency range; whereas for having desirable controllability of the vehicle in a frequency near the normal frequency of the wheel, a hard spring (bigger k_s) is needed. For decreasing the range of vehicle body vibrations in a frequency near the natural vehicle body frequency, a greater damping ratio is needed; whereas, for this in a frequency range higher than the normal frequency of the vehicle body a smaller damping ratio is needed. In other words, for reaching the desirable controllability of vehicle in high frequencies there is a need to use greater damping ratio. By the use of an inactive suspension system it is impossible to meet both needs which are vibration isolation and controllability of vehicle; to this aim using an active suspension system is suggested.

In below figure the structure of an active suspension system is shown:
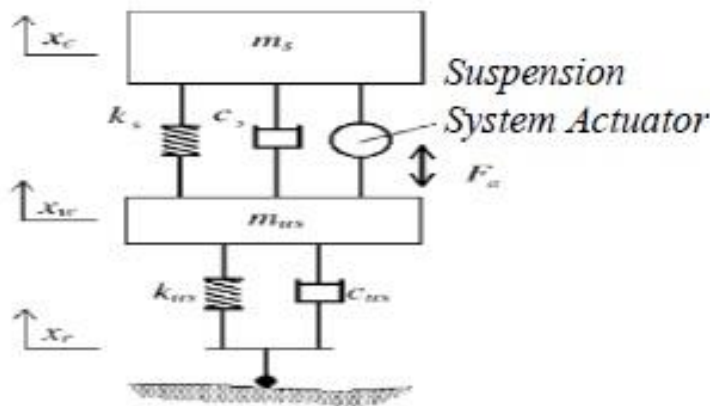

Figure 1: Model of active suspension system.

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In this model the performance of vehicle is repeatedly measured by embedded sensors and applied to control system and output controller of the control system creates a force in hydraulic actuator in order to increase the ride quality, vibration isolation and controllability of the vehicle.

Components of an electro-hydraulic actuator are shown in below figure:
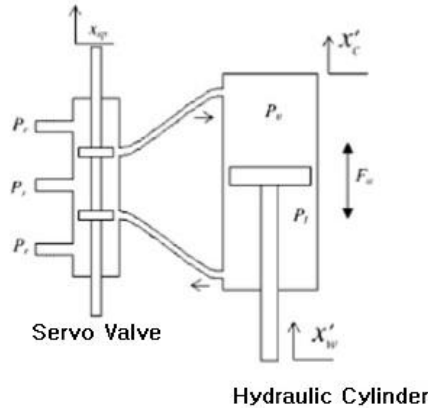

Figure 2: Components of an electro-hydraulic actuator

Generally, the aim of optimal control of a suspension system is decreasing three following characteristics:
1. RMS values of vehicle vertical acceleration.
2. RMS values of suspension displacement.
3. RMS values of tire dynamic compression.

Other duties of active suspension system include controlling height, anti-roll and anti-dive.

The aim of controlling height is holding the driving height of the vehicle body regardless of load changes; and this result in creation of a desirable suspension for road vibrations isolation. In order to decrease the aerodynamic resistance in high speeds, eliminating the roughness of road, and suspension displacement an active suspension system must be developed to achieve a desirable driving performance. An active suspension system, an external source of force actuator, is very important; the existence of such source results in increasing the system cost in terms of price, weight, system complexity and reliability.

