

ESTIMATION OF BEARING CAPACITY AND SETTLEMENT OF SPREAD FOOTINGS OVER STONE-COLUMN-REINFORCED CLAY USING FUZZY MODELS AND ARTIFICIAL NEURAL NETWORKS

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ABSTRACT

Stone columns, also known as granular piles, have been extensively used to improve bearing capacity and reduce settlement of foundations lying over soft soils. However, due to uncertainties existing in behavior of the column's material and its surrounding soil, mechanical response of this composite ground is not fully understood. Recently, some efforts have been made to estimate bearing capacity and settlement of spread footings on the composite ground more accurately using statistical analyses. Although significant results have been reported in these studies, the proposed statistical models may have some shortcomings and do not fit well to the experimental data. In current study, fuzzy models and artificial neural networks have been used to evaluate mechanical behavior of the composite ground. In all cases, an information criterion is applied to obtain the most optimized architecture of the models. This is particularly recommended when a few number of high quality data is available for training and testing the models. It is shown, through comparison with experimentally measured data, that the proposed models can accurately estimate bearing capacity and settlement of foundations on the ground reinforced with stone columns.

Keywords: *Stone Column; Bearing Capacity; Clayey Soil; ANFIS; Artificial Neural Networks*

INTRODUCTION

Clay soils are of lower strength compared with granular soils such as sand and gravel. This implies that when a heavy building is constructed over a clayey bed, either the soil is failed in shear or significant settlement is occurred both of which result in serious problems in operation of the structure. One way of solving this problem is drilling some holes at specified intervals and filling them with granular material prior to construction of the foundation. These structures are terminologically called stone columns or aggregate piers and the set of soil - stone columns is known as composite or reinforced ground. The bearing capacity and settlement of the composite ground may not be easily determined since the mechanical behavior of these two basic elements, i.e. clay and stone columns, is of high complexity and cannot be adequately explained through theoretical methods such as the theory of elasticity. Moreover, the combination itself and the resulting interactions should be taken into consideration. In such circumstances, usually the numerical modeling techniques, e.g. finite element method, are regarded as the first and cheapest method of analysis. The other and more reliable solution is conducting field tests on the real ground and assessing its behavior. However, such tests are expensive and require a wide range of equipments as well as skilled and experienced manpower. Development of accurate and applied techniques for estimating the bearing capacity and settlement of clay ground reinforced with stone columns is the issue addressed in this paper.

As one of the earliest attempts for analyzing the behavior of stone columns, Greenwood (Greenwood, 1970) studied the behavior of a single stone column using the theory of plasticity. Afterwards, Brauns (1978) and Van *et al.*, (1997) have proposed modifications to this initial equation. In addition, Fox and Cowell (1998) and Wissmann (1999) presented some equations similar to the well-known relations used to evaluate the bearing capacity of shallow foundation (such as Terzaghi's relations (1943)). On the other hand, Balaam *et al.*, (1977) and Balaam and Booker (1988) were the first who analyzed the behavior of these columns using simple assumptions of the theory of elasticity. Noorzad *et al.*, (1997) assumed the behavior to be elasto-plastic and calculated the total settlement of the ground. Guetif *et al.*, (2007)

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investigated the response of the clayey bed reinforced with stone columns using vibro-compaction technique. The main objective of this study was estimation of the Young's modulus of the composite ground which was achieved through a finite element analysis using PLAXIS software. This study revealed that construction of stone columns using the aforementioned technique results in initial pore water pressure in the clay environment which is gradually vanished over the time. Consequently, the properties of consolidated clay (including deformability and Young's modulus) are improved.

Based on the results of modified triaxial tests on the soil samples reinforced with cemented piles, Juran and Riccobono (1991) assessed the influence of cementation as well as the grouping effect on the settlement of the composite ground. After analyzing the relationship between axial strain and stress ratio and the relationship between axial strain and volumetric strain, Pooroshasb and Meyerhof (1997) derived a correlation between the applied load and the settlement for the stone-columns-reinforced grounds.

The idea of encasing the stone columns has been discussed in recent years (Gniel, 2009). This is especially important in very soft soils where either the construction of ordinary stone columns is not generally feasible or the columns bugle under the loading effects making them practically useless. Other solutions have been also proposed for very soft soils, such as putting horizontal geogrid layers inside the stone columns (Sharma *et al.*, 2004). Nevertheless, such techniques of implementation cause practical problems which may reduce relative merits of stone columns, therefore, these techniques are only recommended in the cases where the surrounding clay is of very low strength.

During past decade, computational intelligence techniques have been successfully applied for modeling complex systems in science and engineering. In current study, mechanical behavior (bearing capacity and settlement) of clay bed reinforced with stone columns is evaluated using fuzzy models as well as artificial neural networks. The data required for training and verifying the proposed models are collected from published reports in the literature. In addition, the results are compared with recently published researches and their relevance is demonstrated.

Stone Columns

Stone columns also known as granular piles and aggregate piers are one of the techniques used to improve soft soils and loose strata. Stone columns are cylindrical elements which gain their strength and stiffness from the confinement provided by the surrounding soil. This implies that the lateral (radial) strain developed under loading and the resulting interaction between soil and column mobilize the strength of stone-column-improved ground.

