

ACCURATE ESTIMATION OF SCALE INDEPENDENT JOINT ROUGHNESS COEFFICIENT, JRC, BY OPTIMIZED BOX COUNTING ALGORITHM

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ABSTRACT

Mechanical behavior of rock mass is effectively related to the shear movements of in situ joints. Roughness is known as a parameter which strongly affects the stability of rock mass by increasing the shear strength. There are some difficulties in obtaining a good approximation to JRC such as inappropriate precision in measuring joint surface topography, time-consuming nature of this measurements and some challenges involved in the expression of roughness, e.g. visual analogy, scale effects and unaffordability. Despite several attempts to handle such challenges, still there is the lack of a practical way to obtain such a correct approximation. This paper tries to take the advantages of combining digital image processing and scale independent Box Counting fractal method by employing the most efficient edge detection techniques to propose an ideal and practical algorithm which overcome these defects. Furthermore, the performance of the proposed approach is gained by identifying the probable factors affecting the results and doing the analysis of their sensitivity on JRC values. So, by employing a single image of joint trace, the algorithm offers an accurate scale independent joint roughness coefficient in an efficient time. Finally, as a case study the technique is successfully applied to JRC estimation in Choghart iron mine.

Keywords: Joint Roughness Coefficient, Box Counting, Image Processing, Choghart Iron Mine

INTRODUCTION

The mechanical behavior of rock mass is complex due to the presence of discontinuities. These structures have a major influence on the deformational behavior of rock systems. Roughness, which influences the friction angle, the dilatancy and the peak shear strength, refers to the local departures from planarity at both small and large scales.

The development of research into roughness dates back to the 1930s. It has long been recognized that the roughness of rock discontinuities, when clean and unfilled, can have a significant impact on both the hydraulic and shear strength characteristics of discontinuous rock mass (Bryan and Grasselli, 2010; Giovanni and Egger, 2003; Mariusz, 2010). So Several criteria have been proposed in the past to identify the strength of a rough rock joints. These criteria delineate the state of stress that separates pre-sliding and post-sliding of the joint. Despite providing more complicated rough joint models such as Ladanyi's empirical (Hsiung, 1993), Amadei-Saeb's analytical (Aydan *et al.*, 1996; Jing *et al.*, 1992) and Plesha's theoretical (Kulatilake *et al.*, 1995) etc., in engineering practice, the shear strength criterion proposed by Bartonis widely adopted and used (Nicholas, 1973; Yang *et al.*, 2001). The joint roughness coefficient (JRC) presented in this model and several empirical formulas were connected the rock mass mechanical parameters e.g. the shear stiffness and joint aperture related to the JRC. This coefficient scales the joint roughness in the range from 20 (rough) to 0 (smooth) and can be determined either by tilt, push or pull tests on rock samples (Fardin *et al.*, 2001). Nevertheless in order to simplify the process of JRC measurement, Barton *et al.*, offered a technique in which, the JRC value for a given joint profile can be estimated visibly by comparing it with ten JRC profiles. This set of profiles has subsequently been adopted as a standard by the ISRM (Yang *et al.*, 2001).

However, in practice it may be difficult to determine the proper JRC due to the inductive nature.

Shortly after this, several alternate approaches have been proposed to estimate joint roughness coefficient with the aim to put down the analogy, so that recent years have witnessed a rapid development of new

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methods and variety of parameters has been studied. In this regard, addition to the empirical, one can found numerous statistical and fractal methods. As the most notable ones, Grasselli *et al.*, (2002) was introduced a quantitative three-dimensional description of a rough surface (Grasselli *et al.*, 2002). Another interesting investigation has been performed by Grasselli *et al.*, (2010), who provided a new 2D discontinuity roughness parameter and its correlation with JRC (Bryan and Grasselli, 2010). Tse and Cruden's (1979) introduced an empirical statistical relationship between the JRC and the root mean square (r.m.s) of the tangents to the slope angles along the profile to calculate typical JRC values (Tse and Cruden, 1979). The objective of the research presented by Rasouli *et al.*, (2010) was the investigation of a good relationship between the roughness parameter D_{R1} and rough joint shear strength by the analysis of unit normal vectors in terms of directional statistics in Riemannian geometry (Vamegh and Harrison, 2010). The purpose of these all studies was to remove some of the problems that existed in the ISRM standard technique. Indeed the presence of large-scale irregularities in the joint surfaces known as waviness, resulting in joint roughness coefficient to be dependent on the measurement scale, where by the effect of roughness is seen to reduce as the scale of sampling increases, and periodicity, where a longer wavelength variation is superimposed on short-wavelength roughness (Fardin *et al.*, 2001; Barton, 1971; Fecker and Rengers, 1971; Rasouli and Harrison, 2000; Brady, 2004; Nader *et al.*, 2004). [14] As well, detailed laboratory investigations confirmed JRC scale dependency (Fardin *et al.*, 2001). The other key problem is to work out methods to adequately interpret such large packets of data (Mariusz, 2010). Nowadays, although the presence of several precise methods for mapping the joint profiles and surfaces, none of them can directly estimate a proper value for the roughness of surfaces (Mariusz, 2010; Fardin *et al.*, 2001; Grasselli *et al.*, 2002). Also, the limitation of traditional JRC and the conventional statistical parameters in joint roughness quantification have been reported (Kulatilake *et al.*, 1995; Fardin *et al.*, 2001; Vamegh and Harrison, 2010). Therefore, the rock joint roughness needs to be characterized using a scale invariant parameter, taking into account measurement accuracy through a fully applicable way. This study has the aim to develop such an accurate method without any computational complexities.

Fractal Analysis of Roughness

Fractal theory is a technique that can remove “Barton analogy” in addition to JRC scale dependency. It is clear that rock joints and broken rock surfaces like several natural objects are commonly rough, fragmented, and composed of mountains and valleys. Fractal geometry introduced by Mandelbrot (1983), gives a new approach to describe geometrical irregular shapes and provides a general framework for the study of such irregular sets (Issa *et al.*, 2003; Yujing *et al.*, 2006). Fractal quantifications are related to various types of patterns on a wide range of scales (Mark and Kruhl, 2009) and can disclosure the essential relations or rules between local and global structures (Yujing *et al.*, 2006).