Quarter Car Model with Hydraulic Actuator Dynamics

For modelling a car, different mathematical models can be used regarding the type of application; for example, for anti-roll and anti-dive, half-car model must be used in order to also have the equations related to torque; but since our aim in this project is controlling the vehicle vertical acceleration, we must use quarter-car model. Models used for quarter-car are usually linear and they are different for active and inactive suspension systems. Quarter-car model considering hydraulic actuator dynamics is shown in below figure.
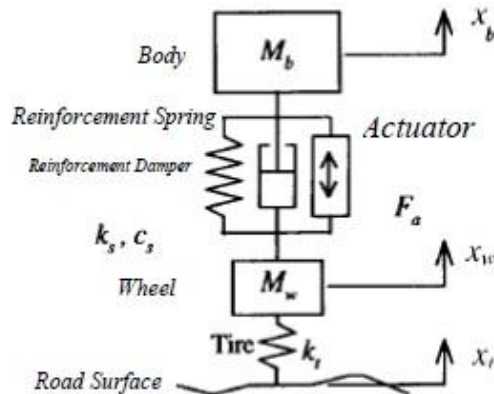

Figure 3: Quarter-car model with hydraulic actuator dynamics

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As it is observable in the figure, in this model no damper is used between wheel and road surface. With these interpretations and by the use of vector system model, the state variables are defined as follows:

$$X = \begin{bmatrix} x_s - x_w \\ \dot{x}_s \\ x_w - x_r \\ \dot{x}_w \\ p_t \\ x_v \end{bmatrix}$$

(1)

Thus the state system form is as follows:

$$\begin{cases} \dot{x}_1 = x_2 - x_4 \\ \dot{x}_2 = -\dfrac{1}{M_b}\left[K_s x_1 + c_s(x - x_4) - A x_5\right] \\ \dot{x}_{3=x_4-\dot{x}_r} \\ \dot{x}_4 = \dfrac{1}{M_w}\left[k\, x_1 + c_s(x_2 - x_4) - k_t x_3 - A x_5\right] \\ \dot{x}_5 = -\beta x_5 - \alpha A(x_2 - x_4) + \gamma x_6 w_3 \\ \dot{x}_6 = \dfrac{1}{\tau}(-x_6 + u) \end{cases}$$

(2)

In which x_1 is suspension displacement, x_2 is body vertical speed, x_3 is tire compression, x_4 is tire vertical speed, x_5 which is p_t is pressure drop across the piston, x_6 which is x_7 is the valve displacement to its closed state and x_r is road surface entrance.

Also $W_3 = \sqrt{p_s - sign(x_6)x_5}$ and Mb is body mass, Mw is wheel mass, Ks is spring constant of suspension, Kt is wheel spring constant, and cs is the damper characteristic of suspension.

A is piston surface in which:

$$\alpha = 4\beta_e / v_t$$

$$\beta = \alpha c_{tp}$$

$$\gamma = \alpha c_d w \sqrt{1/\rho}$$

(3)

Be is effective volume correction factor, vt is total volume of actuator, Q is hydraulic load discharge, ctp is total piston leakage rate, w is slope of surface of valve spool, ps is actuator supply pressure, p is density of hydraulic fluid and cd is discharge coefficient.

As it is observable the above mentioned system is degree-6 and its four state variables are related to suspension system and two state variables are related to hydraulic actuator dynamics existing in the system.

Optimal Linear Control Method

Assuming an invariant linear system with time and following state equations:

$$\dot{x}(t) = Ax(t) + B_u U(t) \, ; \, X(0) = X_0$$

$$y(t) = C_y x(t) \text{ (5-1)}$$

(4)

In which A is a n×n matrix, Bu is a n×1 matrix, X(t) is the state vector, y(t) is output vector and u(t) is control signal.

Our controlling aim is minimizing the following cost function:

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$$J(y,u) = \frac{1}{2} y^T (\mathrm{t}_f) \mathrm{H}_y\, y(t_f) + \frac{1}{2} \int_0^{t_f} (y^T(t)Q_y y(t) + \mathrm{u}^T(t)Ru(t))\,\mathrm{dt} \qquad (5)$$

In which Q_y, R, and H matrices are positive definite, square, symmetric matrices with appropriate dimensions.