Due to their efficiency, easy construction and availability of the material with low price, stone columns are regarded as a very common method of soil improvement. These columns are a vertical support for their overlying embankments and structures. They decrease the settlement and increase the bearing capacity. In fine-grained impermeable soils, they act as vertical drains and hence reduce the time of consolidation. Moreover, if the soil is cohesion-less, like sands and silts, this drainage function helps to mitigate liquefaction potential under cyclic loading of earthquake.

Stone columns may be used either in linear configuration to support walls and strip foundations or as triangular/rectangular group to support mat foundations and embankments. In the cases which stone columns are employed to support a vertical load (of a foundation for example), the bearing capacity of column itself is of importance. On the other hand, in some occasions such as earth slopes, stone columns rely on their drainage capabilities to increase the stability of the slope.

Stone columns may be constructed through a variety of techniques such as vibro-replacement, vibro-flotation, pre-drilling and encased borehole methods. As shown in Figure 1, in vibro-replacement method, a long torpedo-shaped probe is driven into the soil either by the vibration only (dry method) or by a combination of vibration and water jet (wet method). Once the probe excavated the ground up to the desired depth, the aggregates are poured into the hole and compacted by the vibratory probe. Pre-drilling method has been used during past decade as a relatively low-cost alternative of other techniques which require expensive equipment and skilled workforce. In this method, the aggregate material are dumped into a hole, which is previously excavated using auger, and then compacted by falling weights. More details on construction of stone columns may be found in the literature (Barksdale and Bachus, 1983).

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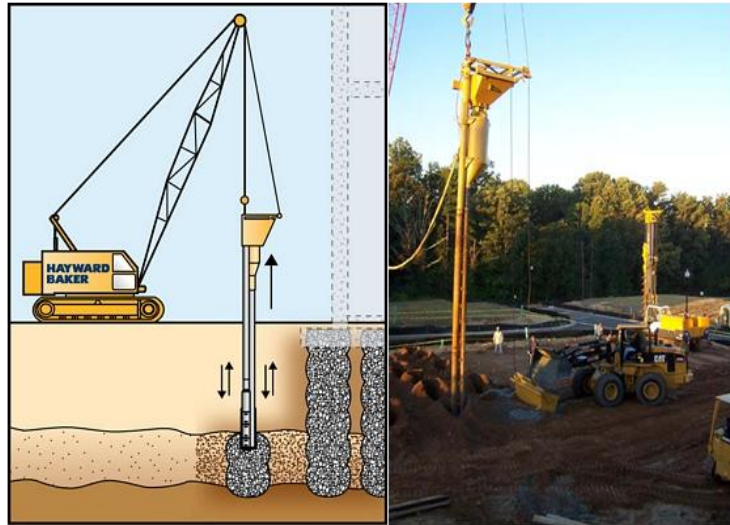


Figure 1: Construction of stone columns by vibro-replacement technique (after Hayward Baker, 2012)

Analyzing the Behavior of Stone Columns

Bearing Capacity Estimation

Brauns (1978) demonstrated that the ultimate bearing capacity of stone column with the surcharge q over the surrounding soil can be estimated as:

$$q_{ult} = \left(q + \frac{2s_u}{\sin 2\delta} \right) \left(1 + \frac{\tan \delta_s}{\tan \delta} \right) \tan^2 \delta_p \quad (1)$$

Where s_u is undrained shear strength of the native soil; δ_p and δ_s are failure angle in aggregate material of the column and the surrounding soil respectively; δ is the conical angle of shearing in the native soil. It can be shown that for a typical value of $\delta_p = 45^\circ$, the conical angle of friction equals to 62° .

Mitchell (1981) proposed a simple empirical method to estimate the ultimate capacity of a single isolated stone column:

$$q_{ult} = s_u \cdot N_p \quad (2)$$

where N_p is bearing capacity factor recommended to be 25. Barksdale and Bachus (1983) suggested N_p to be between 18 and 22 for stone columns constructed by vibration based on back- analysis of field tests results. Moreover, Bergado and Lam (1987) recommended N_p ranging from 15 to 18 for stone columns constructed using falling weights (rammed method). Stuedlein and Holtz (2013) proposed the following empirical modification to equation (above):

$$N_p = \exp(-0.0096 \cdot s_u + 3.5) \quad (3)$$

Barksdale and Bachus (1983) also proposed the following equation to evaluate the bearing capacity of a group of stone columns:

$$q_{ult,group} = \sigma_3 \tan^2 \left(45 + \frac{\phi_{ave}}{2} \right) + 2s_{u,ave} \tan \left(45 + \frac{\phi_{ave}}{2} \right) \quad (4)$$

Where ϕ_{ave} and $s_{u,ave}$ are friction angle and undrained shear strength of the composite ground respectively. σ_3 is minor principal stress applied radially around the stone column. Stuedlein and Holtz (2013) performed multiple linear regression (MLR) modeling of the bearing capacity of stone-column-treated ground and obtained the following equation:

$$\ln(q_{ult}) = 4.756 + 0.013S_{rp} + 1.914a_r + 0.07d_f S_{rp} + \frac{-13.71}{\tau_{mp}} + 0.005\tau_{mp} \quad (5)$$

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Where S_{rp} is the slenderness ratio of the column, i.e. the ratio of its lengths to diameter. It will be noted that S_{rp} plays a significant role in the value of bearing capacity; a_r is area replacement ratio which is defined as the ratio of column's cross section to the area of unit cell (see the next section); d_f is depth of footing embedment; and \square_{mp} is the native soil shear mass participation factor defined as the ratio of s_u to a_r .