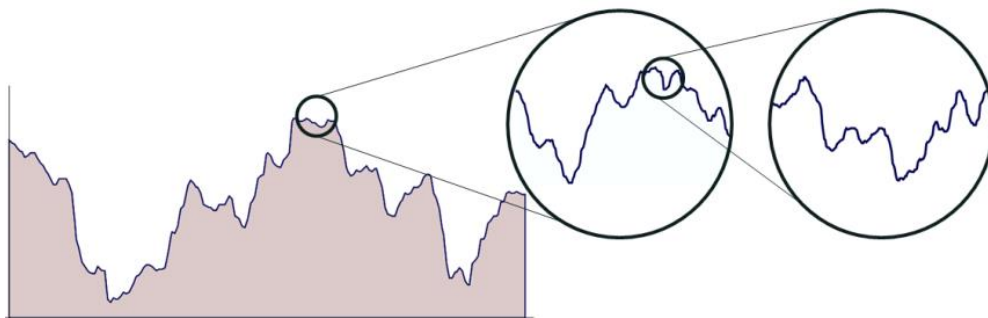


Figure 1: Schematic exaggerated model of self-similarity in the rock joint surfaces (Yuan *et al.*, 2003)

So the attraction of fractal models lies in the abilities to predict scaling behavior (Fardin *et al.*, 2001). Fractal geometry interacts with the shapes having quasi-infinite details and statistical self-similar characteristics (Dianliang *et al.*, 2009). Numerous researchers have applied the concepts of fractals to rock joints (Yang *et al.*, 2001). In recent years, because of the preliminary efforts of Mandelbrot (1983,

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1967), there have been a number of studies investigating the applicability of fractal models to characterize roughness of joint surfaces (Fardin *et al.*, 2001), and it has been repeatedly shown that geo-materials, like many other natural as well as artificial ones, show fractal, i.e. statistically self-similar, patterns which can be quantified and studied by fractal techniques most effectively (Figure 1) (Mark and Kruhl, 2009; Feder, 1988; Mandelbrot, 1977; Kaye, 2008; Turcotte, 1989; Yuan *et al.*, 2003).

Based on Mandelbort, Self-similarity and fractal dimension are the most important features of fractal patterns. So in fractals, a scale invariant coefficient called the fractal dimension (D) can be estimated. About rock joint surfaces, there are several studies in literatures that have demonstrated the constant fractal dimension and subsequently, the applicability of fractal geometry for roughness estimation (Wu, 2000; Lee *et al.*, 1990; Zhao, 1998; Hsiung *et al.*, 1995; Muralha, 1995; Odling, 1994; Carr and Warriner, 1987). In this regard as one of the most basic relationships, the equation (1) has been offered by Lee *et al.*, (1990):

$$JRC = -0.8780 + 37.78 \times \left(\frac{D-1}{0.015} \right) - 16.9304 \times \left(\frac{D-1}{0.015} \right)^2 \quad (1)$$

In this equation, JRC is the joint roughness coefficient and D is the fractal dimension. Apart from this, many other new equations presented in the same act.

The estimation of fractal dimension can be done by different techniques. Sofar, several methods have been suggested to estimate this dimension of rock joints. The divider, box counting (Charkaluk *et al.*, 1998), variogram (Orey, 1970), spectral (Berry and Lewis, 1980), roughness-length (Malinverno, 1990) and line scaling (Mitsugu and Ouchi, 1989) is some of these methods. By comparing these, however, it can be found that even for the same rough profiles or surfaces, controversial and anomalous estimations were made by different researchers or by employing different techniques and scale parameters (Heping *et al.*, 1997; Yujing *et al.*, 2006). Furthermore, in most cases, it can be seen that different fractal roughness in being reported from a similar profile. These contradictions originate from the lack in presence of standard method to measure fractal roughness taking in to account the parameters affecting these calculation methods. Recently, based on some comprehensive studies, Jimenez *et al.*, (2012), asserted the need of focusing on different techniques to measure fractal dimension and to introduce the relationship equations in consistent with these techniques (Jimenez and Miras, 2012). Nevertheless, despite the presentation of numerous relationships, there are no studies discussing the methods and those standards features in estimation of fractal dimension especially in JRC calculation. In this study, first the Box Counting algorithm is used as a universal technique in combination with image processing, to measure the fractal dimension accurately. Also, with identification of some basic parameters affecting the results, some standard features are introduced and the standard scale independent and operative technique for measuring the joint roughness coefficient is presented.

Algorithm Development

Based on Hirata (1989), Kruhl and Nega (1996), Velde (1999), Gonzato (2000), Bonnet (2001), Kruhl (2004) and many other researchers, “Box Counting” is one of the most widely used methods providing fractal dimension (Mark and Kruhl, 2009), although, in our opinion, that it might be better to dedicate the “box-counting dimension (D_b)”. In order to estimate the box-counting dimension, in a cyclic process the Euclidean space containing the pattern is divided in to a grid of boxes of size $\eta(i)$ and those boxes $N_\eta(i)$ are counted which contain at least one point (pixel) of the pattern (Takayasu, 1990). The box sizes are then progressively reduced and the corresponding number of non-empty boxes is counted again. $N_\eta(i)$ is plotted vs. $\frac{1}{\eta(i)}$ in a double-logarithmic diagram. If the pattern is being self-similar, the data points show a linear distribution within a certain interval. This proved such a pattern to be self-similar in that range. The slope of the resulting regression line is equal to D_b (Mark and Kruhl, 2009; Feder, 1988; Mandelbrot, 1977), so for 2D fractal profile, the regression relation can be written as:

$$N_\eta(i) = k \times \left(\frac{1}{\eta(i)} \right)^{D_b} \quad (2)$$

Research Article

For the implementation procedure of this technique in relation to rock joints, 2D joint roughness profiles should be converted into digital data. To develop a practical, simple and fast determination, an algorithm was written in combination with digital image processing. So, the roughness topography measurements are driven by taking a digital joint cross picture, and quantification are done by using Box Counting method. Figure (2) shows the flowchart designed in this study.