Now the above mentioned cost function is explained based on input; to this aim the following equation is written:

$$y(\mathrm{t}) = \mathrm{C}_y\, x(t)$$

Then we have:

$$J(\mathrm{x,u}) = \frac{1}{2} x^T (t_f) H x(t_f) + \frac{1}{2} \int_0^{t_f} (x^T(\mathrm{t})Qx(t) + u^T(t)Ru(t))\,dt \qquad (6)$$

In which we have:

$$H = C_y^T H_y C_y$$

$$Q = C_y^T Q_y C_y \qquad (7)$$

The optimization is a constrained optimization with dynamic equations related to the system; and by the use of Lagrange multipliers method it is converted to unconstrained optimization. To this aim the system dynamic equations are added to a cost function by the use of a coefficient named Lagrange multipliers vector:

$$J_a(x,u,p(t)) = J(x,u) + \int_0^t p^T(t)(Ax(t) + B_u u(t) - \dot{x}(t))\,dt \qquad (8)$$

In which p(t) is Lagrange multipliers vector (semi-state).

The calculus of variations method is used for system optimal control; and in this method the function of variations is as follows:

$$\delta J_a(x,u,p,\delta x,\delta u,\delta p) = \frac{d}{d\alpha} J_a(x + \alpha \delta x, u + \alpha \delta u + \alpha \delta p) \qquad (9)$$

In which δx are variations of x, δu is variations of u and δp is variations of p.

For achieving the optimal control system input, since the amount of function of variations of δj_a in optimal point equals zero, thus 9 relationship is expanded and we have:

$$\delta J_a(x,u,p,\delta x,\delta u,\delta p) = (x^T(t_f)H - p^T(t_f)) +$$

$$\int_0^t \{(x^T(t)Q + p^T(t)A + p(t)\delta x + (u^T(t)R + p^T(t)B_u)\delta u + (Ax(t) + B_u u(t) - x(t)) \qquad (10)$$

Since the amount of function of variations of δj_a at optimal point equals zero, thus coefficients of δx, δu and وδp also equal zero.

As a result the following relationships are achieved:

$$\begin{cases} \dot{P}(t) = -x^T(t)Q - P^T(t)A \\ \quad P^T(t_f) = x^{TT}(t_f)H \\ \quad u(t) = -R^{-1}B_u^{-1}p(t) \\ \quad \dot{x}(t) = Ax(t) + B_u u(t) \end{cases} \qquad (11)$$

(5-7)

And if the u control input is eliminated in the above mentioned relationships, then we have:

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$$\begin{bmatrix} \dot{x}(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} A & -B_u R^{-1} B_u^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} \tag{12}$$

(5-8)

And by defining the Z vector as follows we have:

$$z = \begin{bmatrix} A & -BR^{-1}B_u^T \\ -Q & -A^T \end{bmatrix} \qquad\qquad \begin{bmatrix} \dot{x}(t) \\ \dot{p}(t) \end{bmatrix} = z \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} \tag{13}$$

In which the boundary conditions will be as follows:

$$\begin{cases} x(0) = x_0 \\ p(t_f) = Hx(t_f) \end{cases} \tag{14}$$

This optimal control system is a Hamiltonian System and Matrix Z is called a Hamiltonian Matrix. By solving the above mentioned state equations, amounts of x(t) and p(t0 are achieved and since our control aim is achieving the optimal control input for minimizing the cost function, by putting the above mentioned values in 10 relationship the optimal control input values of u(t) are also achieved.