Settlement Analysis

So far several approaches have been proposed to estimate the settlement of foundations resting over stone columns, e. g. (Fox and Cowell, 1998; Lawton *et al.*, 1994; White *et al.*, 2006). Most of these methods assume that the unit cell concept, which is described in detail by Barksdale and Bachus (1983), is valid. This theory states that the behavior of a group of columns beneath a uniformly loaded area can be simplified to a single column installed at the centre of a cylinder of soil representing the column's zone of influence.

In order to calculate the settlement of stone column improved ground, Lawton *et al.*, (1994) and Fox and Cowell (1998) proposed the ground to be divided into two separate zones namely the upper and lower zone. The settlement of the upper zone is estimated assuming that the surrounding soil and the stone column act as independent elastic springs. On the other hand, settlement of the lower zone is evaluated either with Westergaard stress distributions or by assuming the two-layered soil profile (a stiff, infinite, uniform elastic upper layer and a lower layer representing the unimproved soil).

A MLR analysis on the measured displacements ranging from two to 50 mm with respect to common design variables is reported by Stuedlein and Holtz (2014):

$$\ln(q_s) = b_0 + b_1 a_r + b_2 L_p + b_3 S_{rp} + b_4 \tau_{mp} + b_5 d_f S_{rp} \quad (6)$$

Where b_i ($i = 0,5$) are statistical constants; L_p is the length of stone column. Stuedlein and Holtz (2014) showed that the MLR model conforms well to the measured values of bearing pressures (q_{\square}) for a wide range of displacements and various configurations.

Computational Intelligence Systems

Artificial Neural Networks

From a general point of view, ANNs can be divided into two types namely Feed-forward Neural Networks (FNN) and Recurrent Neural Networks (RNN). In FNNs the input signal only passes forward while in RNNs neurons send feedback signals to each other. The main advantage of FNNs is their ease of implementation and estimation of inputs and outputs. However, they slow down the training procedure and require a large amount of training data. On the other hand, RNNs take advantage of their internal state known as "memory" to rapidly extract the relationship between the data (White, 1996).

Probabilistic Neural Network (PNN) introduced by Specht (1990) is a kind of FNN, which performs classification tasks where the target variable is categorical.

PNN can sustainably increase the learning speed in comparison with conventional algorithms such as back propagation. The use of a PNN is especially advantageous due to its ability to converge the underlying function when only few training samples are available. Generalized Regression Neural Network (GRNN) in particular, is of the same architecture as PNN except that they perform regression where the target variable is continuous.

Adaptive Neuro-Fuzzy Inference System

Fuzzy logic first introduced by Zadeh (1965) is a solution for efficient and flexible modeling of complex systems that cannot be easily modeled using conventional mathematical techniques. The start point of developing a fuzzy model is derivation of fuzzy if-then rules. Due to their training capabilities, ANNs can form an appropriate relationship between input and output variables. On the other hand, fuzzy systems are regarded as ideal tools to deal with nonlinear problems associated with variables of approximate nature. Therefore, combination of fuzzy inference system and artificial neural networks is a brilliant idea to develop a powerful approach of modeling.

Adaptive Neuro Fuzzy Inference System (ANFIS) is such a method in which the relationship between input/output variables is established by the fuzzy portion, and the parameters of fuzzy membership functions are optimized by ANN learning algorithm (Jang, 1995).

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Fuzzy Clustering

Fuzzy clustering, which falls into unsupervised learning paradigm, is an automatic procedure during which the similar data samples are categorized a number of clusters. Similarity may be defined according to various criteria. In distance-based clustering, for instance, the samples which are adjacent to each other are considered to be similar.

Fuzzy C-Means Clustering

Like its classical algorithm, the number of clusters (c) in fuzzy c-means clustering should be specified before. The objective function is as follows:

$$J = \sum_{i=1}^c \sum_{k=1}^n u_{ik}^m d_{ik}^2 = \sum_{i=1}^c \sum_{k=1}^n u_{ik}^m \|x_k - v_i\|^2 \quad (7)$$

where x_k is k -th sample and v_i is the representative or the center of i -th cluster. u_{ik} shows the degree of dependence of i -th sample to the k -th cluster. In this equation, m is a real number greater than 1, and usually set as 2 (Stuedlein and Holtz, 2013). The sign $\|\cdot\|$ denotes a function which expresses the degree of similarity (here the distance) to the cluster center. u_{ik} generates the matrix u with c rows and n columns and the elements taking a value between 0 and 1. If all of the elements take either 0 or 1 the algorithm turns into the classical type.