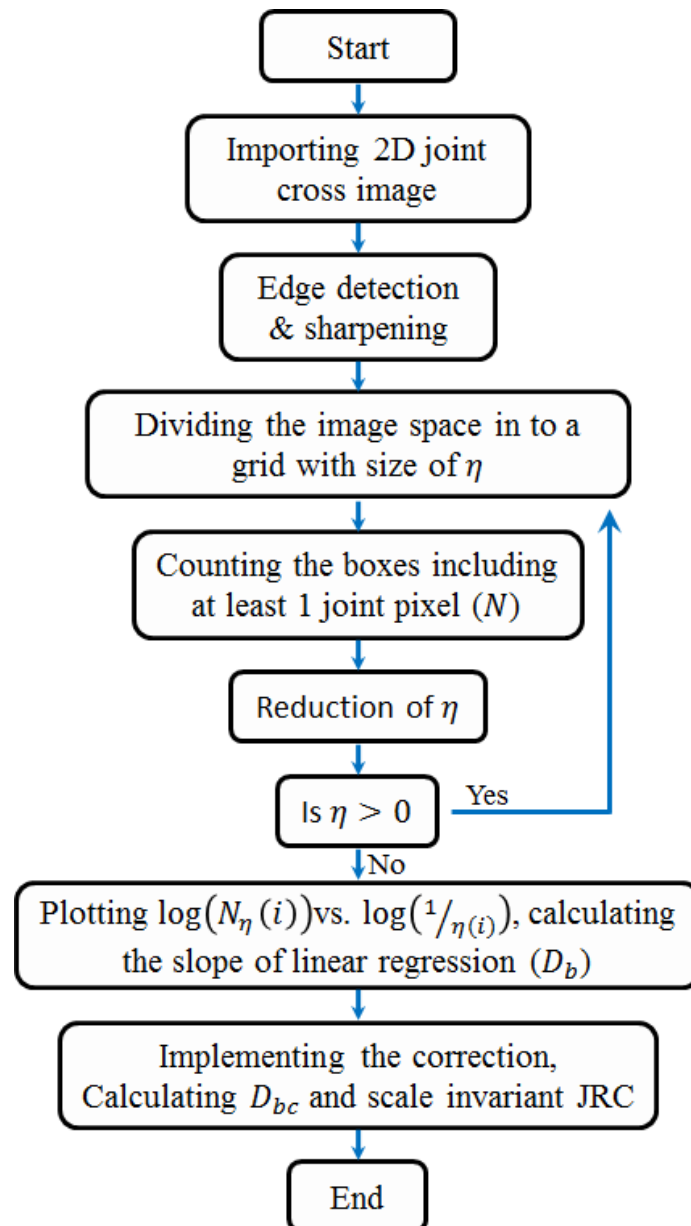


Figure 2: Designed flowchart to calculate scale invariant JRC

As the need to provide a standard technique, and according to presented flowchart, a picture is taken from an arbitrary cross section of joint outcrop in resolution of 1000×1000 pixels. This picture forms the inputs of flowchart. Furthermore, the output images of some other software may be imported too. Using the RGB color system, image pixels dispart to the constituent percentage of red, green and blue and three dimensional matrices are formed with the size of $1000 \times 1000 \times 3$. The other subsequent calculations can be driven by providing this data. A schematic of this process is shown in Figure (3).



Figure 3: Schematics of joint image pixels disparting to the Red, Green, Blue percentage

In the present algorithm, there is the need of joint trace pixels to be sharpened with eliminating the others. So, the operator of color vectors gradient, as one of the basic edge detection algorithm, was proposed and its ability about joint traces was studied. This is a local operator calculating the maximum vector distance among the central and 8 other neighboring pixels. Euclidian distance model of this operator can be written as:

$$E_{VG} = \text{Max}_{i=1:8} \{ |\vec{V}_i(R, G, B) - \vec{V}_0(R, G, B)| \} \quad (3)$$

E_{VG} is the Euclidian distance of color vectors, $\vec{V}_i(x, y)$ is the color vector of neighboring pixels and $\vec{V}_0(x, y)$ is the color vector of central pixel. Furthermore the equation of angular distance model of this operator is:

$$E_{VG} = \text{Max}_{i=1:8} \left[\sqrt{1 - \left(\frac{\vec{V}_i^T(R, G, B) \cdot \vec{V}_0(R, G, B)}{|\vec{V}_i(R, G, B)| \cdot |\vec{V}_0(R, G, B)|} \right)^2} \right] \quad (4)$$

In operating the edge detection process by vectors gradient methods, the entire pixels are being scanned and E_{VG} is being calculated for them (Figure 4). By this, the image background is being faded and the algorithm can detect the joint trace accurately. Figure (5) shows the implementation results of these operators in combination with some other additional smoothing techniques on a joint profile cross images.

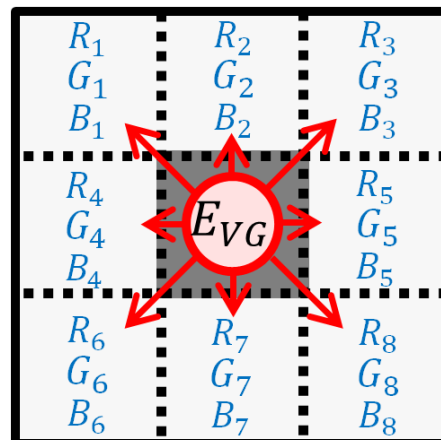


Figure 4: The pattern of gradient vector from adjacent pixels

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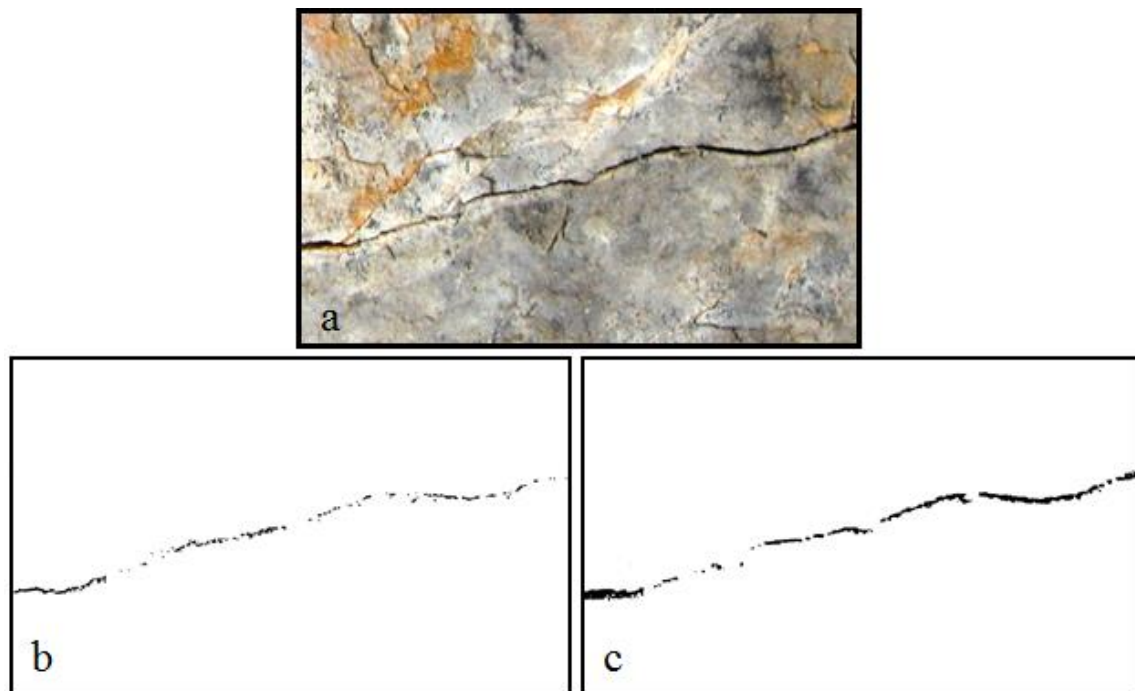


Figure 5: Detection of joint roughness trace by vector gradient operators. a: original image b: Euclidian distance operator c: angular distance operator

Despite a slight difference between the two operators presented in this study, vector distance was selected as the effective one and was employed in the study algorithm.