To this aim, by using the state transition matrix, the following relationship will be achieved:

$$\begin{bmatrix} x(t_f) \\ p(t_f) \end{bmatrix} = e^{Z(t_f - t)} \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} = \begin{bmatrix} \phi_{11}(t_f - t) & \phi_{12}(t_f - t) \\ \phi_{21}(t_f - t) & \phi_{22}(t_f - t) \end{bmatrix} \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} \tag{15}$$

Also, by solving the above-mentioned equations, the p(t) values will be achieved:

$$p(t) = \left[\phi_{22}(t_f - t) - H\phi_{12}(t_f - t) \right]^{-1} \left[H\phi_{11}(t_f - t) - \phi_{21}(t_f - t) \right] x(t) \tag{16}$$

By defining p(t) values we will have:

$$p(t) = \left[\phi_{22}(t_f - t) - H\phi_{12}(t_f - t) \right]^{-1} \left[H\phi_{11}(t_f - t) - \phi_{21}(t_f - t) \right] \tag{17}$$

$$p(t) = p(t)x(t) \tag{18}$$

And ultimately, the optimal control system input is achieved by putting achieved p(t) values in 10 relationship:

$$u(t) = -R^{-1}B_u^T p(t) = -R^{-1}B_u^T p(t)x(t) = -K(t)x(t) \tag{19}$$

In which we have:

$$k(t) = R^{-1}B_u^T p(t) \tag{20}$$

As it is observed, with this control procedure, the optimal control input is achieved for minimizing the system cost function.

Simulation

In this part, the simulations conducted on an active suspension system of quarter-car model with hydraulic actuator are surveyed; and the effects of designing an optimal linear controller on it are shown by the use of simulation and Matlab software.

Dynamic equations related to surveyed active suspension system are as follow:

$$m_s \ddot{z}_s = -b_s (\dot{z}_s - \dot{z}_u) - k_s (z_s - z_u) + f_a \tag{21}$$

$$m_u \ddot{z}_u = b_s (\dot{z}_s - \dot{z}_u) + k_s (z_s - z_u) - f_a + b_t (\dot{z}_r - \dot{z}_u) + k_t (z_r - z_u) \tag{22}$$

In which ms, mu, ks, kt, bs and bt are mass, coefficient of hardness and damping coefficient of elements related to system; system consists of a hydraulic actuator placed between ms and mu masses and it applies a fa force between vehicle body and wheel.

Servo valve system formula is as follows:

$$\dot{x}_{sp} = (-x_{sp} + i_{sv}) / \tau \tag{23}$$

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In which, isv is amount of flow and T is the mechanical time constant of Servo valve system.
Dynamic equation of hydraulic system is stated through the following relationship:

$$\dot{f}_a = -\alpha A_p \, (\dot{z}_s - \dot{z}_u) - \beta f_a + \gamma x_{sp} \sqrt{p_s - \text{sgn}(x_{sp}) f_a / A_p} \qquad (24)$$

In which we have:

$$\alpha = 4\beta_e A_p / V_t$$

$$\beta = \alpha C_{tp} A_P$$

$$\gamma = \alpha C_d w A_p \sqrt{1/\rho}$$

In the above mentioned equation, Ap is the piston surface, βe is hydraulic fluid correction coefficient, Vt is total capacity of actuator cylinder chambers, Ctp is piston total leakage rate, Cd is discharge coefficient, W is width of the valve spool, p is relative density of hydraulic fluid.

By defining system states as $x = \begin{bmatrix} z_s & \dot{z}_s & z_u & \dot{z}_u \end{bmatrix}^T$ system state equations are written as follow:

$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t) \qquad (25)$$

In which control signal of u(t) is the same fa.
For designing the LQR controller for the surveyed system, firstly a state feedback gain matrix is calculated in order to minimize the system cost function in order to achieve a stable system. For the above mentioned linear system, the control signal is defined as follows:

$$u(t) = -kx(t) \qquad (26)$$

In which we have:

$$k = R^{-1}B^T X \qquad (27)$$

And X is achieved from solving the below matrix differential Riccati equation (MDRE):

$$-\dot{X}(t) = A^T(t)X(t) + X(t)A^T(t) + Q(t) - X(t)B(t)R^{-1}B^T(t)X(t)$$

Cost function with the aim of designing an optimal controller for minimizing it is defined as follows:

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u)\,dt \qquad (28)$$

In which matrices of Q (state weight matrix) and R (control weight matrix) are positive semi-definite and diagonal matrices and their values for reaching the best answer for above mentioned active suspension system (desirable Q & R) include:

$$Q = \begin{bmatrix} 60000 & 0 & 0 & 0 \\ 0 & 60000 & 0 & 0 \\ 0 & 0 & 60000 & 0 \\ 0 & 0 & 0 & 60000 \end{bmatrix} \qquad (29)$$

R= 1   (30)
Values used for simulating considered suspension system are shown in below table:

Table 1: Suspension system characteristics

$m_s = 243\text{kg}$	$A_p = 3.35 \times 10^{-4} m^2$
$m_u = 40\text{kg}$	$p_s = 10342500\text{Pa}$
$b_s = 370\text{N}/(\text{m}/\text{s})$	$\tau = 30\text{s}$
$b_t = 414\text{N}/(\text{m}/\text{s})$	$\alpha = 4.515 \times 10^{13}$
$k_s = 14671\text{N}/\text{m}$	$\beta = 1$
	$\gamma = 1.545 \times 10^9$
$k_t = 14671\text{N}/\text{m}$	

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For surveying the effect of Q and R weight matrices on designing the LQR optimal linear controller, values related to each of them are changed in below sample states and their simulation results are considered regarding the road input.

1st state:

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix}, R=10$$

2nd state:

$$Q = \begin{bmatrix} 50000 & 0 & 0 & 0 \\ 0 & 10000 & 0 & 0 \\ 0 & 0 & 10000 & 0 \\ 0 & 0 & 0 & 10000 \end{bmatrix}, R=1$$

3th state:

$$Q = \begin{bmatrix} 60000 & 0 & 0 & 0 \\ 0 & 60000 & 0 & 0 \\ 0 & 0 & 60000 & 0 \\ 0 & 0 & 0 & 60000 \end{bmatrix}, R= 10$$

Simulation Results

Simulation results are states in two active and passive states for states of Q and R weight matrices of values for surveying the sensitivity of designed control system toward change of these parameters and then the total results achieved from this simulation are surveyed.
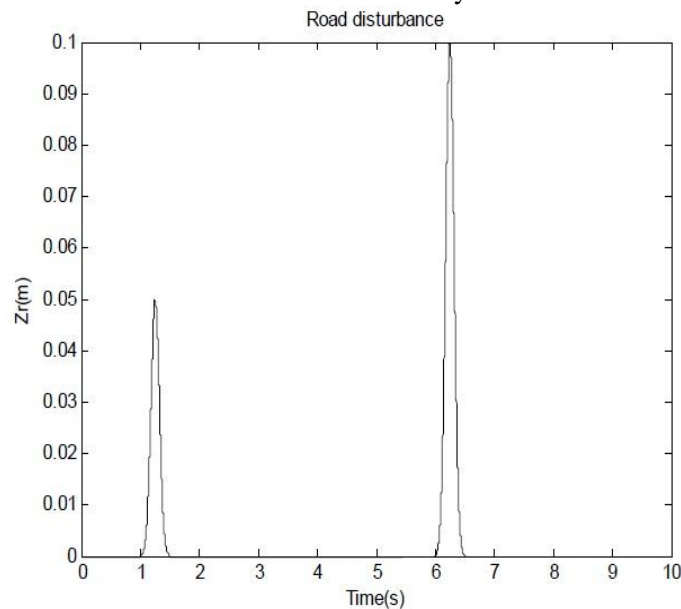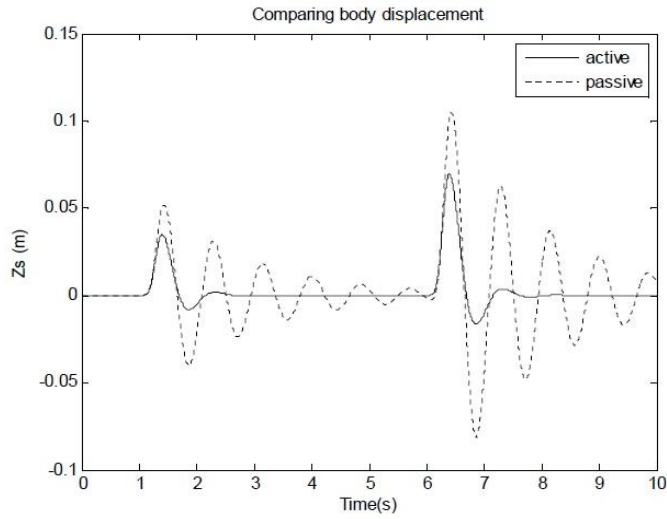