Fuzzy Subtractive Clustering

The main advantage of fuzzy subtractive clustering is preventing from unnecessary and excessive growth of cluster numbers. In this algorithm, each data point (x_i) is regarded as a potential cluster center for which the density measure is calculated using the following equation:

$$D_i = \sum_{j=1}^n \exp\left(-\frac{\|x_i - x_j\|^2}{(r_a/2)^2}\right) \quad (8)$$

where r_a is a constant greater than zero called radius of influence. According to this equation, the point with more samples in their vicinity will have a greater density measure. Once the density measure is calculated for all of the samples, the point with the highest value of density measure is selected as the first cluster center. Subsequently, the density measure for other points is modified using the equation below:

$$D_i = D_i - D_{c1} \exp\left(-\frac{\|x_i - x_{c1}\|^2}{(r_b/2)^2}\right) \quad (9)$$

where r_b is a constant which is recommended to take a value equal to 1.5 (Stuedlein, 2008). Moreover, x_{c1} and D_{c1} are the first cluster center and its density measure respectively. According to this equation, the density measure of the points adjacent to the first cluster center decreases dramatically and, as a result, the probability of being selected as the next cluster center rapidly diminishes for these points. In this manner, the algorithm repeatedly finds other cluster center using the above-mentioned equation.

Information Criteria

Information Criteria (IC) have been successfully used in evaluation of data-driven models as alternative of the laborious cross validation method (Terzaghi, 1943). IC do not split the data into different portions and perform the training over the whole data set. In order to avoid excessive complexity of the model, a penalty function is defined which its value increases as the model becomes more complex. Generally, an information criterion is a function of model's parameters (n) as well as the number of training data (N). As an example consider SRC criterion:

$$SRC(n) = \ln(MSE) + n \cdot (\ln(N)/N) \quad (10)$$

where MSE is mean squared error over the whole data. If MSE is minimized as the only measure of model's performance, the problem of over-fitting will be inevitable especially when a few number of training data is available. However, application of IC such as SRC results in an optimized trade-off between accuracy and complexity (Impe *et al.*, 1997).

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Proposed Intelligent Models

Collecting Experimental Data

Stuedlein (Wang and Yen, 1999) collected results of load tests on stone columns and discussed that such data should fulfill a number of criteria in order to form a reliable database for statistical analysis. These criteria mainly include: adequate representation of soil characteristics and load testing configuration, uniformity of the soil (in failure zone), loading in a rapid manner and possibility of bearing capacity extrapolation from measured displacements. In current research, the method discussed by Studlein (Wang and Yen, 1999) is adopted and results of 29 load tests over reinforced grounds are collected in Table 1. This table presents bearing pressures measured at different settlements (2, 5, 10, 17, 25, 35 and 50 mm) as well as the ultimate bearing capacity of composite ground. The parameters represent ground characteristics and configuration of the reinforcement system (i.e. stone columns) include: undrained shear strength of surrounding soil (s_u), area replacement ratio (a_r), width of the footing (B), depth of footing embedment (d_f), diameter of stone column (d_p) and its length (L_p).

Table 1: Experimental data collected from the literature to train and evaluate proposed intelligent models

s_u (kPa)	a_r (%)	B (m)	d_f (m)	d_p (m)	L_p (m)	Bearing pressure (kPa) for specific settlements (mm)							ultim ate	reference
						2	5	10	17	25	35	50		
30	100	0.3	0.0	0.3	8	133	244	358	462	541	605	660	722	Lillis <i>et al.</i> , (2004)
30	44.4	0.4 5	0.0	0.3	8	66	119	176	227	263	292	316	396	Lillis <i>et al.</i> , (2004)
30	25	0.6	0.0	0.3	8	50	79	109	143	177	216	268	559	Lillis <i>et al.</i> , (2004)
30	16	0.7 5	0.0	0.3	8	36	70	117	173	229	288	358	482	Lillis <i>et al.</i> , (2004)
30	34.6	2.2 9	0.4 6	0.7 6	2.3 3	46	93	140	171	187	198	210	338	Stuedlein, (2008)
30	34.6	2.2 9	0.4 6	0.7 6	4.6 4	51	102	153	191	218	244	276	477	Stuedlein, (2008)
30	100	0.7 6	0.4 6	0.7 6	2.3 3	154	292	398	455	485	510	539	604	Stuedlein, (2008)
30	100	0.7 6	0.4 6	0.7 6	4.6 4	213	368	485	558	599	628	653	664	Stuedlein, (2008)
59	30.2	2.7 4	0.0	0.7 4	4.5 7	42	71	99	127	153	182	219	555	Wang and Yen (1999)
54	24.2	2.7 4	0.0	0.7 4	4.5 7	34	55	80	107	135	167	208	532	Wang and Yen (1999)
59	30.2	2.7 4	0.0	0.7 4	3.0 5	50	96	147	199	246	297	358	645	Wang and Yen (1999)