Now, by detecting the joint roughness traces, the Box Counting technique can be implemented. This process is shown in figure (6). Also, a linear fit to the purpose data in this method is presented in figure (7).

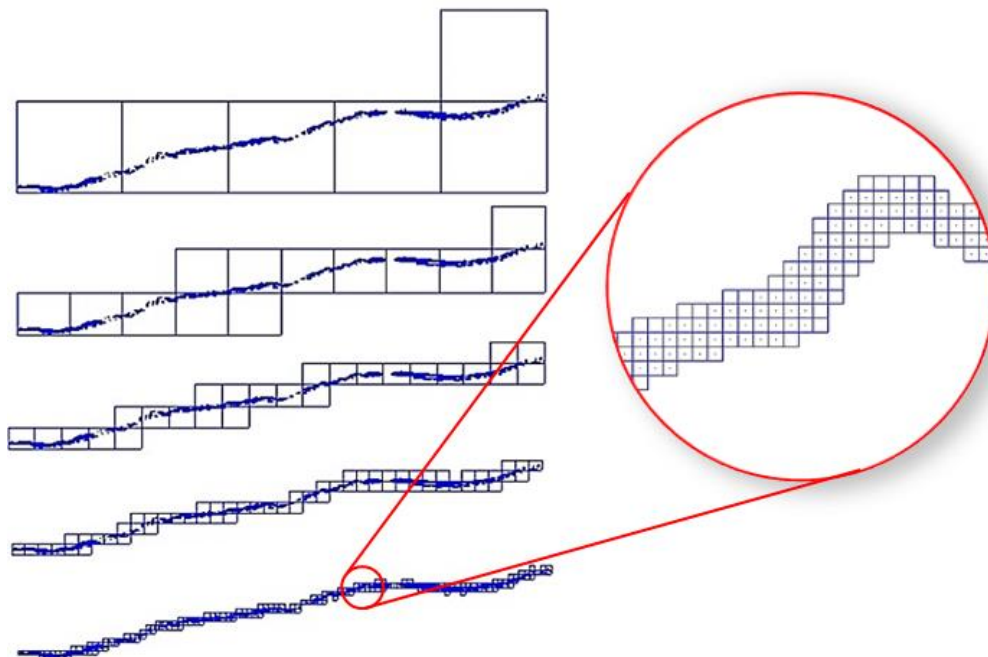


Figure 6: Implementation of Box Counting to the detected joint trace in the proposed algorithm

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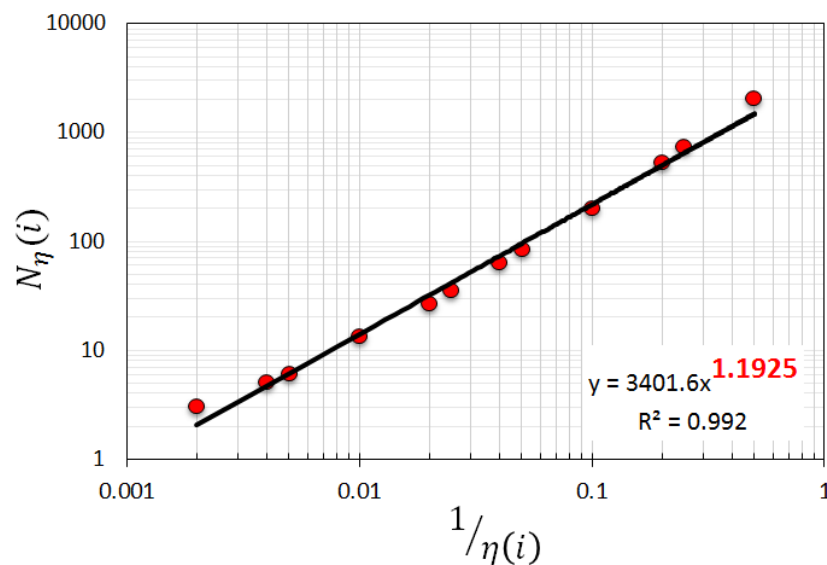


Figure 7: Linear fitting to the $N_{\eta}(i)$ vs $1/\eta(i)$ data in box counting method. $D_b = 1.1925$

From the figure (7), it is shown that an accurate Box Counting fractal dimension can be estimated. However, the resulting D_b can be affected from some of the critical image parameters and should be corrected. Due to this necessity, some of these parameters were detected based on the authors' experiences and their correction factors are being discussed. With this, a standard technique can be introduced and the accurate JRC can be reported independent to the scale of measurements.

Analysis of Parameters Affecting the Results

As mentioned in the previous section, by applying this method for numerous estimations (at least 500 JRC estimation), it was found that some of uncompromised characteristics of the input image are very impressive on the result dimensions. As the most important, joint traces length, position and orientation can be considered. Without this, the results will be unrealistic and non-applicable for JRC estimations.

Joint Trace Length

Based on fractal theory, it is expected that the measurement scale doesn't influence on the results of fractal roughness.