Figure 4: Road displacement

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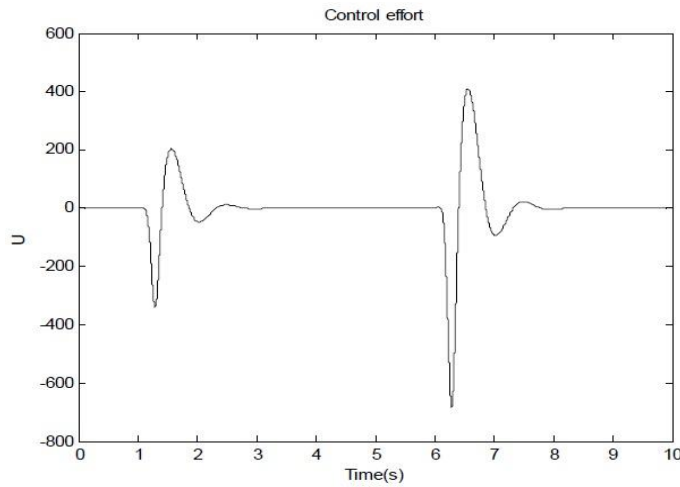**Figure 5: Body displacement for desirable Q & R**

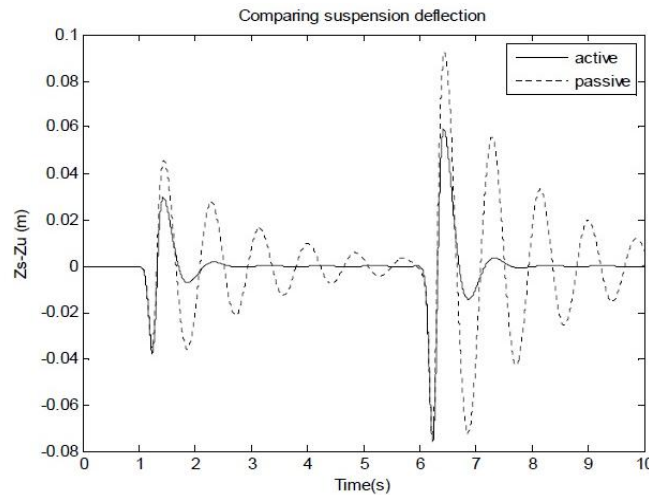**Figure 6: Control effort of active suspension system for desirable Q & R**

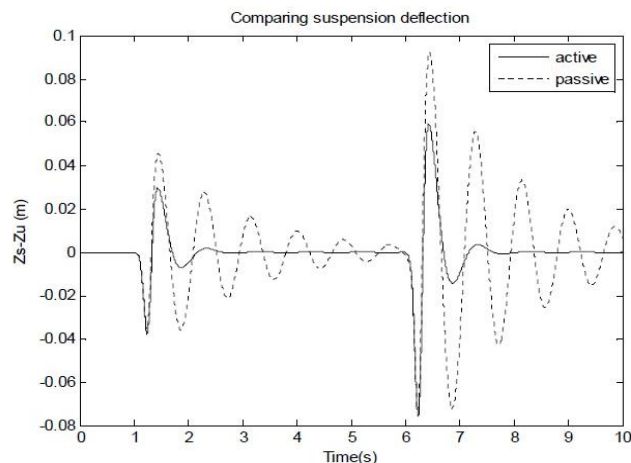**Figure 7: Body vertical acceleration for desirable Q & R**