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75	30.2	2.7 4	0.0	0.7 6	4.5 7	45	94	149	196	234	272	319	624	Wang and Yen (1999)
65	30.2	2.7 4	0.0	0.7 4	4.5 7	48	95	145	194	238	282	335	615	Wang and Yen (1999)
65	100	0.7 6	0.6 1	0.7 6	3.0 5	210	348	461	552	624	694	774	1096	Wang and Yen (1999)
69	100	0.7 6	0.6 1	0.7 6	3.0 5	139	258	364	443	498	546	599	1006	Wang and Yen (1999)
67	100	0.7 6	0.6 1	0.7 6	4.5 7	239	433	601	724	808	881	957	1132	Wang and Yen (1999)
70	100	0.7 6	0.6 1	0.7 6	4.5 7	163	281	377	449	502	552	612	1202	Wang and Yen 1999)
57	95	0.7 6	0.6 1	0.7 4	3.0 5	135	245	375	506	616	721	835	1115	Wang and Yen (1999)
61	100	0.7 6	0.6 1	0.7 6	3.0 5	167	329	493	628	727	812	901	1093	Wang and Yen (1999)
63	88	0.7 6	0.6 1	0.7 1	3.0 5	173	310	463	607	721	819	916	1067	Wang and Yen (1999)
61	95	0.7 6	0.6 1	0.7 4	4.5 7	244	426	621	797	929	1038	1139	1214	Wang and Yen (1999)
53	95	0.7 6	0.6 1	0.7 4	4.5 7	206	341	481	609	709	797	887	1071	Wang and Yen (1999)
52	95	0.7 6	0.6 1	0.7 4	4.5 7	66	142	232	318	388	454	532	1106	Wang and Yen (1999)
12	46.8	1.3 7	0.0	1	5	102	146	173	183	185	185	186	189	White <i>et al.</i> , (2007)
44	40.1	0.9 1	0.6 1	0.6 1	2.9	80	141	200	249	282	307	325	399	Wissman n (1999)
12	36	1.2 5	0.0	0.8 5	14	34	51	65	78	90	101	114	177	Zadeh (1965)
12	36	1.2 5	0.0	0.8 5	14	19	36	54	70	85	99	117	252	Zadeh (1965)
12	100	0.8 5	0.0	0.8 5	14	94	130	156	179	199	218	241	378	Zadeh (1965)
100	100	0.6 1	0.0	0.6 1	3.0 5	43	116	236	390	542	694	860	1346	[34]

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Evaluating the Bearing Capacity

In this section, four intelligent models including two fuzzy models, ANFIS based on fuzzy c-means clustering ANFIS (FCM) and subtractive clustering method ANFIS (SCM), and two neural network models, generalized regression neural network (GRNN) and feed-forward neural network (FNN), are utilized to evaluate ultimate bearing capacity of spread footing on composite ground reinforced with stone columns.