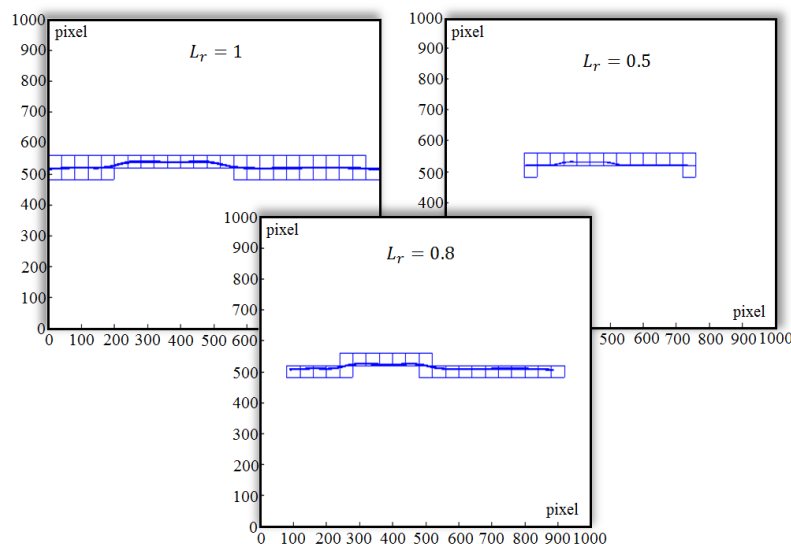


Figure 8: Schematics of the sensivity analysis on the joint trace length ratio (L_r)

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We found that this is true just in the case in which the width of the input image be occupied by the joint profile. However, any reason (large scales, the presence of non-persistence joint, inaccessibility to take a picture, etc.) that makes the picture un-covered by the joint trace causes the results to be scale dependent. So here, some sensitivity analyses are conducted with respect to this parameters with the aim to offering the corresponding correction factors.

For this, other influential parameters were kept constant and by changing the profiles scale, the situations in which the joint trace length ratio (L_r) lies in the range of $0.1 \times W$ to W being considered where W is the input picture width.

The process schematic of this sensitivity analysis is presented in figure (8) for several values of trace length in a single trace.

Figure (9) shows the sensitivity of the algorithm to this parameter that can be introduced by the logarithmic interpolation having the correlation coefficient of 0.98.

From Mandelbort (1967), it is shown that the fractal dimension of a 2D profile is in the range of 1 to 2 (Yujing *et al.*, 2006). In fact for the straight line, the fractal dimension is equal to the Euclidean dimension that is exactly 1.

We use this issue to calibrate this developing algorithm with regard to L_r and reporting $L_r = 0.5$ as a standard trace length ratio.

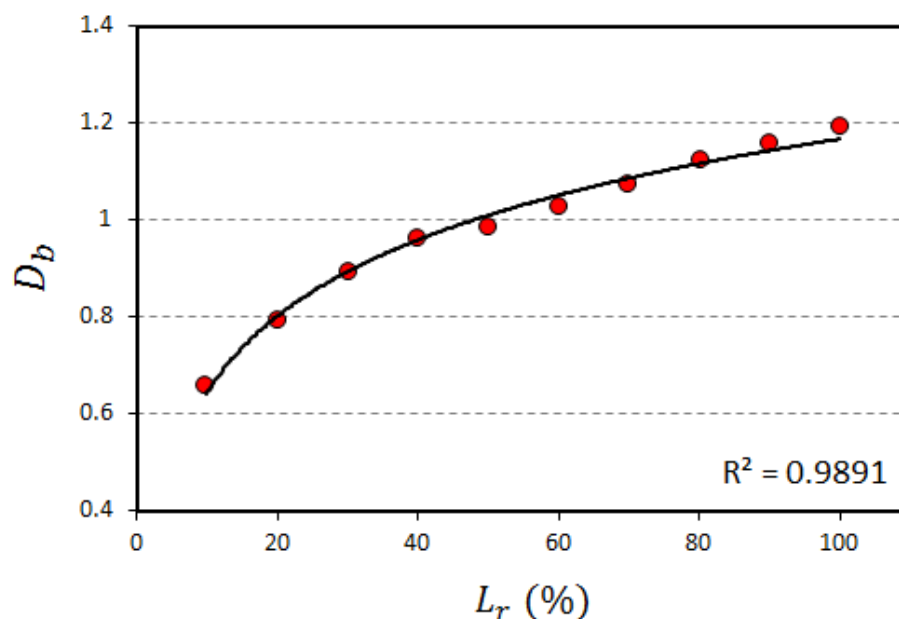


Figure 9: Sensivity of L_r on the D_b values for a stright line

Joint Trace Position

From the numerous measurements conducted on the natural joints, trace position in the joint cross picture, specified as the second basic parameters affecting the JRC results. In fact these traces can be placed in anywhere in the taking picture.

This causes the Box Counting fractal dimensions to get different unrealistic values. So keeping $L_r = 0.5$, the sensitivity analysis was developed about joint trace position.

This was done by taking a straight line horizontally in different lateral distance (LD) from the central horizontal line.

Due to the lateral symmetric condition in the picture box, the ratio of $LD_r = LD/W$ is being changed only in the range of 0 to 0.4 from the central line toward the top. The results of Box counting dimension with respect to the variability of L_d are present in Figure (10). A logarithmic interpolation can represent the changes with correlation coefficient equals to 0.9965.

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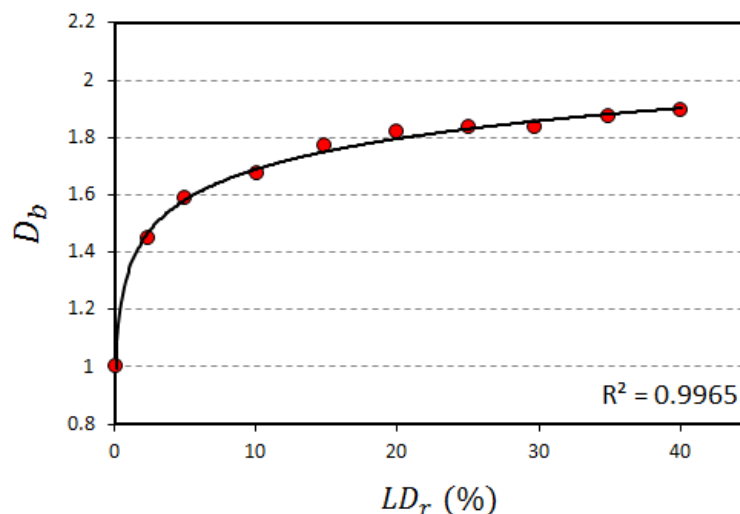


Figure 10: Sensivity of LD_r on the D_b value for a stright line

Joint Trace Orientation

As the natural joint surfaces orient in variety of range, they can have different angles relative to the horizontal axis. Various algorithms for calculating the fractal roughness ignore this issue while there is one the critical parameter influencing the Box Counting results. Here, joint trace orientation is being changed in the range of 0 to 90 degrees in relation to the horizontal central image axes and the calculations are being conducted. In Figure (11), the sensivity results of Box Counting dimension (D_b) to the trace orientation (θ_r) is presented. These changes can be approximated by multi-linear relationship in different range of rotations.