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Figure 8: Suspension displacement for desirable Q & R
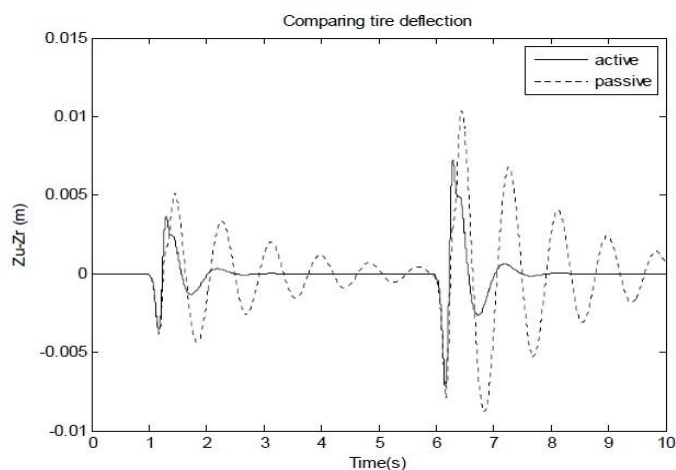

Figure 9: Tire dynamic compression for desirable Q & R

RESULTS AND DISCUSSION
Results
Simulation results for surveying the performance of control system comparing at two open-loop and closed-loop states are provided. In simulation conducted by the use of Matlab software, a specific input is applied as the roughness of road on active suspension system; and regarding the diagram of road displacement, the body displacement, body vertical acceleration, suspension displacement and tire dynamic compression diagrams are drawn at two active and passive states. Surveying the drawn diagrams at two open-loop and closed-loop states, it could be seen that system performance at active state (after applying control signal) had a significant improvement compared to passive state.

The achieved improvement is caused by the improvement in below indicators:
Decreased Vehicle Body Vertical Acceleration Range
In diagram drawn related to vehicle body vertical acceleration it could be observed that vehicle body vertical acceleration range at active state had significantly decreased compared to passive state. Thus the main aim of designing LQR optimal controller which is decreasing the vehicle body vertical acceleration and as a result, decreasing the vertical forced applied on passenger is realized.

Also the time period of reaching a permanent state at active state decreases compared to the passive state. As a result, the passenger's comfort at active state and after designing the controller against road vibration, had a considerable improvement

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Decreased Suspension Displacement Range

Surveying the diagram drawn for suspension displacement state, it could be concluded that suspension displacement range after designing the controller had a significant decrease; thus another important aim of designing the vehicle active suspension system which is increasing the ride quality and vehicle stability as a result of controlling suspension displacement is significantly improved by decreasing the suspension displacement range.

Decreased Tire Dynamic Compression Range

This characteristic is also considered in designing the vehicle suspension system controller in order to increase the ride quality and observing the diagram of tire dynamic compression it could be seen that at active state, the tire dynamic compression range had a considerable decrease compared to the passive state and as a result it is observed that designing LQR controller for suspension system results in increased ride quality and vehicle stability; thus regarding the simulation results, it could be said that an active suspension system could adapt itself with the road conditions.

Diagram of tire dynamic deflection right at the time of colliding with vibrations, show little distortions and also suspension displacement after designing control had a very small increase and also the control effort range had a little decrease. Optimal control starts working during collision with roughness of the road and as figures show, both demands of passenger's comfort and vehicle steering capability are provided. Using and implementing the designed control for vehicle suspension system is very simple. Designing classic controllers has a very effective role in controlling most dynamic systems. As a result, using these controllers could be very beneficial.

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