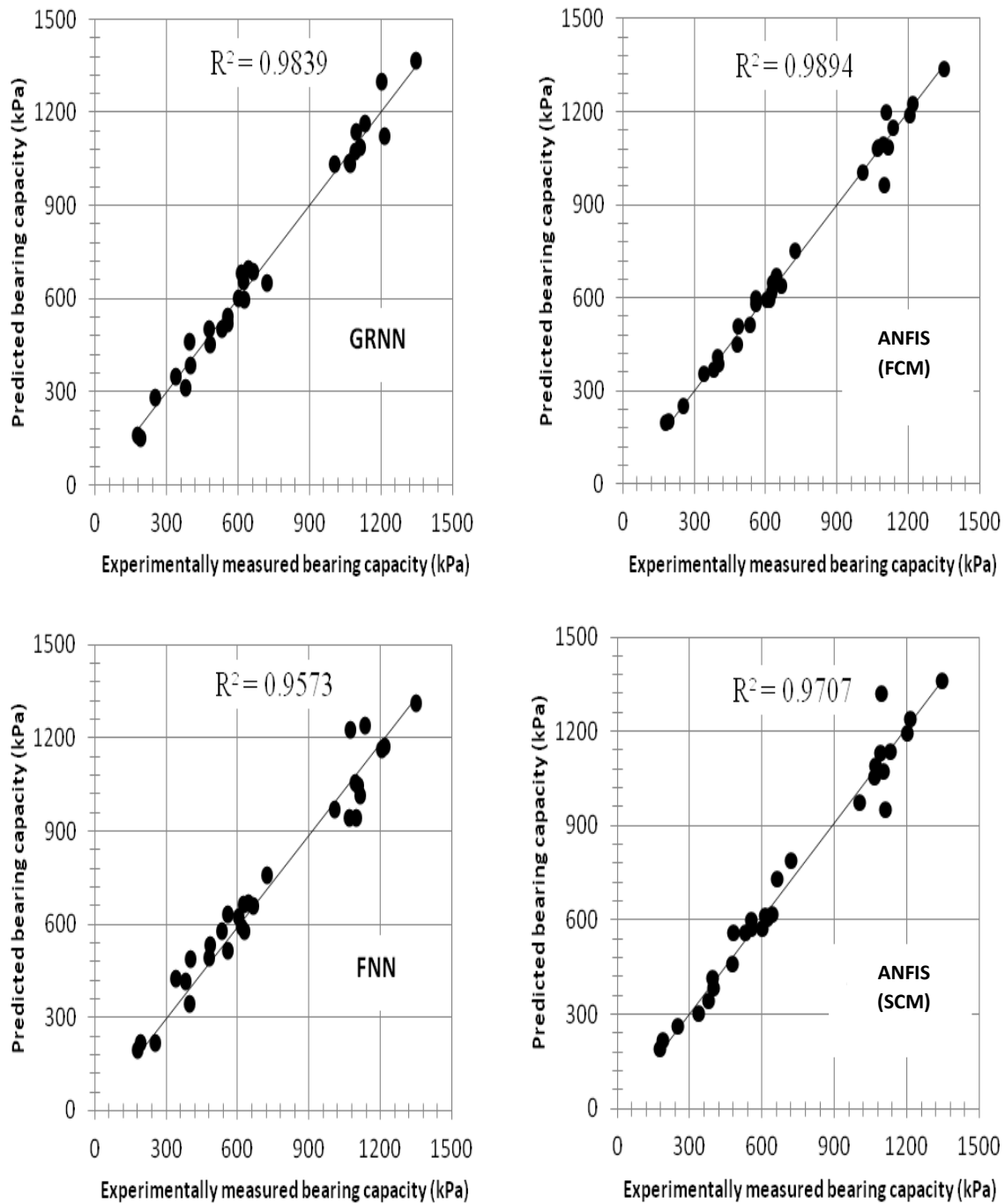


Figure 2: Experimental bearing capacity values versus predictions of proposed models

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All of the models have been developed using MATLAB software. With regard to Eq. 5 the first six columns of Table 1 are selected as input parameters for the intelligent models. Furthermore, the information criterion given by Eq. 10 is used in training and verification of the models. As stated earlier, there is no need to split the data into training and testing parts when an information criterion is applied. Predictions of the four intelligent models are illustrated versus experimental bearing capacity in Figure 2. Results of MLR model proposed by Stuedlein and Holtz (2013) are also shown in Figure 3. According to these two figures, all of the proposed intelligent models are able to precisely capture the value of ultimate bearing capacity of reinforced grounds. In addition, GRNN and ANFIS (FCM) are of higher accuracy compared with other two models.

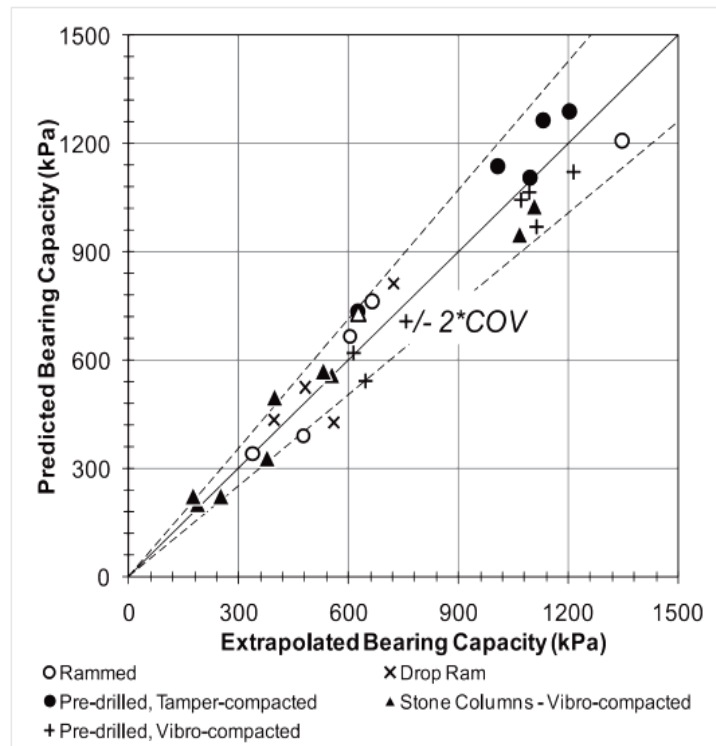


Figure 3: Comparison of experimental bearing capacity values with predictions MLR model proposed by Stuedlein and Holtz (2013)

Evaluating the Settlement

The modeling approach introduced in previous section is also used to estimate bearing pressures corresponding to specific settlements. Input parameters are those of the models for bearing capacity in addition to the corresponding settlement. As a result, each model can evaluate the bearing pressure of composite ground.

Figure 4 depicts the outputs of models in comparison to experimental data. It is evident from this figure that estimations of the intelligent models coincide well with experimental measurements. Predictions of MLR model derived by Stuedlein and Holtz (2014) are illustrated in Figure 5 against real values of bearing pressure at different settlements. Comparison of this model with those illustrated in Figure 4 demonstrates high accuracy of the proposed intelligent models.

In addition, the settlement of spread footing over composite ground is evaluated in this section using afore-mentioned models. The values of target function, i.e. the settlement, are interpolated from the data given in Table 1 in regard to a factor of safety equal to three (F.S. = 3). Figure 6 presents the outputs of proposed models against interpolated settlements. It can be seen from this figure that the proposed models can accurately evaluate the settlement.