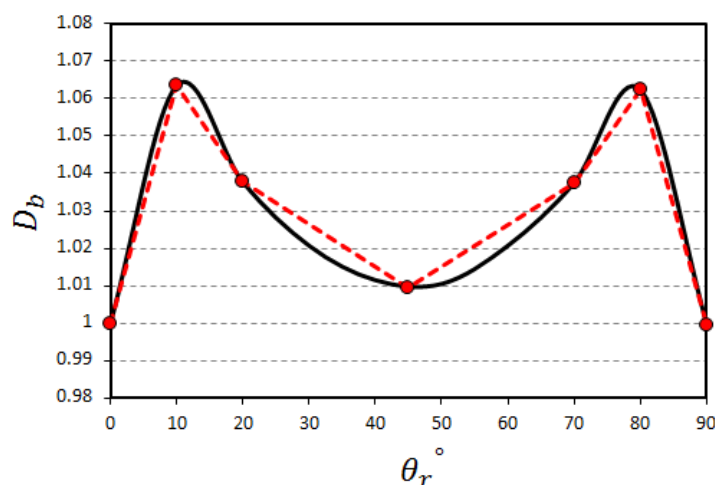


Figure 11: Sensivity of θ_r on the D_b value for a stright line

RESULTS AND DISCUSSION

In the present study, the most important features affecting the values of Box Counting fractal dimension was studied. As was observed, these parameters can have a significant influence on the results. About the joint trace length, increasing causes the fractal dimension to be increased in a semi-logarithmic pattern. The correction for this parameter can be explained according to Equation (5).

$$D_{bc}^L = D_b - 0.2287 \times \ln(100 \times L_r) + 1.053 \quad (5)$$

Where D_{bc}^L is the corrected Box Counting fractal dimension with respect to joint trace length, D_b is the measured Box counting fractal dimension and L_r is the ratio of the trace length to the picture width.

Research Article

Furthermore, deviation of joint profile location perpendicular about the central horizontal line of the taken picture, leading in increasing D_b in a semi-logarithmic trend as the previous. With a logarithmic regression fitting, the correction equation can be written as:

$$D_{bc}^{LD} = D_b - 0.1518 \times \ln(100 \times LD_r) \quad (6)$$

In which D_{bc}^{LD} is the corrected Box Counting fractal dimension with respect to lateral distance from picture central horizontal line, and LD_r is the ratio of this distance to the picture width. About the joint profile rotations, more complex sensitivity trend was recognized. In fact, in the sectors of 45 degrees, the parts of change are symmetric to each other. So in order to provide the correction relationships, the approximations can be done by fitting some multi-linear trends, which is present in equation (7).

$$D_{bc}^r = \begin{cases} 0.00635 \times \theta_r + 1 \rightarrow 0^\circ \leq \theta_r < 10^\circ \\ -0.00258 \times \theta_r + 1.0893 \rightarrow 10^\circ \leq \theta_r < 20^\circ \\ -0.001128 \times \theta_r + 1.06026 \rightarrow 20^\circ \leq \theta_r \leq 45^\circ \end{cases} \quad (7)$$

D_{bc}^r is the corrected Box Counting fractal dimension with respect to the rotation angle. Due to the symmetrical nature, we introduce θ_r as the minimum angle between the joint trace and the horizontal or vertical central image axes. So θ_r can be changed in the range of 0 to 45 degrees.

These multiple relationships can be mentioned as the most necessary and critical corrections that should be applied to the outputs of the proposed image processing based box counting algorithm introduced in this paper. For this, the standard features (no correction situation) of the box counting algorithm were defined according to the table (1).

Table 1: The standard critical features of image processing based box counting algorithm

Picture size (pixels)	L_r	LD_r	θ_r
1000 × 1000	0.5	0	0

The algorithm developed in this study, has the ability of searching these affecting parameters automatically, and hence using the above equations, presenting the corrected results for box counting fractal dimension in non-standard conditions. By providing the standard algorithm, accurate estimations of fractal dimension will be possible for any 2D fractal structures. Now, for accurate calculation of JRC, it is necessary for this optimized algorithm to be implemented on the standard JRC classes introduced by Barton.

The results of such a procedure are reported in table (2). A linear regression can be used for the approximation of the relation between these two standard values (Eq 8).

$$JRC = 383.87 \times D_{bc} - 429.69 \quad (8)$$

Here, JRC is the joint roughness coefficient and D_{bc} is the corrected final value of Box Counting fractal dimension. This linear fitting is shown in figure (12).

Table 2: The standard box counting fractal dimension of standard roughness profiles

The standard profile number	JRC average	D_{bc}
1	1	1.1183
2	3	1.1328
3	5	1.1367
4	7	1.1423
5	9	1.1358
6	11	1.1457
7	13	1.1577
8	15	1.1580
9	17	1.1593
10	19	1.1674

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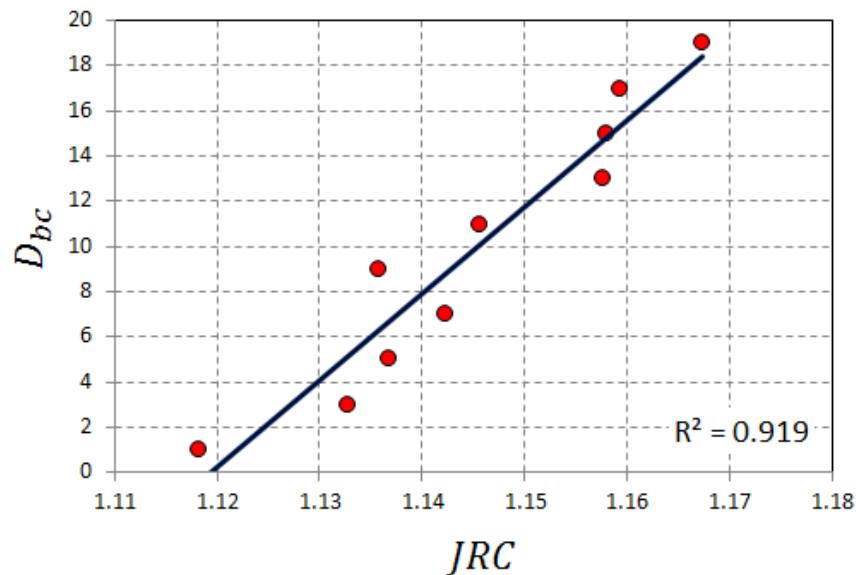


Figure 12: Linear regression between D_{bc} and JRC

JRC Estimation in Choghart Iron Ore

Choghart is known as one of the largest and oldest iron ores in the central Iran zone. This project is located at 132 kilometers South East of Yazd city in the Yazd province, Iran (Figure 13). Ore conditions in this region have led to development of the pit cavity having more than 500 meters depth. So the stability of the existing wall has become a critical issue. In order to provide require data for analysis of the stability in the northern walls located in this cavity, some of the roughness coefficient were calculated by using the developed algorithm to have a reliable results.