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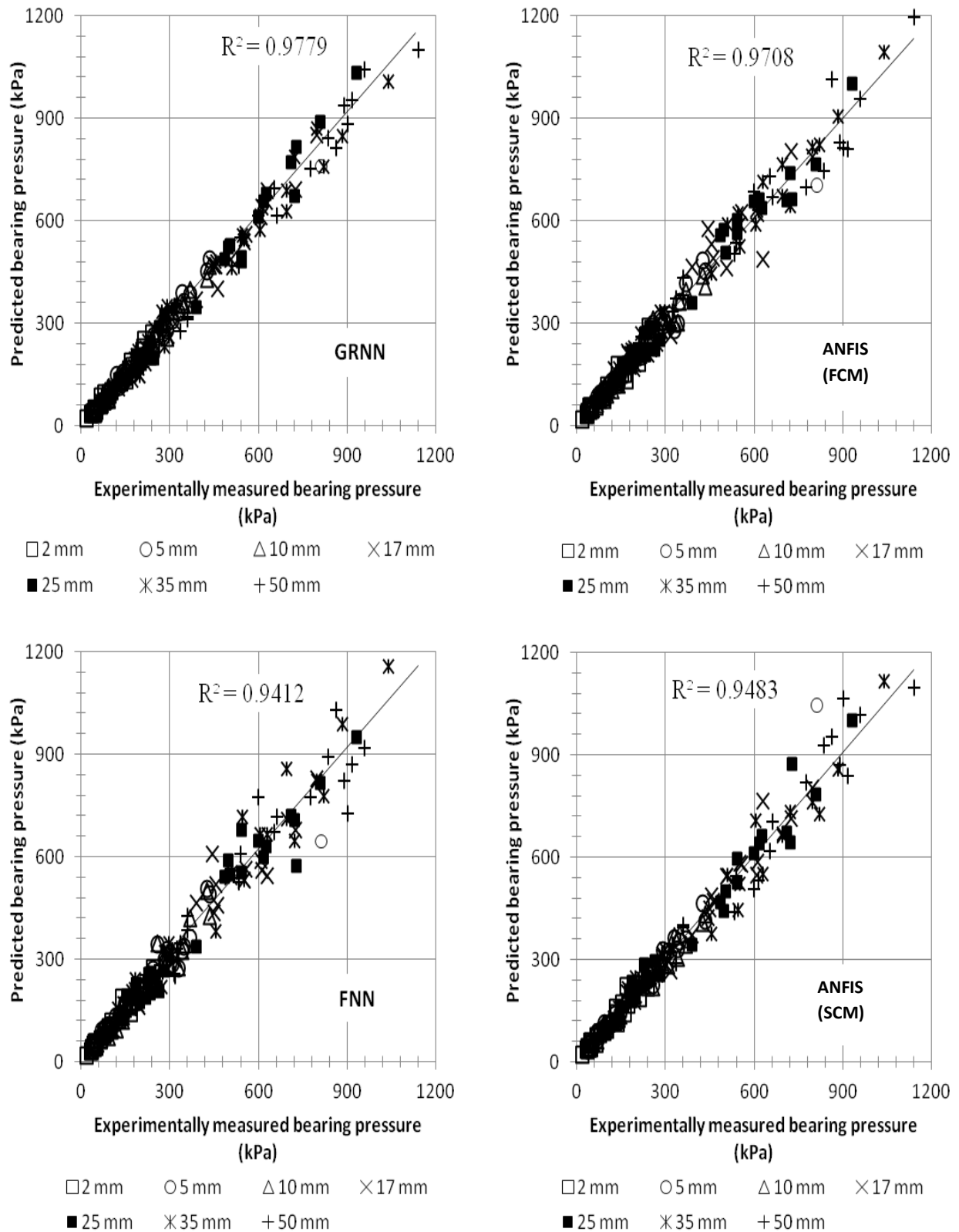


Figure 4: Experimental bearing pressures (corresponding to specific settlements) versus predictions of proposed models

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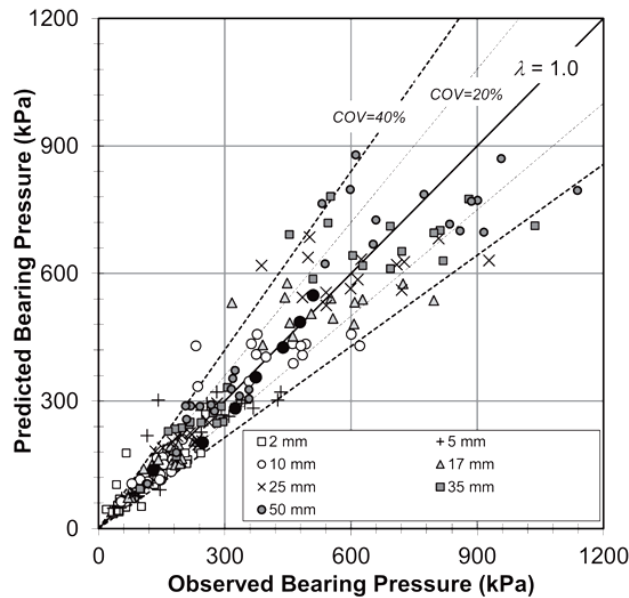