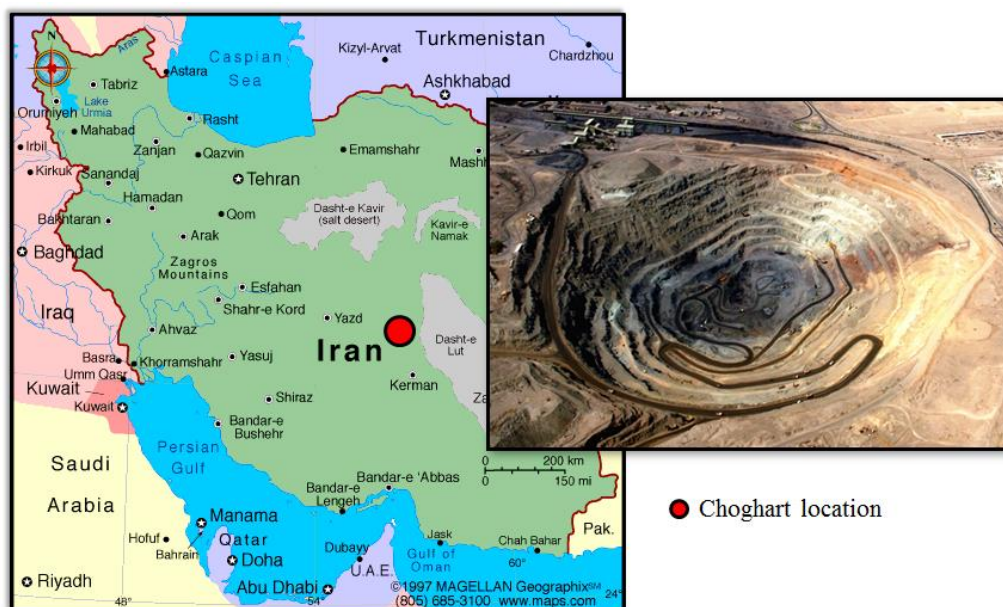


Figure 13: Choghart iron ore location and mining pit

For this purpose, the photography procedures were conducted in three parts of North West, North and North East scan lines. For this, the images were taken for any individual joints intersecting the scan lines; among them total number of 497 ideal images was selected for JRC analysis (figure 14). The selected

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traces were analyzed using the presented algorithm and the required corrections including trace length ratio, lateral distance ratio and the trace rotation were performed. By these the JRC were calculated for them in an accurate way. Figure (15-17) shows the histograms of calculated JRC values and the best fitting distributions for three measurements scan lines. Furthermore the detail descriptions of these histograms are present in table (3).

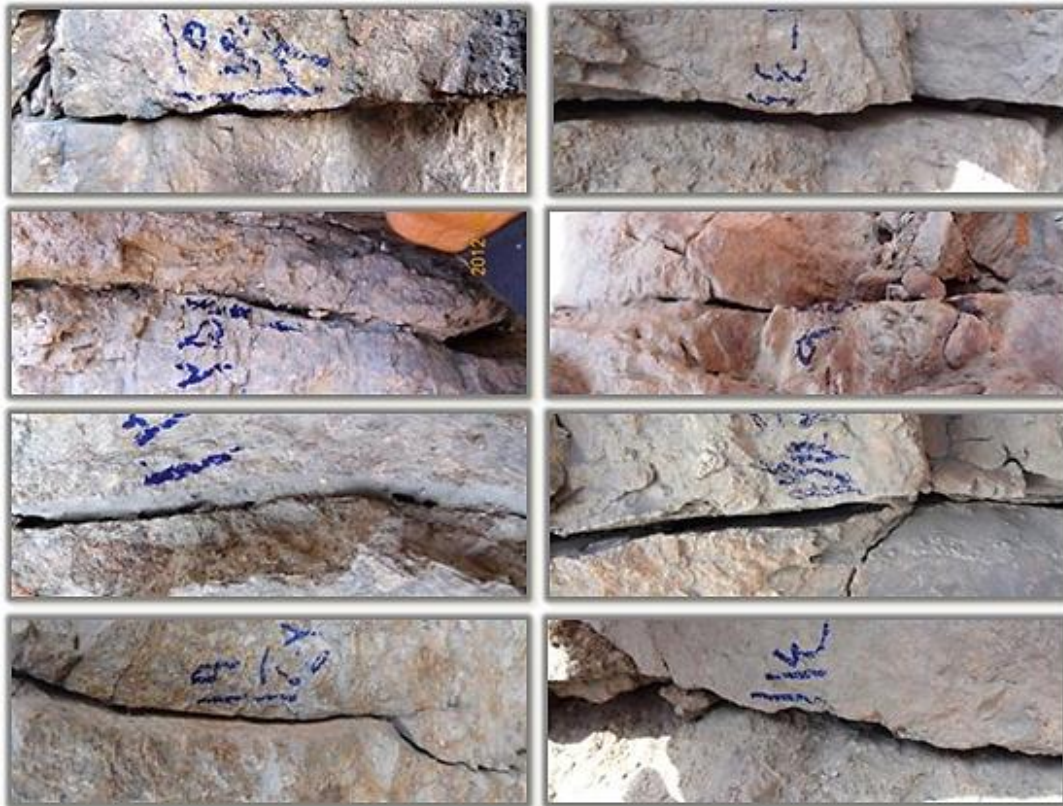


Figure 14: Some cropped of the selected joint traces for JRC analysis in Choghart northern walls