Figure 5: Comparison of experimental bearing pressures with predictions MLR model proposed by Stuedlein and Holtz (2014)

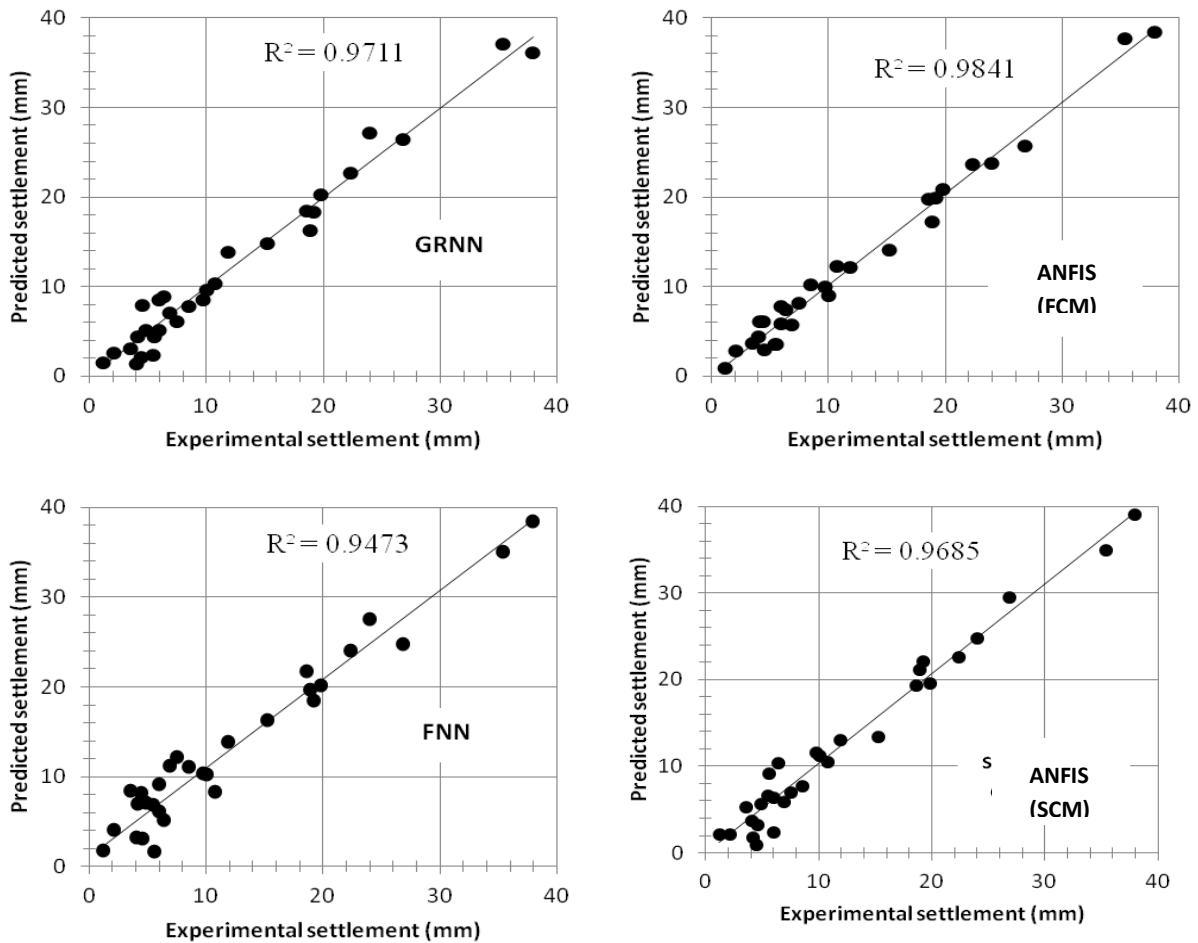


Figure 6: Interpolated settlements (with S.F=3) versus predictions of proposed models

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Concluding Remarks

In current study, four intelligent models were proposed to evaluate bearing capacity and settlement of spread footing on clay ground reinforced by stone columns. The models include two ANFIS models (with c-means and subtractive clustering algorithms) and two neural networks (feed-forward and generalized regression). Moreover, optimized design of models was achieved through an information criterion namely SRC. Use of information criteria is particularly helpful when a small number of data, in regard to the model's parameters, is available (as is the case in current study).

The data required for training and verification of the models were collected from high quality results published in the literature. Due to application of information criterion, the database should not split into separate parts. It was depicted that the four intelligent models are able to precisely estimate bearing capacity as well as bearing pressure of spread footing lying over clay ground improved by granular piles. All of the models are of considerably higher accuracy compared with recently developed MLR models. Furthermore, the models were able to precisely estimate the settlement with a safety factor equal to three. Among the proposed models, GRNN and ANFIS with c-means clustering showed the best performance in all three modeling tasks.

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