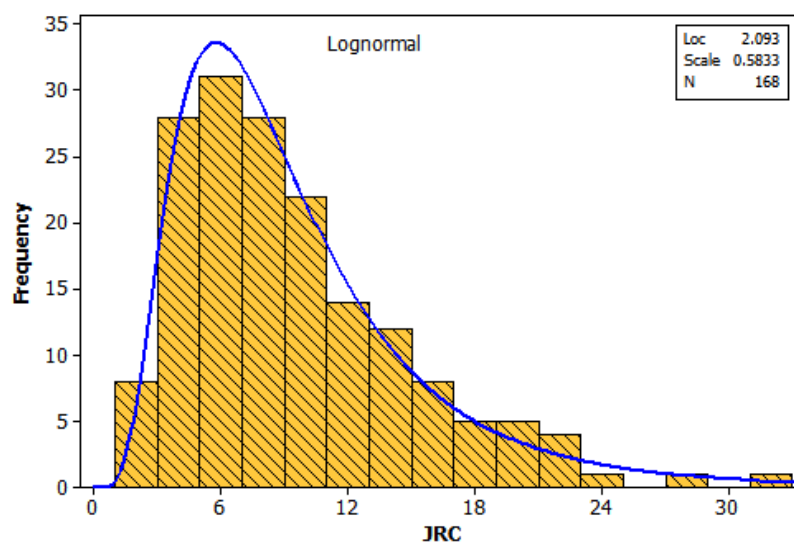


Figure 15: Histogram and the best fitting distribution of JRC in North-West walls of Choghart mine

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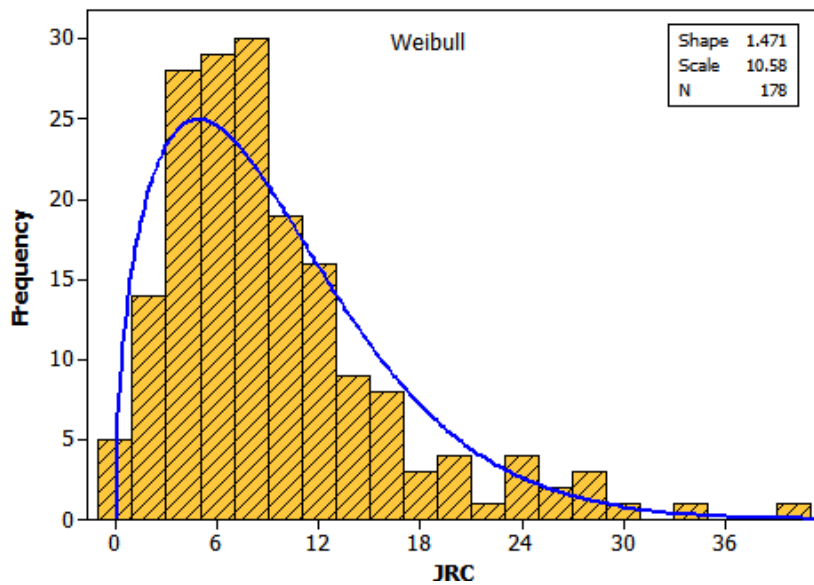


Figure 16: Histogram and the best fitting distribution of JRC in North walls of Choghart mine

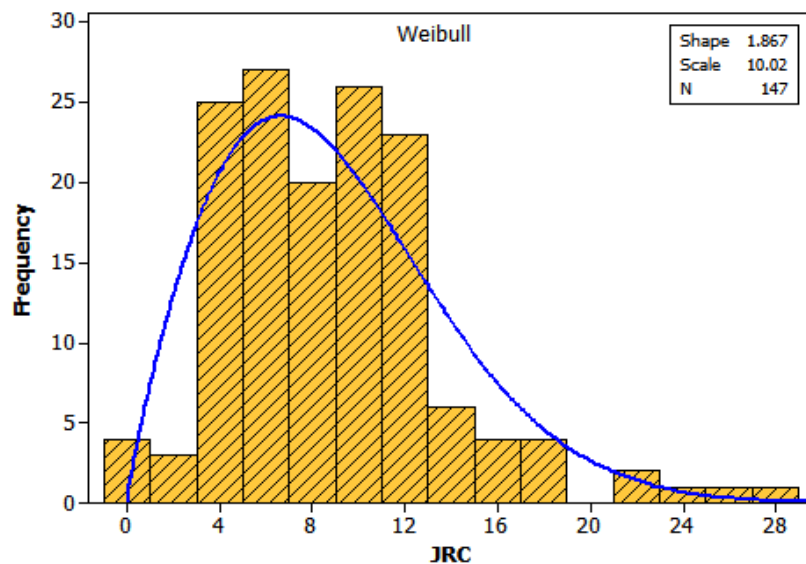


Figure 17: Histogram and the best fitting distribution of JRC in North-East walls of Choghart mine

Table 3: The statistical parameters of the JRC variability in Choghart mine Northern walls

Wall location	Best fitting distribution	JRC Distribution parameters				
		Lognormal			Weibul	
		Mean	Standard deviation	Mod	Scale (λ)	Shape (k)
North	Lognormal	9.54	11.26	5.73	10.78	1.863
North-East	Weibul	10.33	14.75	3.521	10.58	1.471
North-West	weibul	9.30	11.69	4.63	1.02	1.867

Conclusion

The present study aims to provide a quantitative, accurate and scale independent practical method for estimation of joint roughness coefficient without any computational complexity. For this, by introducing

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the fractal theory, the box counting algorithm is being developed in combination with joint digital image processing. Based on the results, in edge detection stage of the proposed algorithm, deployment of the angular distance operator suitably enhanced detection process. By detecting some critical parameters affecting the results, the standard features of the algorithm together with their correction equations were provided. These features introduced in table 1. In fact, using this algorithm, some raws fractal dimensions are produced that are needed to be corrected by consecutive employments of Eq (5 - 7). Final corrected fractal results can be participated in the joint roughness coefficient estimations by using equation (8). In this study, the algorithm was successfully used in JRC analysis of northern walls in Choghart iron mine in Iran as a case study.

Despite the significant changes that occurred in sensitivity analysis, it is certain that there are no studies have addressed these issues. So this standardization guarantees the equality and consonance of the fractal dimension reporting and consequently the correctness of the JRC results in different image-based measurements. Moreover this coefficient will not be affected from the measurements scale and the result accuracy will be fully reached due to the Barton analogy elimination. It may be argued that the two-dimensional algorithms cannot express JRC comprehensively. Although in the last decade increasing attention has been given to 3D characterization of fracture surface roughness and its link to the behavior of rock discontinuities, the characterization of linear profiles remains important for applications such as empirical predictions of shear strength (Rasouli and Harrison, 2000). As noted by recent publications by the authors, the majority of discontinuity roughness evaluations to date have been based on the analysis of 2D profiles rather than the 3D surface topography (Tatone, 2009; Bryan and Grasselli, 2009; Bryan and Grasselli, 2010). Finally, it is suggested for this optimized algorithm to be used in JRC estimation because of the simplicity, the speed of implementation, low measurement cost, the accuracy and no scale dependency.

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